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Living Standards Analysis Model: The First Prototype

Anita King

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New Zealand Government

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Living Standards Analysis Model: The First Prototype

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AUTHORS
Anita King
Senior Analyst
New Zealand Treasury
No. 1 The Terrace
Wellington
New Zealand
Email: anita.king@treasury.govt.nz
Telephone: ++64 +4 917 6138

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NZ TREASURY
New Zealand Treasury
PO Box 3724
Wellington 6008
NEW ZEALAND
Email: information@treasury.govt.nz
Telephone: +64 4 472 2733
Website: www.treasury.govt.nz

Abstract

How do we understand the synergies and trade-offs of a given policy on area beyond that policy, such as the effect of housing on health and on income? How do we choose between policies in completely different areas, such as an education policy and a health policy? Treasury's Living Standards Framework provides one possible starting point, but it provides little assistance tracing the many dependencies between policy areas. A model that includes those dependencies could help.

The Living Standards Analysis Model (LSAM) is designed to do this. A first prototype of the model has just been developed. This model includes all eleven aspects of wellbeing as described by the OECD's How's Life? framework and linkages between the different aspects for a small open economy. Most models for studying wellbeing only include one or two aspects, missing the rich set of interactions that can occur with greater coverage. As an early prototype, this version of the model does have many flaws and requires significant further development, but it forms a basis for creating an improved model as well as providing some qualitatively useful results. The model is loosely based on a stocks-and-flows type of model, with a small general equilibrium model covering the market economy part of the model. This paper is focussed on the description of the model.

JEL Classification: C650, D580, H500, I310, Q210, Q310

Keywords: wellbeing, wellbeing model, living standards, sustainability, stock and flow, CGE, health, education, environment, housing, civic engagement, jobs, income, wealth, life satisfaction, work-life balance, social connection, safety, Better Life Index

Executive Summary

This paper presents the first prototype of a model of wellbeing. This model is designed to support policy development and decisions in New Zealand by providing a tool for thinking about changes in aspects of wellbeing including trade-offs and synergies for policies. It does this by modelling the linkages between the eleven aspects of wellbeing used by the OECD in their How's Life? framework, in combination with a small general equilibrium model and stock-and-flow equations. A model of this nature is a new addition to the policy modelling toolkit.

It is clear that this model, perhaps more than most, has many flaws and unrealistic assumptions. However, it is the first attempt at such a model, and its existence has several uses. Firstly, it shows that such a model is possible, and with a level of complexity that, if not low, is low enough to be manageable and no higher than many CGE (computable general equilibirum) models used for policy purposes. Secondly, it provides a basis for criticism and improvement. Third, despite its flaws, it is useful for policy analysis as it provides a tool for thinking about the trade-offs and synergies between different aspects of wellbeing and policy that currently does not exist.

Given the prototype nature of the model described in this paper, there is clearly room for significant further work. The model and the results it gives should undergo serious critical analysis to understand how the model behaves, what assumptions are critical, and where the model gives unrealistic and unhelpful results. These results will help inform improvements to the model and where they will be most important. Data will also be a challenge for this model, as it requires a large number of parameters and initial values. Much of the data is available (with varying quality), but sensitivity analysis will be an important part of using the model.

There is also much to learn about policy by applying the model. Without a model of this type, higher order effects (particularly those in different policy areas to the original) are very difficult to anticipate. This model allows those effects to be recognised, as well as obtain some understanding of how significant they are. For example, consider an increase in health spending by government. The model results indicate a small negative effect on the natural environment. Tracing through the model shows this is not spurious: more health spending results in better health which results in higher productivity, higher productivity results in more consumption and production which results in more pollution and therefore a poorer natural environment. This is a chain of impacts that is too long for most people to perform mentally without the aid of a model, but the consequences could be important.

The model can be used for both single policies and for policy packages. In the case of policy packages, it can be used to see how the effects of different policies balance each other out, and which policies should be included in a package. It provides a consistent framework for policies from different fields, e.g. education and health, to be compared with each other and in a broader way than tools such as cost-benefit analysis allow. As well, policies can be chosen on their ability to mitigate some of the undesirable trade-offs inherent in the choice of policies that are the cornerstones of a package.

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Living Standards Analysis Model: The First Prototype

1 Introduction

How do we understand the synergies and trade-offs of a given policy on area beyond that policy, such as the effect of housing on health and on income? How do we choose between policies in completely different areas, such as an education policy and a health policy? How do wellbeing outcomes change if the government focusses on wellbeing rather than GDP? Treasury's Living Standards Framework provides one possible starting point, but it provides little assistance tracing the many dependencies between policy areas. A model that includes those dependencies could help.

The first prototype of the Living Standards Analysis Model (LSAM) described in this paper is designed to do this. This model includes all eleven aspects of wellbeing as described by the OECD's How's Life? framework and linkages between the different aspects for a small open economy. Most models for studying wellbeing only include one or two aspects, missing the rich set of interactions that can occur with greater coverage. As an early prototype, this version of the model does have many flaws and requires significant further development, but it forms a basis for creating an improved model as well as providing some qualitatively useful results.

1.1 What is Wellbeing?

There are several different ways the term "wellbeing" is used. Often it is used to refer to subjective wellbeing, which in turn can refer to positive and negative affect[1], life evaluation (commonly measured as life satisfaction), or eudaimonia[2]. Sometimes is it used primarily in relation to health and particularly mental health. It is also used in a more holistic way to cover the breadth of both objective and subjective conditions that lead to what might be described as "the good life". It is in this last sense that wellbeing is used in this paper.

Traditionally, GDP has been used as a measure of wellbeing when comparing countries, partially due to its standardised measurement and single value, and partially due to the belief that income level is a good proxy for wellbeing. There have been many challenges to the use of GDP for this purpose (for example

1 Positive and negative affect refers to experiences of (respectively) pleasant emotional states such as joy or peace; and unpleasant emotional states such as fear or sadness

2 Eudaimonia relates to the sense of purpose or value in one's life

Nordhaus and Tobin, 1972; Waring and Steinem, 1988; and see Bergh, 2009 for a brief review of the literature). In the last decade or so, the belief in income as a good proxy for wellbeing has been increasingly challenged by a variety of authors, and perhaps most prominently by Stiglitz, Sen, and Fitoussi in their 2009 report to the French president.

At an individual level, wellbeing encompasses the things an individual values as contributing to their quality of life. This includes their material conditions such as their financial wealth and their consumption, but it also includes things money can't always buy, such as good health, clean air, and quality friendships. Individuals will value these things differently according to their preferences, which then makes it difficult to come up with a single value-neutral number representing wellbeing.

It is also useful to think about the distinction between utility and capability. Both are important. Utility can be described as the value we get from outcomes—how much we can consume, how healthy we are, how many quality friendships we have etc. The capability approach (as described by Sen in his body of work such as (Sen, 1999) and (Sen, 2009)), is more about the process and choices available to us, such as the freedom good health gives us to choose to earn more income or to further our education. The capability approach highlights the importance of the linkages between different aspects of wellbeing. The greater our capability, the more opportunity we have to make choices that maximise our utility even if we change our preferences.

At the societal level, we face the additional challenge of aggregation when attempting to understand wellbeing, and this, like combining different aspects of well-being, is not value-neutral. Do we follow Rawls (1971), and take the wellbeing of the worst-off member of the society as the representative wellbeing; or Bentham (1789) considering the (additive) total of wellbeing of all members of society; or something in-between (which can be described mathematically by the isoelastic formulation); or something else altogether?

If wellbeing is considered in the sphere of public policy, it is also important to consider the role of government—another value judgement. Is the role of government and public policy to "improve people's lives, now and into the future" as suggested by Karacaoglu (2015, p. 1)? Or is it much narrower, limited to the minimal state providing basic protections against fraud, theft, and violence as described by Nozick (1974)? In a democracy such as New Zealand, the answers to these questions are determined by those in power with the consent of the population.

1.2 Frameworks, Measurements, and Models

Much of the current effort studying wellbeing is in developing frameworks and defining and measuring wellbeing. This has led to a proliferation of indexes, many covering a similar set of aspects of wellbeing, but dividing it into different categories and using different measures and weights. Each index is designed with a different purpose in mind and makes different value judgement. No one measure or framework is dominant yet.

Prominent frameworks by intergovernmental organisations include the OECD's How's Life? framework and related Better Life Index (OECD, 2011; OECD, 2017), the UNDP's Human Development Index (UNDP, 1990), and measurements against the UN's Sustainable Development Goals (UN, 2015). The World Bank uses Genuine Savings (Hamilton, 2000, also known as Adjusted Net Savings), which emphasises sustainability through measuring changes to capital stocks. Some for-profit and many non-governmental organisations have developed their own measures, including the Legatum Institute's Prosperity Index (The Legatum Institute, 2016) and the New Economics Foundation's Happy Planet Index (Abdallah et al., 2009). There are also regular reports ranking the "best" cities to live in, such as the Global Liveability Ranking (The Economist Intelligence Unit, 2016), and in New Zealand The Treasury have used their Living Standards Framework (Gleisner, Llewellyn-Fowler, and McAlister, 2011) as a qualitative tool for policy analysis.

There has also been significant econometric work looking at the interaction between (typically) two aspects of wellbeing at a time. This could be the interaction between health and income (Karanikolos et al., 2016), or subjective wellbeing and income (Layard, 2005), or education and social connection (Helliwell and Putnam, 2007), for example. This type of work is highly important for identifying the factors that contribute to different aspects of wellbeing and finding some of the connections between those different aspects.

But measurement (and to a lesser degree regression) doesn't let us experiment with different settings and understand their interaction, and there is much less literature on creating a model of wellbeing. Most of the current and historical work is theoretical and only brings one or two aspects of wellbeing together, most commonly growth or income and the environment. This includes applying viability theory to explore the boundaries of sustainable satisficing in fisheries (Krawczyk et al., 2013), Arrow et al's work on "comprehensive consumption" (Arrow et al., 2012; Arrow et al., 2013), "clean" and "dirty" technologies (Acemoglu et al., 2016), and adapting growth theory to include the environment (Dasgupta and Heal, 1974). In health economics, the Grossman model (Grossman, 1972) describes some of the interaction between health and income. Karacaoglu (2015) pulls together some of these theoretical strands of work to create a more comprehensive theoretical model appropriate for a small open economy such as New Zealand; but this model has not been implemented (and is not designed to be implemented) as a practical tool for answering policy questions.

1.3 The Prototype Living Standards Framework Model

The prototype model described in this paper is unique, both in the holistic treatment of wellbeing, and in its applied nature. It includes all eleven of the OECD's How's Life? framework aspects of wellbeing; and is intended for implementation in a computational form for use in policy analysis. As this prototype was developed before the current version of the Treasurys Living Standards Framework, it does not include a culture aspect of wellbeing, but this will be considered for future versions of the model.

The purpose of this model is to support policy development and decisions in New Zealand by providing a tool for thinking about changes in aspects of wellbeing including trade-offs and synergies for policies. This means it must be able to answer practical policy questions in a suitable timeframe. As a tool for thinking it need not provide the "right answer", but it should raise new questions, make implicit assumptions explicit, allow the anticipation of otherwise unanticipated consequences, and break down silos between different policy areas. As the model is focussed on the trade-offs and synergies between the different aspects of wellbeing, it must cover a significant breadth of subjects. To remain tractable and simple enough to be useful, this breadth must come at the sacrifice of depth and detail in any given area. As the model is intended for use in New Zealand, it models a developed country that is geographically isolated and has a small, open economy. The How's Life? framework has been chosen for its full coverage of all aspects of wellbeing and as an internationally-used measure that focusses on wellbeing in developed countries.

In The Idea of Justice, Amartya Sen said "It is possible to be at once deeply appreciative and seriously critical of a theory" (2009, p.58). This paper describes a model that, as a prototype, has many flaws but provides a framework for improvement. One of the purposes of creating a prototype model is to discover flaws, and to learn which are important. Another purpose is to provide something concrete that can be criticised, and then use that criticism to improve future prototypes. This paper is written in that spirit, and the author would appreciate constructive feedback on this prototype model.

1.4 Model Overview

The model described in this paper is a top-down stock-and-flow model including a basic general equilibrium open economy. Households experience eleven aspects of wellbeing (corresponding to those in the OECD's How's Life? framework), and there are multiple household types. Households can produce goods and services at home, have them supplied by the government, or purchase them from firms. Firms buy labour and capital from households for use in production, and can import and export goods. The government taxes income and consumption, and uses those funds to provide goods and services to households, pay interest on debt, and invest in physical infrastructure.

This is a complex model. It aims to incorporate the major links between all aspects of wellbeing, as well as the significant drivers of those aspects. As can be seen in Figure 1, not every aspect of wellbeing is directly linked to every other aspect, but they are far from independent from each other.

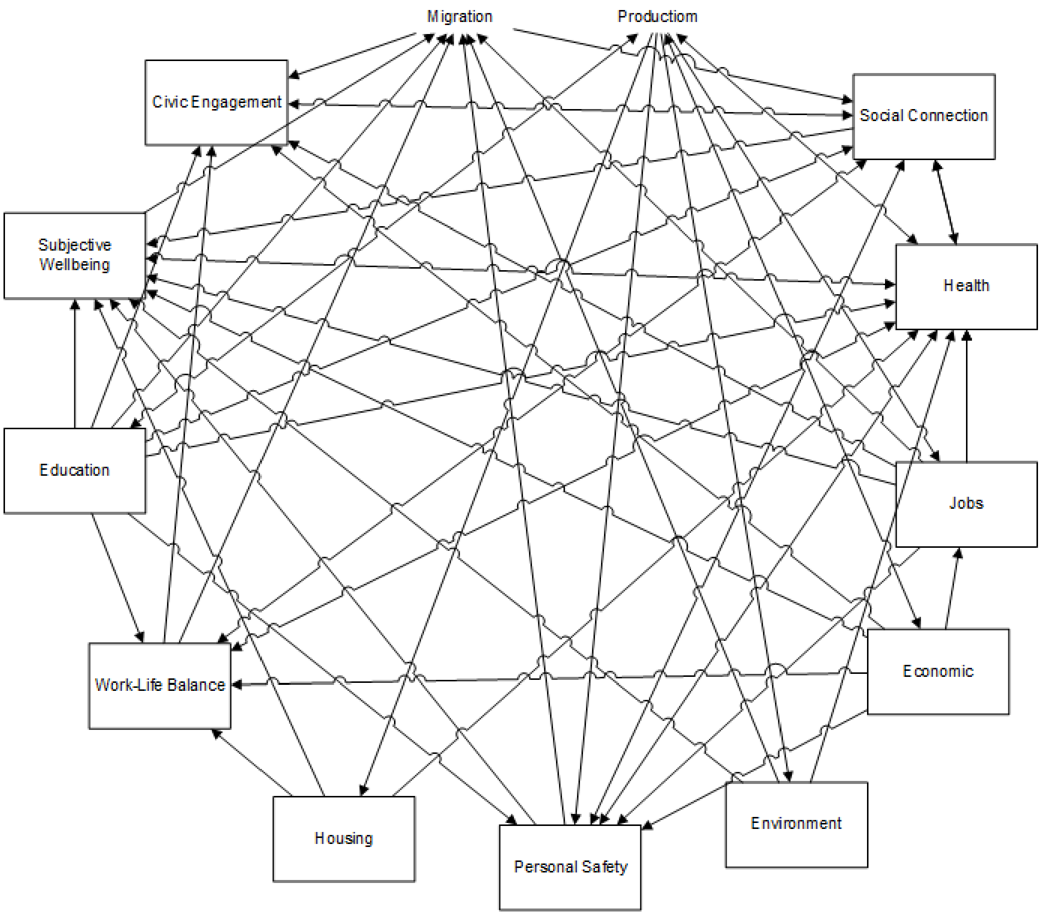

Figure 1 – Links between the different aspects of wellbeing in the prototype model

Many of the interactions between different aspects of wellbeing occur in the "investment" equations of the model. These investment equations describe how the stocks (also called capitals) in the model change from one time period to the next, typically in response to changes in the stocks that relate to other aspects of wellbeing. Everything else in the model happens within a time period or refers back to previous time periods—only the changes in the stocks move the calculations forward to the next time period. A time period could, in theory, be any length provided the parameter values were chosen appropriately, but in practise one time period is likely to be a year long, with the model running for decades (i.e. tens of time periods). This is limited by if the model reaches limits such as negative wealth (sometimes caused by insufficient feedback loops in the model), or by when the time elapsed makes the projection meaningless—after about 50 years it could easily be argued that technology and society will have changed more than the model can sensibly accommodate.

As well as labelling the model equations as "stock" and "investment (flow)" equations, the model equations could also be labelled as "accounting" and "behaviour" equations. In this description, accounting equations must hold by definition—for example, the sum of the time taken for all activities during the day must add to 24hrs—but behaviour equations describe choices by households, firms, and others. Behaviour equations encode our assumptions about how the world works, but there is also a trade-off with model complexity, and they are the equations most likely to be changed in future versions of the model. Currently, most behaviour in the model is assumed to either optimise narrowly-defined utility functions, or to be constant (and exogenous) over time; this need not be the case in future versions of the model, nor need behaviour be fixed to just one option. The model results are the logical implications of the assumptions that go into the model including the constraints that the accounting equations imply.

The value of this model is in understanding how changes in one aspect of wellbeing flow through to other aspects of wellbeing. This requires the inclusion of many aspects of wellbeing and the links between them, which makes the model complex. All other modelling decisions must then be made to avoid adding further complexity without compromising this aim. As such, this model is not optimised across time (it is myopic), nor is it fully optimised within a time period. This may also partially reflect reality. Economics traditionally assumes that all agents optimise their utility, which in this case is equivalent to their wellbeing (either lifetime or in a given time period). While it is likely the case that agents optimise more of their wellbeing than is modelled here, it is also likely (based on results from behavioural economics) that for at least some aspects of wellbeing they use heuristics or satisficing rules rather than optimisation.

The relationships between different variables in this model are primarily identified based on face validity, with some input from those who are experts or familiar with the literature for each aspect of wellbeing. This has allowed the model to be developed relatively quickly, albeit at the cost of empirical rigour. The expectation is that future versions of the model will include much more rigourous evidence from the literature and empirical research, which may change some of the relationships included in this version of the model. Where relationships that turn out to be weak have been included, this is usually easy to deal with in this version of the model by setting the associated parameter to a low value (or, in some cases, to zero). Where important relationships have not been included, this requires changing the equations and represents a more significant limitation of this version of the model.

A frequent question that arises when measuring wellbeing is that of how (or whether) the different aspects of wellbeing can be directly compared or combined into a single value. In this model, an indexation and aggregation approach is proposed, but it is not essential to the model and should be downplayed. Much more important is the results for each aspect of wellbeing, and how they are caused by and relate to the other aspects of wellbeing. These are best measured in the natural units of the variable. A single measure of wellbeing is generally of little use in policy settings as any changes in that value can only be understood in the context of the cause of those changes.

This paper describes the design and equations of the model (applications of the model including calibration, validation, results, and sensitivity analyses will be presented in future papers). The next sections describe the modelling of the different aspects of wellbeing, then the modelling of the supporting elements such as the production sector. The paper concludes with a description of the anticipated uses of the model and plans for future improvements. Additionally, the appendices have a list of all the mathematical notation used in this paper with precise definitions, a list of the equations in the order they are solved in, and selected derivations of those equations.

2 The Eleven Aspects of Wellbeing

The OECD How's Life? framework describes eleven aspects of wellbeing that provide near-complete coverage of what many people considers makes up wellbeing, as described earlier. These eleven aspects and the interactions between them form the basis of this model.

2.1 Civic Engagement

Civic engagement is the term used to describe how engaged people in the society are with how they are governed. A society with high civic engagement will have high voter turnout, good stakeholder consultation by government, and a sense that members of the society can influence the behaviour of government. With good civic engagement, governments are held to account and so government spending may be more effective. It is also easier for governments to implement policies that the affected people have been involved in creating. Note that civic engagement does not always make government spending more effective if it takes the form of lobbying for policies that benefit a small group (for example, Federated Farmers, Greypower) rather than everyone.

In this model, civic engagement is treated as a stock akin to social connection that changes as other aspects of society change (in contrast, the OECD treats this as a flow, due to the importance people attach to procedural utility). As such, it obeys the standard stock evolution equation:

$$K_{CG,h,t+1} = (1 - \delta_{CG,h}) \, K_{CG,h,t} + I_{CG,h,t} \qquad \forall h, t \tag{1}$$

where $\delta_{CG,h}$ is the "natural" decline in civic engagement in the absence of any other changes in society and $I_{CG,h,t}$ is the aggregate of the flow during the time period t. The "natural" decline could be taken to reflect the cohort effect that is being observed in a number of countries, where younger people are less likely to vote than older people, and remain less likely to vote even as they age (Konzelmann, Wagner, and Rattinger, 2012). Note that different household types may have different levels of civic engagement, which reflects the fact that some sections of society can be disengaged or unable to engage in a two-way manner with government; it also reflects the fact that the children of civically-engaged parents have a higher level of civic engagement.

The civic engagement aspect of wellbeing is represented solely by the stock of civic engagement:

$$\hat{W}_{CG,h_S,t} = \hat{K}_{CG,h_S,t} \qquad \forall h_S, t \tag{2}$$

The level of civic engagement in a society is affected by the level of education, the net income, the social connection, the level of immigration, and the level of unemployment. It is also affected by the level of leisure time available, without which people may not have the time to engage with government even if they have both the desire and the ability. Thus, the change in the level of civic engagement is given by the change in these factors:

$$
\begin{aligned}
I_{CG,h,t} = & \xi_{CG,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right) + \xi_{CG,y,h}\left(y_{h,t} - y_{h,t-1}\right) \\
& + \xi_{CG,WL,h}\left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,h,t-1}\right) + \xi_{CG,SC,h}\left(K_{SC,h,t} - K_{SC,h,t-1}\right) \\
& - \xi_{CG,J,h}\left(J_{ST,h,t} + J_{LT,h,t} - J_{ST,h,t-1} - J_{LT,h,t-1}\right) \\
& - \xi_{CG,NM,h}\left(N_{M,h,t} - N_{M,h,t-1}\right) \qquad \forall h, t
\end{aligned} \tag{3}
$$

2.2 Economic (Income & Wealth)

Economic wellbeing is an important part of providing ourselves with the necessities of life and material pleasures, and it is an aspect of wellbeing that has been heavily focussed on by governments and economists around the world. A lack of economic wellbeing particularly impacts health, and can also negatively impact many other aspects of wellbeing.

To have good economic wellbeing, a household must have money. This may be in the form of financial wealth, inherited or accumulated over time; or it may be from income from investments or from working in the labour market. This money can then be spent on consumption. Consumption goods and services can also be created by home production, so a low income need not force a low level of consumption if the household has the time and skill to make what they need for themselves.

Economic wellbeing is defined as the simple average of the indexes of the three economic indicators—net income, total consumption, and financial wealth:

$$
\hat{W}_{IW,h_S,t} = \tfrac{1}{3}\left(\hat{y}_{h_S,t} + \hat{C}_{h_S,t} + \hat{K}_{IW,h_S,t}\right) \qquad \forall h_S, t \tag{4}
$$

This follows the method used by the OECD Better Life Index (2017), where the indicators, after conversion to indexes, are combined using a simple arithmetic averages to get the indexes for each aspect of wellbeing.

Although these three indicators heavily overlap and are highly interrelated, they do cover different concepts. Consumption is often smoother than income, as wealth acts as a buffer. Changes in wealth are dependent on the difference between income and consumption, but the level of wealth (and to a lesser extent, income) is heavily influenced by the initial level of wealth. They also match with the first recommendation of the Stiglitz Report (Stiglitz, Sen, and Fitoussi, 2009): "When evaluating material wellbeing, look at income and consumption rather than production" and the third recommendation "Consider income and consumption jointly with wealth".

2.2.1 Income

Gross income comes from returns to capital and wages (both paid by the production sector):

$$Y_{h,t} = P_{Q,K,t}K_{IW,h,t} + P_{L,h,t}T_{IW,h,t} \qquad \forall h,t \tag{5}$$

The time available for work for each household is given by the number of hours worked and the employment rate:

$$T_{IW,h,t} = L_{h,t}J_{E,h,t} \qquad \forall h,t \tag{6}$$

Households chose how much of their time to spend working. In this version of the model, they chose to spend a constant proportion of their time working:

$$L_{h,t} = L_{h,t=0} \qquad \forall h,t \tag{7}$$

This choice simplifies the model and represents the strong habit persistence that can occur in the proportion of time spent working. However, it means that households cannot choose to change their hours of work to increase their income or their leisure time. It is common in static CGE (computable general equilibirum) models for the total amount of labour to be fixed—it is part of the "closure" for such models.

The total labour supply (per household) for production is adjusted for the productivity of the workers from health and education:

$$\tilde{L}_{S,h,t} = A_{\tilde{L},h}K_{ESF,h,t}{}^{\eta_{L,ESF,h}}K_{ESS,h,t}{}^{\eta_{L,ESS,h}}K_{HS,h,t}{}^{\eta_{L,HS,h}}T_{IW,h,t}\frac{K_{N,h,t}}{K_{N,S,t}} \qquad \forall h,t \tag{8}$$

However, gross income is not the income that affects wellbeing. Households must pay taxes on their income, and may receive transfers from the government. Net income, the income after transfers (including benefit payments) and taxes, is a better measure of economic wellbeing as it represents the money available for households to save or spend on consumption. Net income is given by:

$$\begin{aligned} y_{h,t} = &(1-\tau_{Y,h})\,Y_{h,t} + \Gamma_J\,(J_{ST,h,t} + J_{LT,h,t}) \\ &+ \gamma_{NW,h}\Gamma_{NW}J_{NW,h,t} + \gamma_{\tau,h}\Gamma_{\tau,t}K_{N,S,t}{}^{-1} \qquad \forall h,t \end{aligned} \tag{9}$$

2.2.2 Consumption

Households consume goods produced by the market and created by home production. However, not all consumption is desirable—consumption of goods and services such as transport and personal safety are more about negating problems rather than being attractive for consumption in their own right. Thus, the total consumption defined here for use in calculating wellbeing excludes undesirable consumption:

$$C_{h,t} = \sum_{k \notin \{PS, HOT\}} C_{h,k,t} + \sum_{h_1} \sum_{k \notin \{PS, HOT\}} Q_{h_1,h,k,t} \qquad \forall h, t \tag{10}$$

The processes for creating market goods and home production is described later, in sections 3.1 and 3.3 respectively. However, the purchasing decisions for market goods and services are made by the household, according to behavioural rules.

In this version of the model, two simple rules are used. First, the amount of money the household spends on consumption remains constant:

$$B_{h,t} = B_{h,t=0} \qquad \forall h, t \tag{11}$$

This is a very simple choice, and suggests a very strong level of habit persistence in the consumption behaviour of households. However, it is not very realistic as it assumes income and wealth have no effect on prices. It is also more likely that level of consumption, rather than budget of consumption, is the subject of habit persistence.

Secondly, the household gets "utility" from consumption that they try to maximise. This utility is given by a Cobb-Douglas function:

$$U_{C,h,t} = \prod_{k \neq HOT} C_{h,k,t}{}^{\alpha_{C,h,k}} \qquad \forall h, t \tag{12}$$

Note that transport consumption is excluded from this calculation as it is obtained in the housing section 2.6 from time spent travelling (e.g. commuting), but it includes all other types of consumption, essential and non-essential, desirable and undesirable. This is solely for modelling reasons, i.e. not calcuating the same variable twice.

A Cobb-Douglas utility function implies that although we get less value from each additional unit of consumption, more consumption always gives greater utility. This is certainly realistic up to a point, but it is not clear that it holds for all levels of consumption. In principle, households should get utility from all aspects of wellbeing and optimise their behaviour across this. However, this substantially complicates the problem for both households and the modeller. Using the method described here allows the allocation of consumption across different categories, without commenting on choices around other aspects of wellbeing.

In maximising their utility, households face a budget constraint given by:

$$B_{h,t} = \sum_{k} \left(1 + \tau_{C,k}\right) P_{C,k,t} C_{h,k,t} \qquad \forall h, t \tag{13}$$

which must include transport spending as it is part of consumption. For all non-transport goods and services, consumption is given by:

$$C_{h,k,t} = \frac{\alpha_{C,h,k} \left(B_{h,t} - \left(1 + \tau_{C,k_2=HOT}\right) P_{C,k_2=HOT,t} C_{h,k_2=HOT,t}\right)}{\left(1 + \tau_{C,k}\right) P_{C,k,t}} \qquad \forall h, k \neq HOT, t \tag{14}$$

Thus, in this model households always spend the same share of their after-transport consumption budget on each good or service.

2.2.3 Wealth

Household financial wealth is a stock that can be changed by the behaviour of the household. As such, it obeys the standard stock equation

$$K_{IW,h,t+1} = \left(1 - \delta_{IW,h}\right) K_{IW,h,t} + I_{IW,h,t} \qquad \forall h, t \tag{15}$$

Changes in wealth are due to inflation $\delta_{IW,h}$ and the difference between net income and spending:

$$I_{IW,h,t} = y_{h,t} - B_{h,t} \qquad \forall h, t \tag{16}$$

This assumes all household wealth is held in (or can be liquidised to) financial forms rather than in physical forms such as housing. While not particularly realistic, it matches with the treatment of housing in the current version of the model.

2.3 Education

Many people value education, knowledge, and skills for their own sake. Learning new skills and knowledge often creates a sense of achievement and satisfies our curiosity, as well as increasing employability, making labour more productive, and improving our ability to look after our health, the environment, and participate in civic issues.

However, it takes a significant investment of both time and money to obtain a good education and reasonable level of skills. People spend an average of 17.5 years in formal education in OECD countries (OECD, 2017) mostly before entering the workforce. They then continue to develop skills on the job (and elsewhere) throughout their working career and sometimes beyond. Typically, the government will pay most of the cost of primary and secondary education, but a much smaller proportion of tertiary education.

In this model, we will consider two types of education-related human capital—skills and formal education. Formal education occurs outside the workplace, typically (but not exclusively) at a school, university, or polytechnic. It takes time away from other activities a household may perform (including earning an income), and must be paid for either by the government or the household. It often leads to a qualification.

Skills are defined as the result of on-the-job training and experience and are harder to measure. They are effectively paid for by producers, who must lose productive time from their employees to invest in their skills. However, producers benefit from the increase in skills through improved productivity. As much of this training is provided on-the-job or as experience, there are no direct costs to be paid.

Both skills and formal education evolve over time according to the standard stock equation:

$$K_{ESF,h,t+1} = (1 - \delta_{ESF,h})\, K_{ESF,h,t} + I_{ESF,h,t} \qquad \forall h,t \tag{17}$$

$$K_{ESS,h,t+1} = (1 - \delta_{ESS,h})\, K_{ESS,h,t} + I_{ESS,h,t} \qquad \forall h,t \tag{18}$$

where $\delta_{ESF,h}$ and $\delta_{ESS,h}$ are the rates at which we forget what we've learnt (or our knowledge becomes irrelevant), and $I_{ESF,h,t}$ and $I_{ESS,h,t}$ are the results of the effort put into increasing formal education and skills respectively. These two stocks are the indicators for the education aspect of wellbeing, and are combined into the wellbeing index using a simple average of the indexes of the two stocks:

$$\hat{W}_{ES,h_S,t} = \tfrac{1}{2}\left(\hat{K}_{ESF,h_S,t} + \hat{K}_{ESS,h_S,t}\right) \qquad \forall h_S,t \tag{19}$$

2.3.1 Formal Education

The quantity of formal education demanded is determined in this model by the supply of education. Education can be supplied by the government, by the market, and by home production:

$$Q_{ESF,h,t} = \sum_{h_1} Q_{h_1,h,k=ESF,t} + C_{h,k=ESF,t} + \frac{C_{G,k=ESF,t}}{K_{N,h,t}} \frac{\tilde{S}_{G,ES,h,t}}{\sum_h \tilde{S}_{G,ES,h,t}} \qquad \forall h,t \tag{20}$$

This supply of formal education is converted to an increase in formal education level with an efficiency that depends on the existing education level:

$$I_{ESF,h,t} = A_{ESF,h} K_{ESF,h,t}{}^{\eta_{ESF,h}} Q_{ESF,h,t} \qquad \forall h, t \tag{21}$$

Formal study also requires time that cannot then be used for leisure, work, or other purposes. We estimate the (health-adjusted) amount of time spent studying based on the investment in formal education:

$$\tilde{T}_{ES,h,t} = A_{ES,T,h} I_{ESF,h,t} \qquad \forall h, t \tag{22}$$

Removing the health adjustment means the time required to consume these education services is given by:

$$T_{ES,h,t} = A_{HS,ES,h} K_{HS,h,t}{}^{-\eta_{HS,ES,h}} \tilde{T}_{ES,h,t} \qquad \forall h, t \tag{23}$$

2.3.2 Skills

Training for skills are provided by producers to the labour they employ. This means that, of the time they pay wages for, a proportion will be dedicated to skills training, and the remainder to production:

$$\tilde{L}_{ESS,h,e,q,t} = \xi_{Q,ESS,h,e,q} \tilde{L}_{Q,h,e,q,t} \qquad \forall h, e, q, t \tag{24}$$

This supply of skills training is converted to an increase in skill level:

$$I_{ESS,h,t} = A_{ESS,h} \sum_{e} \sum_{q} \tilde{L}_{ESS,h,e,q,t} \qquad \forall h, t \tag{25}$$

The efficiency with which this can be done (incorporating current education, skill, and health levels) is already incorporated in the time allowed.

2.4 Environment

The quality of the environment we live in affects our wellbeing in several different ways. High levels of pollution negatively impact physical health. Lack of green spaces negatively affects mental health. Production depends on accessing natural resources and being able to emit pollution to natural sinks. Biodiversity contributes to technological progress (for example, biomining for new drugs; biomimicry in engineering). A natural, unpolluted environment also provides a variety of recreational opportunities and its existence is valued for its own sake by many people who may never visit an untouched habitat.

In this model, three aspects of the environment are considered: resources, pollution, and biodiversity. The level of resources is important for production, but does not directly impact on the environment aspect of wellbeing. However, the pollution level and the biodiversity level are both used as indicators for the environment aspect of wellbeing:

$$\hat{W}_{EQ,h_S,t} = \frac{1}{2}\left(\hat{K}_{EQ,h_S,t} + \hat{K}_{\Psi,h_S,t}\right) \qquad \forall h_S, t \tag{26}$$

2.4.1 Resources

Resources are defined as goods provided by the natural environment that are valued for their use in production. In this model, two types of resources are recognised: stock resources and flow resources. A flow resource is always renewable and is generally difficult to store—sunlight being an excellent example. Use of a flow resource for production is constrained by the cost of extracting it and the rate of flow; and it is not affected by the level of use in the previous time period. For notational purposes it is still necessary to define the stock level of flow resource, and so (using the standard evolution equation) it is given as:

$$K_{j=EQF,t+1} = \left(1 - \delta_{EQ,j=EQF}\right) K_{j=EQF,t} + I_{j=EQF,t} \qquad \forall t \tag{27}$$

with $\delta_{EQ,j=EQF} = 1$. The rate of flow is constant over time, so:

$$I_{j=EQF,t} = I_{j=EQF,t=0} \qquad \forall t \tag{28}$$

and this defines an upper bound on the amount of flow resource that can be extracted, given by:

$$Q_{j=EQF,t} \leq I_{j=EQF,t} \qquad \forall t \tag{29}$$

A stock resource, such as fish or coal, may or may not be renewable, but is characterised by being able to be stored. We assume the stocks do not deteriorate or decline with storage. However, this stored resource may run out if it is used faster than it regenerates (in the case of a renewable resource) or within a finite time period (in the case of a non-renewable resource). The level of the stock resource is also given by the standard evolution equation:

$$K_{j=EQS,t+1} = \left(1 - \delta_{EQ,j=EQS}\right) K_{j=EQS,t} + I_{j=EQS,t} \qquad \forall t \tag{30}$$

where $\delta_{EQ,j=EQS} = 0$ for non-renewable stocks, and $\delta_{EQ,j=EQS} < 0$ is the regeneration rate for renewable stocks. Note that this means renewable stocks grow exponentially without limit if they are not used, which is clearly unrealistic. However, we assume that the range of values that $K_{j=EQS,t}$ take remain in a realistic range over the timeframe the model is run for.

The “investment” in the level of the stock resource is simply the reduction due to the extraction of the resource:

$$I_{j=EQS,t} = -Q_{j=EQS,t} \qquad \forall t \tag{31}$$

These equations define the resources as they are before human intervention. They must then be extracted (as described in the production section 3.1) for use in production where they become valuable for human wellbeing.

2.4.2 Pollution

Pollution is emitted into the natural environment by production. The environment can absorb and neutralise some pollution, but pollution flows in excess of this level build up in the environment, damaging biodiversity and creating health risks. Most types of pollution form geographic concentrations, often in the area they are emitted, but sometimes elsewhere such as polluted streams emptying into a lake. A few forms of pollution do not become geographically concentrated, but have global impacts, such as carbon dioxide.

The total level of pollution experienced by a society is governed by the standard evolution equation:

$$K_{\Psi,S,t+1} = (1 - \delta_{\Psi,S})\, K_{\Psi,S,t} + I_{\Psi,S,t} \qquad \forall t \tag{32}$$

where $\delta_{\Psi,S} > 0$ is the rate the natural environment can neutralise pollution. Note that this assumes the more pollution there is, the more pollution can be neutralised. This assumption is appropriate for low levels of pollution, but the converse is much more likely to be true in highly polluted environments which may become overloaded.

Apart from natural absorption, there are two factors that change the level of pollution: the level of pollution emitted by production in each time period, and the spending (and effectiveness of that spending) by government primarily on clean-up. This gives the “investment” in the pollution level:

$$I_{\Psi,S,t} = \sum_{q} \sum_{e} \Psi_{e,q,t} - \xi_{\Psi,S} \tilde{S}_{G,\Psi,t} \qquad \forall t \tag{33}$$

As this model does not contain geographic information, it is assumed that all households experience the same (societal) level of pollution, which is given by:

$$K_{\Psi,h,t} = K_{\Psi,S,t} \qquad \forall h, t \tag{34}$$

2.4.3 Biodiversity

Biodiversity is also an environmental stock. It is valuable as an enabler for production and for health, as well as in its own right. As a stock, it is governed by the standard evolution equation:

$$K_{EQ,S,t+1} = (1 - \delta_{EQ,S})\, K_{EQ,S,t} + I_{EQ,S,t} \qquad \forall t \tag{35}$$

Like a renewable stock resource, it is assumed that biodiversity stocks can regenerate themselves (i.e. $\delta_{EQ,S} < 0$), although in the absence of other effects that reduce biodiversity this again results in unlimited exponential growth. In this model other effects come in two types: spending by government on increasing biodiversity, and the negative effects of pollution on biodiversity. This results in the additional "investment" in biodiversity given by:

$$I_{EQ,S,t} = \xi_{EQ,S} \tilde{S}_{G,EQ,t} - \zeta_{EQ,S} K_{\Psi,S,t} \qquad \forall t \tag{36}$$

It is also assumed that all households experience the same level of biodiversity, which is given by:

$$K_{EQ,h,t} = K_{EQ,S,t} \qquad \forall h, t \tag{37}$$

This is a more realistic assumption than for pollution, as the location of biodiversity relative to households has much less impact on different household types than the geographic distribution of pollution.

2.5 Health

Good health is a crucial part of wellbeing. Without it, we feel uncomfortable at best, and it also prevents us improving other aspects of our wellbeing. In the worst case scenario, poor health leads to (early) death.

It also has large impacts on how we can spend our time. Poor health effectively reduces our ability to use our time well, reducing the intensity with which we can work, play, and study, and increasing the time needed for personal care including resting and receiving medical treatments and monitoring. This applies both to physical and to mental health.

Our health level is affected by many factors, some of which also relate to other aspects of our wellbeing. There is a strong degree of continuity, with people who currently have good health likely to continue to have good health in the future, and those who do not continuing to have poor health. When we do get sick, the level of healthcare we have access to (relative to the level we need) determines how quickly (or whether) we get better. Wealth, education, leisure, social connections, and housing can all improve health for the better, as can life satisfaction and personal safety. Unemployment, pollution, and a degraded natural environment can make it worse.

The average level of health is modelled here as a stock, and evolves over time according to the standard stock equation:

$$K_{HS,h,t+1} = (1 - \delta_{HS,h})\, K_{HS,h,t} + I_{HS,h,t} \qquad \forall h, t \tag{38}$$

where $\delta_{HS,h}$ is the natural decline in health over time in the absence of healthcare provision or any other drivers of health change. This value should vary depending on the age of the people in the household. All other changes in health (both improvements and declines) are incorporated into the health investment term $I_{HS,h,t}$. This health level is the only indicator used for the health aspect of wellbeing:

$$\hat{W}_{HS,h_S,t} = \hat{K}_{HS,h_S,t} \qquad \forall h_S, t \tag{39}$$

The health investment term—change in health level—is determined by adding the change in each of the factors that affect health, multiplied by an effectiveness parameter for that factor. This assumes that each of these factors act independently and linearly on health. These factors that affect health are the excess of demand for health services above supply, the level of biodiversity, the wealth, formal education level, unemployment level of the household type, life satisfaction, social connection, and personal safety of the household. Note that for a few variables, i.e. level of pollution, housing quality and quantity, and health-adjusted leisure, health changes are due to the level of these variables, and not the change as for other factors that affect health:

$$\begin{aligned}
I_{HS,h,t} =& \xi_{HS,service,h}\left(Q_{HS,h,t} - C_{HS,h,t}\right) + \xi_{HS,EQ,h}\left(K_{EQ,h,t} - K_{EQ,h,t-1}\right) \\
& - \xi_{HS,\Psi,h} K_{\Psi,h,t} - \xi_{HS,J,h}\left(J_{ST,h,t} + J_{LT,h,t} - J_{ST,h,t-1} - J_{LT,h,t-1}\right) \\
& + \xi_{HS,IW,h}\left(K_{IW,h,t} - K_{IW,h,t-1}\right) + \xi_{HS,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right) \\
& + \xi_{HS,SW,h}\left(\Lambda_{SW,h,t} - \Lambda_{SW,h,t-1}\right) + \xi_{HS,PS,h}\left(K_{PS,h,t} - K_{PS,h,t-1}\right) \\
& + \xi_{HS,SC,h}\left(K_{SC,h,t} - K_{SC,h,t-1}\right) + \xi_{HS,HO,h}\left(C_{HO,h,t} - C_{HO,h,t-1}\right) \\
& - \xi_{HS,\Xi,h}\left(1 - \Xi_{HO,h,t}\right) + \xi_{HS,WL,h}\left(\tilde{T}_{WL,h,t} - A_{HS,T,h}\right) \qquad \forall h, t
\end{aligned} \tag{40}$$

Most of these values are determined in other parts of the model, demand for and supply of healthcare being the exception. Demand for healthcare is modelled as a function of current health, with healthier people requiring less healthcare than those in poorer health:

$$C_{HS,h,t} = A_{HS,h} K_{HS,h,t}{}^{-\eta_{HS,h}} \qquad \forall h, t \tag{41}$$

This way of modelling healthcare assumes that demand for healthcare increases rapidly as health levels decline. Note that it does not take account of the increase in demand for healthcare services due to greater wealth or income which can create higher expectations. Healthcare supply, on the other hand, is given by the total of home production (e.g. carers), market purchases of healthcare, and government supply:

$$Q_{HS,h,t} = \sum_{h_1} Q_{h_1,h,k=HS,t} + C_{h,k=HS,t} + \frac{C_{G,k=HS,t}}{K_{N,h,t}} \frac{\tilde{S}_{G,HS,h,t}}{\sum_{h} \tilde{S}_{G,HS,h,t}} \qquad \forall h, t \tag{42}$$

Thus, supply and demand of healthcare services do not necessarily match, and if demand for healthcare services exceeds supply, this will have a negative impact on health. Note that health is modelled in an analogous way to personal safety.

2.6 Housing

The need for shelter is one of the basic necessities of life, and the quality of that shelter can have a large impact on wellbeing. In the New Zealand climate, good quality housing is warm and dry and should not have the drafts, damp, and mould that can cause or exacerbate health problems such as asthma. The location of the housing is important—if it is not close to friends and families, schools, workplaces, and amenities, too much time and money must be spent on transport or alternatively social connections, education, and income will suffer. This is also dependent on the type of transport available, as it is much easier to travel 10km to work each day if you can take a train or drive on an uncongested road than if you are constrained to walking.

Good quality housing also has enough room for its occupants. Overcrowding increases the risk of infectious diseases and can also be stressful and unpleasant for the occupants. The quantity of housing is also closely linked to the price of housing—the cheaper housing is, the less likely it will be overcrowded to save money. The price of housing is also important in its own right. Because housing is a necessity, it is usually a high priority in a household's budget, and expensive housing reduces the amount of money available for other consumption including food, healthcare, and education.

The indexes of these four indicators—quality, quantity, price, and location (proxied by transport time)—are used to calculate the housing aspect of wellbeing:

$$\hat{W}_{HO,h_S,t} = \frac{1}{4}\left(\hat{\Xi}_{HO,h_S,t} + \hat{C}_{HO,h_S,t} + \hat{P}_{HO,h_S,t} + \hat{T}_{HO,h_S,t}\right) \qquad \forall h_S, t \tag{43}$$

In this version of the model, households spend a constant proportion of their time travelling:

$$T_{HO,h,t} = T_{HO,h,t=0} \qquad \forall h, t \tag{44}$$

While a useful simplification for modelling purposes, this is quite restrictive. It does not allow for many things that can change transport time for households including moving to a better location (possibly increasing housing costs to decrease transport time and costs), changing transport mode (such as from walking to public transport), or environmental changes such as an increase in congestion.

As well as using time, most transport methods cost money. This is modelled by assuming that the consumption of the specific final transport good is proportional to the time spent travelling:

$$C_{h,k=HOT,t} = A_{HOT,h} T_{HO,h,t} - \sum_{h_1} Q_{h_1,h,k=HOT,t} \qquad \forall h, t \tag{45}$$

Note that home production of transport could include active modes like walking or cycling, which would have a positive impact on health. This link has not been included in this version of the model.

Housing itself is supplied by the production of two specific final goods—good quality housing and bad quality housing. These are produced and sold in the same way as any other final good. This simplifies the modelling considerably, but ignores the fact that housing is a durable good and so misses many important aspects of the housing market that can have large effects on the price of housing. It does however have the advantage of automatically including running costs (e.g. heating) in the housing price naturally and without any further adjustments.

Distinguishing good quality housing (HOG) from bad quality housing (HOB) allows the calculation of a quality indicator, defined as the proportion of good quality housing to total housing:

$$\Xi_{HO,h,t} = \frac{C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t}}{C_{h,k=HOB,t} + C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOB,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t}} \qquad \forall h, t \tag{46}$$

The meaning of “good quality housing” does not affect these equations, and can be chosen to fit the data and needs of the user. However, it will generally be used to here to mean housing that does not have a major problem with damp, cold, or mould, as these are major problems with the quality of some New Zealand homes.

The quantity of housing is important for indicating overcrowding, and as such it is measured as the number of rooms per person in the household:

$$C_{HO,h,t} = {n_{h,t}}^{-1} \left(C_{h,k=HOB,t} + C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOB,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t} \right) \quad \forall h, t \tag{47}$$

The number of people per household is assumed to be constant:

$$n_{h,t} = n_{h,t=0} \qquad \forall h, t \tag{48}$$

Finally, for the purposes of the housing aspect of wellbeing, it is the price of housing as a proportion of (net) income that is important. Thus the price of housing indicator is given by:

$$P_{HO,h,t} = {y_{h,t}}^{-1} \left(P_{C,k=HOB,t} C_{h,k=HOB,t} + P_{C,k=HOG,t} C_{h,k=HOG,t} \right) \qquad \forall h, t \tag{49}$$

2.7 Jobs

While many people bemoan the need to have a job, it is generally recognised that the benefits of having a good-quality job extend beyond the income it provides. A good-quality job can provides people with meaning and purpose, independence, social connections, stability, status, and skills development; as well as the means to provide for their economic wellbeing. Being unemployed, i.e. willing to work but not having a job, is generally considered bad for one's wellbeing, creating financial stress, boredom, and negatively affecting self-esteem.

Not all jobs are good quality, and for some people the disadvantages of having even a good-quality job outweigh the advantages. These people may also be able to achieve fulfilment and the other benefits of jobs by other means, and are considered "not in the workforce". This includes people unable to work due to health or disability, parents of young children and other carers, children and full-time students, and retired people.

A good-quality job is one that is well-matched to the worker's skills, pays well, is respected, and contributes meaningfully to society. The hours of work are reasonable (neither too few nor too many, and at an appropriate time of day), the tenure secure (so little or no concern of being fired or made redundant), and the workplace pleasant and no more dangerous that strictly necessary. A poor quality job will be missing many of these attributes. Highly skilled or educated people are better able to obtain good quality jobs, but the number of jobs available is determined primarily by the state of the economy.

All people in the society fit into one of three categories—employed (including part-time), unemployed, or not in the workforce. The unemployed can be further divided into short-term unemployed, who have been unemployed for less than a year, and long-term unemployed. While short-term unemployed people can often find a job on their own in a short time if the general economic conditions are good enough, long-term unemployed people are more likely to suffer from complex issues that make them less employable. Thus, the short-term unemployment rate can be related to the business cycle, but the level of long-term unemployment reflects longer-term issues.

Using these ideas, the proportion of people that are short-term unemployed, long-term unemployed, and not-in-the-workforce can be related to the level of employment in the society. First, if short-term unemployment is determined by the business cycle, then it can be modelled by assuming it is connected to employment by a constant elasticity of substitution:

$$J_{ST,h,t} = A_{J,ST,h} {J_{E,h,t}}^{-1/\eta_{ST,h}} \qquad \forall h, t \tag{50}$$

This assumes that most of the changes in employment due to the business cycle are absorbed by changes in short-term unemployment. The rest of any changes in employment must be absorbed by the long-term unemployed, and those not-in-the-workforce. Both of these groups can be difficult to get into work (for very different reasons), but we assume that the people in these two groups enter or leave the workforce in a fixed ratio $\phi_{J,h}$. This means the long-term unemployment rate is given by:

$$J_{LT,h,t} = \left(1 + \phi_{J,h}\right)^{-1} \left(A_{J,LT,h} - J_{E,h,t} - J_{ST,h,t}\right) \qquad \forall h, t \tag{51}$$

and the proportion of people not-in-the-workforce is given by:

$$J_{NW,h,t} = \left(1 + {\phi_{J,h}}^{-1}\right)^{-1} \left(A_{J,NW,h} - J_{E,h,t} - J_{ST,h,t}\right) \qquad \forall h, t \tag{52}$$

In this version of the model, the highly unrealistic assumption that employment is constant is made for simplicity:

$$J_{E,h,t} = J_{E,h,t=0} \qquad \forall h, t \tag{53}$$

This assumption is often implicitly made in static CGE models that hold labour supply fixed as part of their closure. Later versions of the model will need to link the employment rate to the demand for labour for production.

The employment rate and the long-term unemployment rate comprise two of the indicators for the jobs aspect of wellbeing. The other two indicators are wage rates and a measure of job security. Thus, the index for the jobs aspect of wellbeing is given by:

$$\hat{W}_{JE,h_S,t} = \frac{1}{4}\left(\hat{J}_{E,h_S,t} + \hat{J}_{LT,h_S,t} + \hat{P}_{L,h_S,t} + \hat{\Omega}_{JE,h_S,t}\right) \qquad \forall h_S, t \tag{54}$$

Job security is not modelled in this version of the model, and so

$$\Omega_{JE,h,t} = \Omega_{JE,h,t=0} \qquad \forall h, t \tag{55}$$

As the wages paid by the production sector are adjusted for health and education, this adjustment must be removed:

$$P_{L,h,t} = A_{\tilde{L},h} K_{ESF,h,t}{}^{\eta_{L,ESF,h}} K_{ESS,h,t}{}^{\eta_{L,ESS,h}} K_{HS,h,t}{}^{\eta_{L,HS,h}} P_{\tilde{L},S,h,t} \qquad \forall h, t \tag{56}$$

While the wage rate is important for determining household income, it is also important in its own right as an indicator of job status and quality—both of which typically (but not always) correspond with the wages paid for the work.

2.8 Personal Safety

Personal safety is primarily an indication of the crime (particularly violent crime) level in the society—the lower the level of crime, the greater the level of personal safety. Obviously, personal safety will also be negatively affected for countries involved in war on their territory, but this is a rare event in New Zealand and some other OECD countries. Levels of corruption and white-collar crime have less effect on personal safety than violent crime and some forms of property crime.

People who have a lack of personal safety will usually be living their lives in fear. This has effects on their physical and mental health. It may restrict their activities, indirectly affecting their ability to earn an income, get educated, and generally enjoy life. Trust is reduced, impacting negatively on communities and business.

The average level of personal safety is modelled here as a stock that can change over time:

$$K_{PS,h,t+1} = \left(1 - \delta_{PS,h}\right) K_{PS,h,t} + I_{PS,h,t} \qquad \forall h, t \tag{57}$$

where $\delta_{PS,h}$ is included for consistency with the other captial evolution equations and could be interpreted as the natural decline in personal safety in the absence of any policing or other drivers of change in personal safety. Note that this ignores the distinction between the true level of crime, and the reported level of crime. As most crime is under-reported, but to varying extents, a change in the measured level in crime may reflect a change in the true level and/or a change in reporting patterns. All other changes in personal safety (both improvements and declines) are incorporated into the personal safety investment term $I_{PS,h,t}$.

This personal safety level is the only indicator used for the personal safety aspect of wellbeing:

$$\hat{W}_{PS,h_S,t} = \hat{K}_{PS,h_S,t} \qquad \forall h_S, t \tag{58}$$

The personal safety investment—change in personal safety—is determined by adding the change in each of the factors that affect personal safety, multiplied by an effectiveness parameter for that factor. This assumes that each of these factors act independently and linearly on personal safety. These factors that affect personal safety are the excess need for policing and other crime prevention above the supply of policing and crime prevention services, social connection, net income of households, and the level of formal education. Poor mental health and long-term unemployment increase the risk of crime and therefore decrease personal safety. It is less clear whether short-term unemployment and inequality also decrease personal safety. This results in:

$$\begin{aligned} I_{PS,h,t} =& \xi_{PS,service,h}\left(Q_{PS,h,t} - C_{PS,h,t}\right) + \xi_{PS,IW,h}\left(y_{h,t} - y_{h,t-1}\right) \\ &+ \xi_{PS,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right) + \xi_{PS,HS,h}\left(K_{HS,h,t} - K_{HS,h,t-1}\right) \\ &+ \xi_{PS,SC,h}\left(K_{SC,h,t} - K_{SC,h,t-1}\right) - \xi_{PS,JLT,h}\left(J_{LT,h,t} - J_{LT,h,t-1}\right) \\ &- \xi_{PS,JST,h}\left(J_{ST,h,t} - J_{ST,h,t-1}\right) - \xi_{PS,\Upsilon,t}\left(\Upsilon_{A,IWK,t} - \Upsilon_{A,IWK,t-1}\right) \\ &\forall h, t \end{aligned} \tag{59}$$

Most of these values are determined elsewhere in the model, demand and supply for crime prevention being the exception. Demand for crime prevention is modelled as a function of current personal safety, with safer societies requiring less crime prevention than those that are more dangerous:

$$C_{PS,h,t} = A_{PS,h} {K_{PS,h,t}}^{-\eta_{PS,h}} \qquad \forall h, t \tag{60}$$

This way of modelling crime prevention assumes that demand for crime prevention increases rapidly as personal safety levels decline. Note that it does not take account of the increase in demand for crime prevention due to greater wealth or income which can create higher expectations.

Supply of policing and security, on the other hand, is given by the total of home production (e.g. good parenting, escorting family members at night), market purchases of security and prevention, and government supply of crime prevention services including policing, rehabilitation, and prevention:

$$Q_{k=PS,h,t} = \sum_{h_1} Q_{h_1,h,k=PS,t} + C_{h,k=PS,t} + \frac{C_{G,k=PS,t}}{K_{N,h,t}} \frac{\tilde{S}_{G,k=PS,h,t}}{\sum_{h} \tilde{S}_{G,k=PS,h,t}} \quad (61)$$

Thus, supply and demand of crime prevention services do not necessarily match, and if demand for crime prevention services exceeds supply, this will have a negative effect on personal safety. Note that personal safety is modelled in an analogous way to health.

2.9 Social Connection (Community)

Humans are a social species, and as such good social connections are extremely important to our wellbeing. The quality of family and other relationships are one of the most important factors affecting subjective wellbeing. Good social connections can improve our health, increase our resilience, and increase the likelihood of finding a good quality job. Poor social connection leads to isolation and loneliness as well as making it much harder to get things done.

Like many of the other aspects of wellbeing in this model, social connection is modelled as a stock that can be invested in for improvement (or decline):

$$K_{SC,h,t+1} = (1 - \delta_{SC,h})\, K_{SC,h,t} + I_{SC,h,t} \qquad \forall h, t \quad (62)$$

where $\delta_{SC,h}$ is included for consistency with the other captial evolution equations and could be interpreted as the natural rate of decline in social connection. The social connection aspect of wellbeing is then given by the level of social connection:

$$\hat{W}_{SC,h_S,t} = \hat{K}_{SC,h_S,t} \qquad \forall h_S, t \quad (63)$$

While levels of social connection are significantly affected by factors outside this model, such as friendships, there are also a number of factors included in the model which can also improve or worsen the chances of good social connections. A change in any of these can cause a proportional change in social connection. These factors include health-adjusted leisure time, volunteering, employment, formal education, personal safety, health, and civic engagement. A high population, high immigration levels, or high level of wealth inequality can decrease social connection:

$$
\begin{aligned}
I_{SC,h,t} =& \xi_{SC,WL,h}\left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,h,t-1}\right) + \xi_{SC,TQ,h}\left(T_{Q,h,t} - T_{Q,h,t-1}\right) \\
&+ \xi_{SC,J,h}\left(J_{E,h,t} - J_{E,h,t-1}\right) + \xi_{SC,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right) \\
&+ \xi_{SC,PS,h}\left(K_{PS,h,t} - K_{PS,h,t-1}\right) + \xi_{SC,HS,h}\left(K_{HS,h,t} - K_{HS,h,t-1}\right) \\
&+ \xi_{SC,CG,h}\left(K_{CG,h,t} - K_{CG,h,t-1}\right) - \xi_{SC,N,h}\left(K_{N,h,t} - K_{N,h,t-1}\right) \\
&- \xi_{SC,NM,h} N_{M,h,t} - \xi_{SC,\Upsilon,h}\left(\Upsilon_{A,IWK,t} - \Upsilon_{A,IWK,t-1}\right) \qquad \forall h, t
\end{aligned} \tag{64}
$$

Volunteering is defined here as the home production done for others. This is easy to detect in the model if it is done for different household types, but we also assume that some proportion of home production is for different households of the same type, and thus is considered volunteering:

$$
T_{Q,h,t} = \gamma_{TQ,h} \sum_k T_{Q,h,h_2=h,k,t} + \sum_{h_2 \neq h} \sum_k T_{Q,h,h_2,k,t} \qquad \forall h, t \tag{65}
$$

2.10 Subjective Wellbeing

Subjective wellbeing is measured by life satisfaction. It is broader than happiness, and includes how people feel about their lives as a whole rather than their current emotional state. It is commonly measured on a 0–10 scale, where people are asked to rate their life as a whole where zero represents the worst possible life for them and ten represents the best possible life for them.

Subjective measures can be difficult to model, as they can be heavily influenced by random events and differences between personalities. In particular, the level of life satisfaction can be difficult to predict, especially from objective information. In addition, as this is a wellbeing model, the most obvious way to model life satisfaction would be to model wellbeing. If life satisfaction is an aspect of wellbeing, this becomes circular.

However, it has been shown that positive and negative shocks have ongoing effects on life satisfaction, with negative shocks having a larger effect than the equivalent positive shock. This suggests a way of modelling the change in life satisfaction that is distinct from modelling wellbeing as a whole.

Start by assuming that the current level of life satisfaction is a “neutral” level, determined by culture and temperament, but unaffected by shocks from the past. Then changes from this level can be modelled by the effect of shocks, discounted by how long ago they occurred, and weighted differently based on whether they caused a loss or a gain. This last aspect allows us to take account of loss aversion, where a decrease in income (for example) has a larger effect than an equivalent increase in income.

There are many aspects of life that affect life satisfaction and are included in this wellbeing model. It could be argued that all other aspects of wellbeing should be included, although this could reduce the tractability of the model. As such, given the approach taken to modelling life satisfcation described above, the equation looks for variables that might have a persistent affect (a change in the past continues to affect you today, even if you have returned to your pre-change level), or a strong difference between positive and negative shocks. Health, net income, social connection, formal education level, job security, personal safety, and altruism (measured as time spent volunteering) all improve life satisfaction. Unemployment, time spent commuting, pollution, and whether you are less financially wealthy than your neighbours can decrease life satisfaction. This means the investment in life satisfaction is given by:

$$\begin{aligned}\Lambda_{SW,h,t} =& \Lambda_{SW,h,t=0} + \sum_{\Lambda_p}\sum_{s=0}^{s\leq t} {\beta_{SW,\Lambda_p,h}}^{t-s}\varpi_{\Delta,\Lambda_p,h}\xi_{SW,\Lambda_p,h}\left(V_{HS,\Lambda_p,s} - V_{HS,\Lambda_p,s-1}\right)\\ &- \sum_{\Lambda_m}\sum_{s=0}^{s\leq t} {\beta_{SW,\Lambda_m,h}}^{t-s}\varpi_{\Delta,\Lambda_m,h}\xi_{SW,\Lambda_m,h}\left(V_{HS,\Lambda_m,s} - V_{HS,\Lambda_m,s-1}\right) \qquad (66)\\ &\forall h, \quad t>0, \quad V_{\Lambda_p} \in \left\{K_{EQ}, K_{ESF}, K_{HS}, K_{IW}, K_{PS}, K_{SC}, T_Q, y, \Omega_{JE}\right\},\\ &V_{\Lambda_m} \in \left\{J_{LT}, J_{ST}, K_\Psi, T_{HO}, \Theta_{IWK}\right\}\end{aligned}$$

The comparison of wealth for a household type with the rest of society is given by:

$$\Theta_{IWK,h,t} = K_{IW,S,t} - K_{IW,h,t} \qquad \forall h,t \qquad (67)$$

The indicator for the subjective wellbeing aspect of wellbeing is therefore given by

$$\hat{W}_{SW,h_S,t} = \hat{\Lambda}_{SW,h_S,t} \qquad \forall h_S,t \qquad (68)$$

2.11 Work-Life Balance

We all only have twenty-four hours in a day, but we can choose to use that time in many different ways. Some of that time is spent in activities that are directly pleasurable, and some is spent doing things that are necessary for improving our wellbeing such as working. Work-life balance describes the trade-off between two aspects of modern life—the time desired for leisure and personal care on the one hand, and the time that must be spent working (both paid work and home production), on transportation, and on education in order to live and improve other aspects of wellbeing on the other hand.

Time spent working in a paid job $T_{IW,h,t}$ is determined by the job and the need for income to provide for the economic needs of the household. Time spent on home production $T_{Q,h_1,h_2,k,t}$ is determined by what needs to be done around the house as well as unpaid work for others. Time spent on education $T_{ES,h,t}$ leads to improved skills and qualifications. Time spent on transportation $T_{HO,h,t}$ is determined by the location of the home relative to the place of work and other activities. (Note that time spent on transportation and home production are given exogenously in this prototype model). All these uses of time have limited flexibility, and may cause a "time crunch", where the time left over for leisure and personal care (including sleeping) feels insufficient.

In this model, the time available for leisure and personal care is defined as the time left over after the other activities listed above have taken place, as a proportion of time:

$$T_{WL,h,t} = 1 - T_{IW,h,t} - \sum_{h_2} \sum_{k} T_{Q,h,h_2,k,t} - T_{ES,h,t} - T_{HO,h,t} \qquad \forall h,t \tag{69}$$

As in the rest of the model, these values are averaged across a household, and so we do not distinguish individuals from households.

However, equation (69) takes no account of the ability of individuals to take advantage of that time. Health and social connection, in particular, affect the quality of time available to a person both in their ability to enjoy and make the most of that time, and in the amount of time they must dedicate to personal care and rest. Therefore, we apply adjustments for health and social connection to the amount of leisure time a household has available:

$$\tilde{T}_{WL,h,t} = A_{HS,WL} {K_{HS,h,t}}^{\eta_{WL,HS,h}} {K_{SC,h,t}}^{\eta_{WL,SC,h}} T_{WL,h,t} \qquad \forall h,t \tag{70}$$

This then becomes the sole indicator for the work-life balance aspect of wellbeing:

$$\hat{W}_{WL,h_S,t} = \hat{\tilde{T}}_{WL,h_S,t} \qquad \forall h_S,t \tag{71}$$

3 Supporting Elements of the Model

The eleven aspects of wellbeing described in the previous section do not form a complete model by themselves. There are a number of supporting elements required to complete the model and to allow for a variety of policy and other experiments within the model. This includes a production sector and a government sector, and interactions with the rest of the world.

3.1 The Production Sector

The production sector is an important supporting element of wellbeing for households and general society. The production sector provides the jobs that enable job-related wellbeing. Those jobs, and the use of capital, provide the income for the economic aspect of wellbeing. They also provide most of the goods and services for consumption (again contributing to the economic aspect of wellbeing). Housing, healthcare, transportation, policing and security, and education services are provided by the production sector. Production also affects the environment.

Production is the process that converts a certain set of inputs into a different set of outputs. At the highly abstract level required in this model, the inputs include capital and labour, and production is enabled by technology, personal safety, infrastructure, social connections, and the natural environment. Traditionally, many of these enablers are included in multi-factor productivity, but they are made explicit in this model. It then outputs the desirable consumption goods and services, as well as undesired pollution.

In this model, production is treated similarly to how it would be treated in a static CGE model. In fact, the production sector in combination with the household consumption choices form a simple CGE model, with closure provided by a combination of variables exogenous to the whole model and variables calculated using the other parts of the model, particularly the stock and flow equations.

There are several different divisions in the way production is modelled here. First, goods and services are divided into extracted goods that do not have any intermediate inputs and are not available as consumption goods; and produced (final) goods that use extracted goods as intermediate inputs and are available for consumption. Second, goods and services can be produced in different ways, based on the environmental friendliness of the method of production. Finally, many of the produced goods are important for aspects of wellbeing beyond the economic aspect, such as for housing or healthcare, and these are produced by specific production sectors.

3.1.1 Extraction

Extraction goods are used for intermediate consumption by final production sectors, but are not available for final consumption by households or government.

Conceptually, they include primary production goods like coal, fish, water, and sunlight that are available “for free” from the environment, but must be extracted using labour and capital in the form of machinery. For modelling purposes, we assume that no intermediate inputs from other industries are required to produce extraction goods. Production of extraction goods are enabled by levels of social connection, personal safety, biodiversity, and government-provided infrastructure such as roading. They are also enabled (unlike final goods) by the level of resource available, as the more resource there is, the easier it is to extract. These enablers of production are things that are necessary for production, but are neither used up nor directly paid for by extraction producers. This is illustrated in Figure 2.

Two generic extraction goods are modelled—a flow resource and a stock resource. These are described in more detail in the environment sector above, as both require natural resources and impact on the level of those resources.

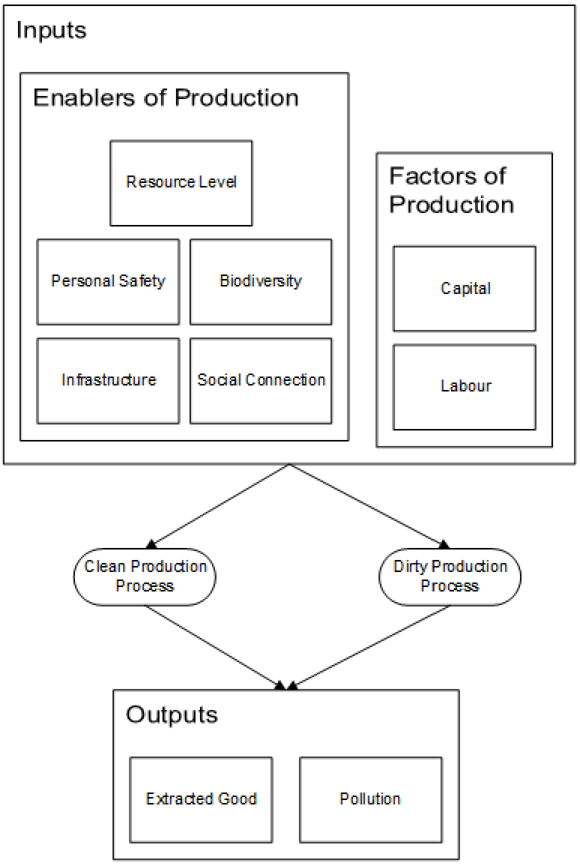

Figure 2 – Production process for extracted goods

3.1.2 Final Production

Final production goods are all other (i.e. excluding extraction) goods and services. They are sold directly to the final consumer, which may be the government or households. They require extraction goods as well as factors of production such as capital and labour for their production. They also require most of the same enablers as extraction production, namely social connection, personal safety, biodiversity, and government-provided infrastructure. This is illustrated in Figure 3.

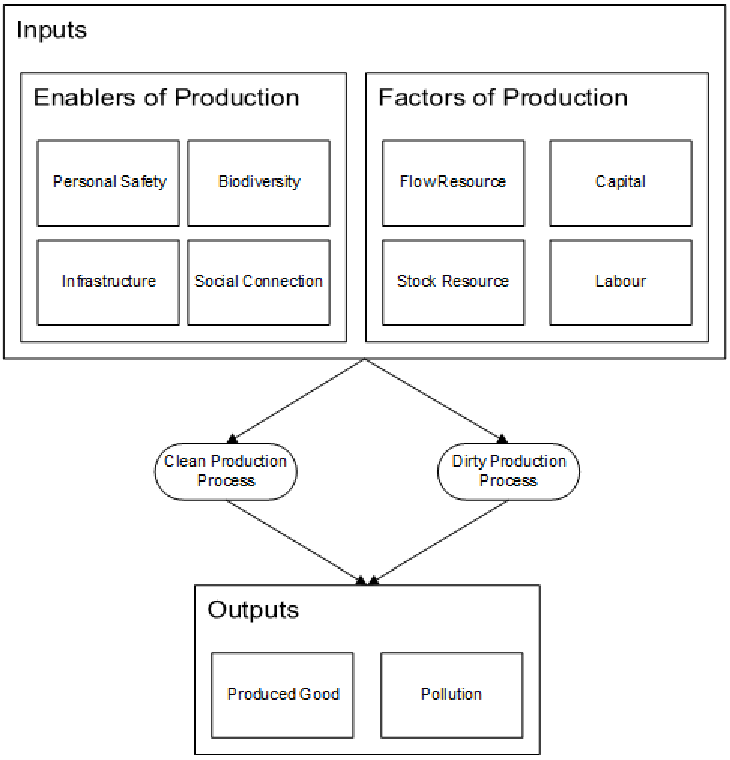

Figure 3 – Production process for final goods

The final goods production has a number of sectors, several of which are used for specific purposes for other parts of the model. Education, healthcare services, transport, policing and security, and good and bad quality housing are all included as final production sectors and described further in the appropriate aspect of wellbeing above. In addition, there is an "other" sector for all other types of goods and services produced for consumption in the economy.

3.1.3 Clean and Dirty Processes

The environmental friendliness of the production process is modelled using "clean" and "dirty" production processes. Any given production process for any particular good sits somewhere on a continuum between "clean", which is low-polluting and generally environmentally friendly (it could even have a positive impact on the environment), and "dirty", which is not. In this model, a dividing line is placed on this continuum, roughly halfway between the two extremes, and all processes on the clean side are grouped as "clean", and all others (on the dirty side) grouped as "dirty". Clean and dirty processes are modelled using the same equations, but the parameters in those equation can take different values, which will be averages of all the different processes in the group. Note that although the dividing line is placed halfway between "clean" and "dirty", this does not necessarily mean that half of production is clean and half is dirty.

3.1.4 Modelling Production

The production process for both extraction goods j and final production goods k is described by a Cobb-Douglas equation including factors of production (labour and capital) and enablers of production (social connection, personal safety, infrastructure, and biodiversity):

$$\begin{gathered} Q_{e,j,t} = A_{Q,e,j,t} \prod_g {V_{g,t}}^{\eta_{g,e,j}} \prod_f {V_{f,e,j,t}}^{\alpha_{f,e,j}} \\ \forall e,j,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{72}$$

Final goods production also includes extraction goods as intermediate inputs:

$$\begin{gathered} Q_{e,k,t} = A_{Q,e,k,t} \prod_g {V_{g,t}}^{\eta_{g,e,k}} \prod_f {V_{f,e,k,t}}^{\alpha_{f,e,k}} \prod_j {V_{j,e,k,t}}^{\alpha_{j,e,k}} \\ \forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{73}$$

We assume that pollution is produced by each process in proportion to the level of production by each process for each extraction or final good q:

$$\Psi_{e,q,t} = \xi_{\Psi,e,q} Q_{e,q,t} \qquad \forall e,q,t \tag{74}$$

Clean processes will produce less pollution per unit of production than dirty processes, but the modelling method is the same for both. Note that a final goods producer could reduce their level of pollution by reducing this parameter (e.g. switching to a cleaner process), but also by reducing the proportion of extraction goods required in production.

The technology used in each production process is assumed to be constant in this version of the model:

$$A_{Q,e,q,t} = A_{Q,e,q,t=0} \qquad \forall e,q,t \tag{75}$$

We also assume all producers price their goods for sale based on the cost of producing those goods:

$$\begin{gathered} P_{Q,e,j,t} Q_{e,j,t} = P_\Psi \Psi_{e,j,t} + \sum_f P_{f,e,j,t} V_{f,e,j,t} \\ \forall e,j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{76}$$

$$\begin{gathered} P_{Q,e,k,t} Q_{e,k,t} = P_\Psi \Psi_{e,k,t} + \sum_f P_{f,e,k,t} V_{f,e,k,t} + \sum_j P_{C,j,t} V_{j,e,k,t} \\ \forall e,k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{77}$$

This assumption will hold true when perfect competition exists, but will understate prices if the situation is closer to a monopoly.

However, goods and services are demanded for consumption, rather than production processes. Thus, the results of clean and dirty production must be combined into a single product for consumption (final production goods) or use in production (extraction goods). As we assume the goods and services created by each production process are imperfect substitutes for each other, a CES (constant elasticity of substitution) function is used to combine them:

$$Q_{q,t} = A_{Q,q}\left(\sum_{e}\gamma_{e,q}Q_{e,q,t}{}^{\frac{\sigma_q-1}{\sigma_q}}\right)^{\frac{\sigma_q}{\sigma_q-1}} \qquad \forall q,t \tag{78}$$

The price of each good is determined by the weighted cost of each method of production:

$$P_{Q,q,t}Q_{q,t} = \sum_{e}P_{Q,e,q,t}Q_{e,q,t} \qquad \forall q,t \tag{79}$$

We have determined the economic environment producers operate in by requiring profits to be zero. The behaviour of the producer within that environment must also be determined. As profits are required to be zero, producers cannot maximise profit. Instead, we assume they maximise production (subject to the profit constraint). This allows us to calculate the demand for both the factors of production and the extraction goods (derivation in Appendix C):

$$\begin{aligned}V_{f,e,q,t} =& \alpha_{f,e,q}P_{f,e,q,t}{}^{-1}\left(\gamma_{e,q}P_{Q,q,t}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} - P_{\Psi}\Psi_{e,q,t}\right)\\ & \forall e,q,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\}\end{aligned} \tag{80}$$

$$\begin{aligned}V_{j,e,k,t} =& \alpha_{j,e,k}P_{C,j,t}{}^{-1}\left(\gamma_{e,k}P_{Q,k,t}Q_{k,t}A_{Q,k}{}^{\frac{\sigma_k-1}{\sigma_k}}\left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} - P_{\Psi}\Psi_{e,k,t}\right)\\ & \forall e,j,k,t\end{aligned} \tag{81}$$

As demand for labour for production is calculated from the level of production, total labour employed by producers for both production and skills development is given by:

$$\tilde{L}_{Q,h,e,q,t} = \frac{V_{f=\tilde{L},h,e,q,t}}{1-\xi_{Q,ESS,h,e,q}} \qquad \forall h,e,q,t \tag{82}$$

Knowing the demand for each factor of production enables the calculation of the amount produced by each type of process:

$$Q_{e,q,t} = Q_{q,t}\gamma_{e,q}{}^{\sigma_q}P_{Q,q,t}^{\sigma_q}A_{Q,q}{}^{\sigma_q-1}X_{e,q,t}{}^{-\sigma_q} \qquad \forall e,q,t \tag{83}$$

where the price factors $X_{e,j,t}$ and $X_{e,k,t}$ are define for notational convenience as:

$$X_{e,j,t} = P_{\Psi}\xi_{\Psi,e,j} + A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}}$$
$$\forall e,j,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \{P_{\tilde{L},Q,h}, P_{C,K}\} \tag{84}$$

$$X_{e,k,t} = P_{\Psi}\xi_{\Psi,e,k} + A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}}$$
$$\times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \tag{85}$$
$$\forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \{P_{\tilde{L},Q,h,e}, P_{C,K,e}\}$$

3.1.5 Market Clearing Conditions

This model also assumes that all markets—both for factors of production and for goods and services—clear. Thus demand equals supply for all factors of production and goods and services, and this is used to determine the price of the factors of production and the quantity of each good or service.

The demand (and therefore supply) for final production goods and services is given by the total consumption demand from households and government:

$$Q_{C,k,t} = \sum_h K_{N,h,t} C_{h,k,t} + C_{G,k,t} \qquad \forall k,t \tag{86}$$

The quantity of extraction goods supplied is the total quantity of extraction goods demanded for use in producing final goods:

$$Q_{C,j,t} = \sum_k \sum_e V_{j,e,k,t} \qquad \forall j,t \tag{87}$$

The total demand for capital by government and firms is given by:

$$K_{IW,C,t} = \sum_e \sum_q V_{f=K,e,q,t} + D_{G,t} \qquad \forall t \tag{88}$$

The price for factors of production can be divided into two groups based on whether the price for the factor of production depends on the process of production. In this version of the model, the price of capital does not depend on the process of production. However, the effective price $P_{\tilde{L},Q,h,e,q,t}$ paid by producers for labour varies by sector and production process, based on what proportion of time the labour spends developing their skills (as described earlier). These prices can be related to the generic price for labour as follows:

$$P_{\tilde{L},Q,h,e,q,t} = \frac{P_{\tilde{L},S,h,t}}{1 - \xi_{Q,ESS,h,e,q}} \qquad \forall h,e,q,t \tag{89}$$

and the generic price of labour is given by:

$$P_{\tilde{L},S,h,t} = \sum_q \sum_e \alpha_{\tilde{L},e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \times \left(K_{N,S,t} \tilde{L}_{S,h,t} \right)^{-1} \qquad \forall t \tag{90}$$

The effective price of capital is the same as the user price of capital for all sectors and production processes:

$$P_{C,K,e,q,t} = P_{C,K,t} \qquad \forall e, q, t \tag{91}$$

3.2 Interactions with the Rest of the World

This model is for a small open economy, and thus interactions with the rest of the world are important. These interactions may take the form of imports and exports of goods and services (both extraction and final), of capital, and of people (migration). There are also other interactions with the rest of the world, most notably the interchange of ideas, that aren't included in this model.

As this model is for a small economy, it is assumed that this economy does not have an effect on international prices (in international currency) or international levels of personal safety etc., and so they are exogenous. For simplicity, they are also chosen to be constant over time:

$$\begin{gathered} V_t = V_{t=0} \qquad \forall t, \\ V \in \{K_{EQ,ROW,h}, K_{ESF,ROW,h}, K_{PS,ROW,h}, K_{\Psi,ROW,h}, P_{L,ROW,h}, P_{ROWM,K}, \\ P_{ROWM,q}, P_{ROWX,K}, P_{ROWX,q}, \tilde{T}_{WL,ROW,h}, \Lambda_{SW,ROW,h}\} \end{gathered} \tag{92}$$

Note that prices paid for imports of goods and capital and prices received for exports of goods and capital need not be the same. However, "exports" and "imports" for wages paid by the rest of the world are not distinguished from each other in this model. The exchange rate is also chosen for simplicity to be exogenous and constant:

$$R_{ROW,t} = R_{ROW,t=0} \qquad \forall t \tag{93}$$

This is in contrast to the floating exchange rate that New Zealand and many other economies use.

The exchange rate can then be used to convert the international prices for imports and exports into local currency:

$$P_{M,b,t} = R_{ROW,t} P_{ROW\,M,b,t} \qquad \forall t, \quad b \in \{j, k, K\} \tag{94}$$

$$P_{X,b,t} = R_{ROW,t} P_{ROW\,X,b,t} \qquad \forall t, \quad b \in \{j, k, K\} \tag{95}$$

and the balance of payments calculated:

$$\begin{aligned} I_{ROW,t} = & R_{ROW,t}{}^{-1} \left(\sum_q P_{X,q,t} Q_{X,q,t} + P_{X,K,t} K_{IW,X,t} \right. \\ & \left. - \sum_q P_{M,q,t} Q_{M,q,t} - P_{M,K,t} K_{IW,M,t} \right) \qquad \forall t \end{aligned} \tag{96}$$

Note that there are no tariffs included in the import prices (or subsidies for exports), as these are minimal in New Zealand.

3.2.1 Imports and Exports of Goods

When an economy produces goods and services (both extraction and final), those goods can either be sold on the domestic market, or exported overseas. Similarly, local consumers can either buy goods produced domestically, or imports. This is shown diagrammatically in Figure 4.

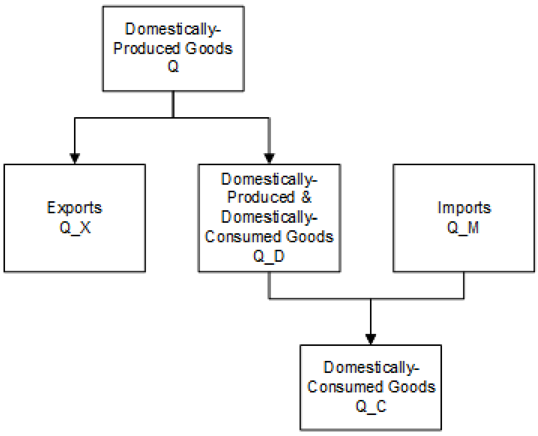

Figure 4 – Imports and Exports of Goods and Services

To determine the level of exports, maximise the revenue of the producers:

$$P_{Q,q,t} Q_{q,t} = P_{X,q,t} Q_{X,q,t} + P_{D,q,t} Q_{D,q,t} \qquad \forall q, t \tag{97}$$

subject to the constant elasticity of transformation (CET) functions:

$$Q_{q,t} = A_{X,q} \left(\gamma_{X,q} Q_{X,q,t}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} + (1 - \gamma_{X,q}) \, Q_{D,q,t}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \qquad \forall q, t \tag{98}$$

The level of exports is then given by:

$$\begin{aligned} Q_{X,q,t} = &\left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{-\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\ &\times {\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} Q_{q,t} \qquad \forall q,t \end{aligned} \tag{99}$$

and the level of (domestically produced) goods and services sold on the domestic market by:

$$\begin{aligned} Q_{D,q,t} = &\left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{-\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\ &\times \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} Q_{q,t} \qquad \forall q,t \end{aligned} \tag{100}$$

From these, the price of domestically-produced goods and services sold domestically is given by:

$$\begin{aligned} P_{D,q,t} = &\left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left(\left(A_{X,q} P_{Q,q,t}\right)^{-\left(\sigma_{X,q}-1\right)} - {\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \\ &\forall q,t \end{aligned} \tag{101}$$

and then the quantity of domestically-produced goods is given by:

$$\begin{aligned} Q_{q,t} = &\left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\ &\times \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} {P_{D,q,t}}^{\sigma_{X,q}} A_{X,q} Q_{D,q,t} \qquad \forall q,t \end{aligned} \tag{102}$$

This then determines the price received for each good:

$$\begin{aligned} P_{Q,q,t} = &{A_{X,q}}^{-1} \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \\ &\forall q,t \end{aligned} \tag{103}$$

Domestically-consumed goods can be sourced from imports or non-exported domestic production, at cost:

$$P_{C,q,t} Q_{C,q,t} = P_{M,q,t} Q_{M,q,t} + P_{D,q,t} Q_{D,q,t} \qquad \forall q,t \tag{104}$$

Assume imports and domestically-produced goods are imperfect substitutes (the Armington assumption). The total goods available for consumption are therefore given by the constant elasticity of substitution (CES) function:

$$Q_{C,q,t} = A_{M,q} \left(\gamma_{M,q} {Q_{M,q,t}}^{\frac{\sigma_{M,q}-1}{\sigma_{M,q}}} + \left(1-\gamma_{M,q}\right) {Q_{D,q,t}}^{\frac{\sigma_{M,q}-1}{\sigma_{M,q}}} \right)^{\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \qquad \forall q,t \tag{105}$$

Minimising the cost (104) subject to the total goods available (105), the level of imports is given by:

$$Q_{M,q,t} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \times \gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \qquad \forall q,t \tag{106}$$

and the level of (domestically produced) good and services sold on the domestic market by:

$$Q_{D,q,t} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \times \left(1-\gamma_{M,q}\right) \sigma_{M,q} P_{D,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \qquad \forall q,t \tag{107}$$

From this, the price paid by consumers for goods and services can be calculated as:

$$P_{C,q,t} = A_{M,q}{}^{-1} \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{\frac{-1}{\sigma_{M,q}-1}} \qquad \forall q,t \tag{108}$$

The full derivation of these equations is given in Appendix C.

3.2.2 Imports and Exports of Financial Capital

Imports and export of financial capital are treated in a similar way to the import and export of production goods and services. As with goods, the domestic supply of capital can be used by domestic firms or invested overseas. Capital demanded for government debt or production can either be supplied from local sources, or received as investment from overseas. This is shown diagrammatically in Figure 5.

To determine the level of exports of capital, maximise the revenue of the suppliers of capital:

$$P_{Q,K,t} K_{IW,S,t} K_{N,S,t} = P_{X,K,t} K_{IW,X,t} + P_{D,K,t} K_{IW,D,t} \qquad \forall t \tag{109}$$

subject to the constant elasticity of transformation (CET) functions:

$$K_{IW,S,t} K_{N,S,t} = A_{XK} \left(\gamma_{XK} K_{IW,X,t}{}^{\frac{\sigma_{XK}-1}{\sigma_{XK}}} + \left(1-\gamma_{XK}\right) K_{IW,D,t}{}^{\frac{\sigma_{XK}-1}{\sigma_{XK}}}\right)^{\frac{\sigma_{XK}}{\sigma_{XK}}-1} \qquad \forall t \tag{110}$$

The level of capital exports is then given by:

$$K_{IW,X,t} = \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-\left(\sigma_{XK}-1\right)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} P_{D,K,t}{}^{-\left(\sigma_{XK}-1\right)}\right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \times \gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-\sigma_{XK}} A_{XK}{}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \tag{111}$$

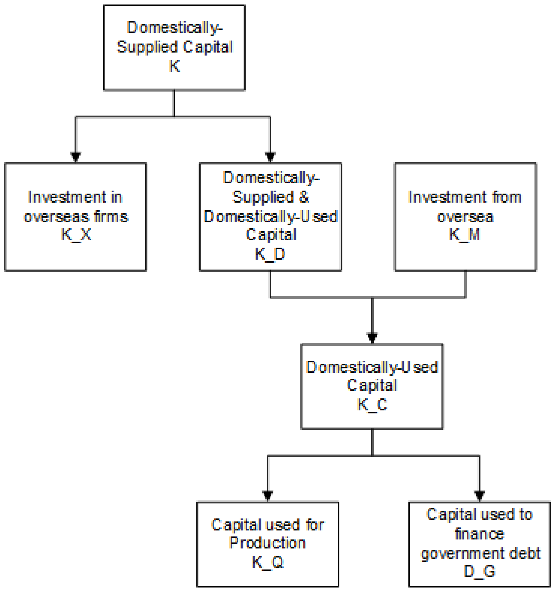

Figure 5 – Imports and Exports of Financial Capital

and the level of (domestically supplied) capital sold on the domestic market by:

$$K_{IW,D,t} = \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-(\sigma_{XK}-1)}\right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \times (1-\gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-\sigma_{XK}} A_{XK}{}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \tag{112}$$

This then determines the price received by domestic suppliers of capital:

$$P_{Q,K,t} = A_{XK}{}^{-1} \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-(\sigma_{XK}-1)}\right)^{\frac{-1}{\sigma_{XK}-1}} \qquad \forall t \tag{113}$$

Domestically-used capital can be sourced from imports or non-exported domestic supply of capital, at cost:

$$P_{C,K,t} K_{IW,C,t} = P_{M,K,t} K_{IW,M,t} + P_{D,K,t} K_{IW,D,t} \qquad \forall t \tag{114}$$

Assume imports and domestically-supplied capital are imperfect substitutes (the Armington assumption). The total capital available for use by government and firms is therefore given by the constant elasticity of substitution (CES) function:

$$K_{IW,C,t} = A_{MK} \left(\gamma_{MK} K_{IW,M,t}{}^{\frac{\sigma_{MK}-1}{\sigma_{MK}}} + (1-\gamma_{MK}) K_{IW,D,t}{}^{\frac{\sigma_{MK}-1}{\sigma_{MK}}}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \qquad \forall t \tag{115}$$

Minimising the cost (114) subject to the total goods available (115), the level of capital imports is given by:

$$K_{IW,M,t} = \left(\gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)} + (1-\gamma_{MK})^{\sigma_{MK}} P_{D,K,t}{}^{-(\sigma_{MK}-1)}\right)^{-\frac{\sigma_{MK}}{\sigma_{MK}-1}} \times \gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-\sigma_{MK}} A_{MK}{}^{-1} K_{IW,C,t} \qquad \forall t \tag{116}$$

and the level of (domestically supplied) capital sold on the domestic market by:

$$P_{D,K,t} = (1-\gamma_{MK})^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \left((A_{MK} P_{C,K,t})^{-(\sigma_{MK}-1)} - \gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)}\right)^{\frac{-1}{\sigma_{MK}-1}} \qquad \forall t \tag{117}$$

From this, the price paid by users of capital can be calculated as:

$$P_{C,K,t} = (1-\gamma_{MK})^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} P_{D,K,t} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}}\right)^{\frac{1}{\sigma_{MK}}} \qquad \forall t \tag{118}$$

The full derivation of these equations is given in Appendix C.

3.2.3 Migration

Migration is treated somewhat differently to imports and exports of capital and goods, as the decisions for migration are rarely made on purely economic grounds, and it can take some time for households to move between countries.

There are many reasons why a household may wish to move from one country to another. These include higher wages or better job opportunities, better lifestyle, educational opportunities, and reuniting with family. New Zealand traditionally has an advantage in lifestyle, and a disadvantage (relative to other developed countries) in wages and job opportunities. As such, people tend to leave New Zealand for job opportunities (the traditional OE (Overseas Experience) being a partial exception), and people tend to come (or return) to New Zealand for greater personal safety, a higher quality natural environment, education, work-life balance, and general subjective wellbeing.

Emigration can therefore be modelled as proportional to the difference between wages overseas and in New Zealand:

$$N_{X,h,t} = \xi_{NX,PL,h} \left(R_{ROW,t} P_{L,ROW,h,t} - P_{L,h,t}\right) \qquad \forall h,t \tag{119}$$

While it is generally expected that international wages will be higher than New Zealand wages, if this is not the case, negative emigration (i.e. immigration) may occur, particularly with New Zealanders returning home.

Similarly, immigration can be modelled as proportional to the difference in lifestyle factors:

$$\begin{aligned}N_{M,h,t} =& \xi_{NM,PS,h} \left(K_{PS,h,t} - K_{PS,ROW,h,t}\right) + \xi_{NM,EQ,h} \left(K_{EQ,h,t} - K_{EQ,ROW,h,t}\right) \\ &+ \xi_{NM,ESF,h} \left(K_{ESF,h,t} - K_{ESF,ROW,h,t}\right) + \xi_{NM,SW,h} \left(\Lambda_{SW,h,t} - \Lambda_{SW,ROW,h,t}\right) \\ &+ \xi_{NM,WL,h} \left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,ROW,h,t}\right) - \xi_{NM,\Psi,h} \left(K_{\Psi,h,t} - K_{\Psi,ROW,h,t}\right) \\ &\forall h,t \end{aligned} \tag{120}$$

Again, it is assumed that New Zealand will be better than the rest of the world in these aspects of wellbeing, but if this is no longer the case negative immigration (i.e. emigration) may occur.

The change in the number of households of any given type is therefore given by the level of net migration:

$$I_{N,h,t} = N_{M,h,t} - N_{X,h,t} \qquad \forall h,t \tag{121}$$

Note that we have assumed for simplicity that migrants have the same characteristics as their household type.

3.3 Home Production

Not all production is done by firms, or is part of the formal economy. There are many tasks that households routinely perform for themselves, that would not be described as leisure. Instead, they contribute to the household's standard of living and can substitute for consumption of market goods and services. Cooking, cleaning, childcare, and many other household tasks that were traditionally the domain of housewives fit into this category; as does gardening and home maintenance work not purchased as market services.

Recipients of home production can be divided into two groups—those in the same household as the producer, and those in a different household to the producer. In both cases, this adds to the wellbeing of the recipient household. In the case where the recipient is in a different household to the producer, it also strengthens communities and improves the life satisfaction of the producer through a sense of altruism.

The main inputs into home production are time and skill. Some materials may also be needed, for example food for cooking, but these are assumed to be part of the household's market consumption. Coming out of home production are consumption goods and services that are substitutes for market goods and services. Home production is modelled using a simple one-input Cobb-Douglas production function:

$$Q_{h_1,h_2,k,t} = A_{h_1,k} \tilde{T}_{Q,h_1,h_2,k,t}^{\eta_{Q,T,h_1,k}} \qquad \forall h_1, h_2, k, t \tag{122}$$

Health- and education-adjusted time input into home production activities $\tilde{T}_{Q,h_1,h_2,k,t}$ is the main factor determining the level of home production output $Q_{h_1,h_2,k,t}$, and the only factor that can change over time. The effect on home production of putting in more time is given by the elasticity of home production with respect to time $\eta_{Q,T,h_1,k}$. This is likely to be close to one, as typically putting more time into say cooking will result in a proportional increase in cooked meals. Skill $A_{h_1,k}$ defines how many cooked meals can be made in a given amount of time and is assumed to be constant over time, but could be different for different household types.

Time put into home production activities is adjusted for health, as people in good health can generally perform home production tasks more efficiently than those in poor health, who may for example need to pause during the activity to rest. It is also adjusted for education and skill levels (as a proxy for intellect or ability to learn), as again people with high education and skills are more likely to be able to perform home production tasks more efficiently, just as their labour productivity is higher. These adjustments are modelled as follows:

$$\begin{aligned} \tilde{T}_{Q,h_1,h_2,k,t} =& K_{HS,h_1,t}^{\eta_{TQ,HS,h_1,k}} K_{ESF,h_1,t}^{\eta_{TQ,ESF,h_1,k}} K_{ESS,h_1,t}^{\eta_{TQ,ESS,h_1,k}} \\ & \times A_{HS,TQ,k} T_{Q,h_1,h_2,k,t} \qquad \forall h_1, h_2, k, t \end{aligned} \tag{123}$$

In order to calculate home production, we therefore need to know how much time is dedicated to home production of each type of production good k by household h_1 for household h_2 (which may or may not be the same as h_1) in each time period t. This is a behavioural choice for the household, and for modelling simplicity it is treated as exogenous and constant:

$$T_{Q,h_1,h_2,k,t} = T_{Q,h_1,h_2,k,t=0} \qquad \forall h_1, h_2, k, t \tag{124}$$

In a future version of the model, skills could also be developed through home production, in a similar way to the development of skills in market production.

3.4 Government

The government makes up a large component of the economy, and it has the ability to influence much of the rest of the economy and society more broadly. It is also tasked with running the country. In a democracy, governments that do a better job of running the country well—presumably by at least maintaining the wellbeing of its citizens—are more likely to be re-elected. Thus, governments are motivated to improve the wellbeing of their citizens, and have a number of tools to achieve this. This means that including a government sector in this wellbeing model is highly important, and even more so as the model is intended for use in policy analysis by government officials!

3.4.1 Fiscal Policy

The main role of the government in the model is to raise income through taxes for spending and redistribution. Three types of taxes are collected in the model: income tax, consumption tax, and pollution tax:

$$Y_{G,t} = \sum_{h} \tau_{Y,h} K_{N,h,t} Y_{h,t} + K_{N,S,t} \sum_{k} \tau_{C,k} P_{C,k,t} C_{S,k,t} + P_{\Psi} \sum_{e} \sum_{q} \Psi_{e,q,t} \qquad \forall t \quad (125)$$

Note that this does not include any form of company or production tax. The government uses this income for spending, investing, and providing transfers back to households. It must also pay interest on any debt it owes. However, it need not maintain a balanced budget and in any given year it can lend or borrow to make up the difference between income and expenditure.

For the purposes of this model, the government requires government debt to be maintained at a fixed ratio of GDP:

$$D_{G,t+1} = \kappa Y_{GDP,t} \qquad \forall t \quad (126)$$

where GDP is calculated as:

$$Y_{GDP,t} = \sum_{k} P_{C,k,t} Q_{C,k,t} + \sum_{k} P_{X,k,t} Q_{X,k,t} - \sum_{k} P_{M,k,t} Q_{M,k,t} \qquad \forall t \quad (127)$$

This is a rigid version of the previous government's aspiration to have net debt at about 20% of GDP in 2020 (Treasury, 2017). It means total expenditure, on spending, investment, interest, and transfers, must be given by:

$$E_{G,t} = Y_{G,t} + (D_{G,t+1} - D_{G,t}) \qquad \forall t \quad (128)$$

3.4.2 Spending

The government spends money for a variety of different purposes, and part of the government budget process is deciding how to divide the available funds. In this version of the model, the government can spend money on decreasing pollution, increasing biodiversity, increasing personal safety, on providing healthcare and formal education services, and on other consumption goods and services. These are decided exogenously, and are kept constant. Most of these are decided at the societal level:

$$S_{G,a,t} = S_{G,a,t=0} \qquad \forall a, t \tag{129}$$

However, education, health, and personal safety spending is broken down by the household type that receives the services:

$$S_{G,k,h,t} = S_{G,k,h,t=0} \qquad \forall t, \quad k \in \{ESF, HS, PS\} \tag{130}$$

Total spending is given by:

$$S_{G,t} = \sum_a S_{G,a,t} + \sum_{k \in \{ESF,HS,PS\}} \sum_h S_{G,k,h,t} \qquad \forall t \tag{131}$$

3.4.3 Government Wealth

The government holds a significant level of wealth, and this is assumed to be physical wealth in the form of infrastructure, which is an enabler of production. As with other stocks in the model, infrastructure levels change according to the standard evolution equation:

$$K_{G,t+1} = (1 - \delta_G) K_{G,t} + I_{G,t} \qquad \forall t \tag{132}$$

There are many different choices governments could make for investment in infrastructure. Here, we assume that infrastructure investment occurs at a rate that counteracts depreciation and allows for population growth:

$$I_{G,t} = (\delta_G + r_{K,G,t}) K_{G,t} \qquad \forall t \tag{133}$$

where:

$$r_{K,G,t} = \frac{K_{N,S,t} - K_{N,S,t-1}}{K_{N,S,t-1}} \qquad \forall t \tag{134}$$

This essentially keeps the level of infrastructure constant relative to population. This provides for a baseline level of maintenance, but does not take into account common variations around this that are typically due to more ad-hoc decisions by the government. Equation (134) could easily be replaced if a different policy for infrastructure investment were to be modelled.

3.4.4 Transfers

Transfers are the cash and in-kind payments made to households usually to redistribute wealth within a society or to alleviate poverty. These are modelled in two groups. First are the transfers that could be defined as social welfare payments that vary according to the conditions in the economy and society. They include unemployment benefits (paid to all who are unemployed), and other benefits such as government-provided superannuation and benefits to people who cannot work:

$$\Gamma_{G,t} = \Gamma_J \sum_h K_{N,h,t} \left(J_{ST,h,t} + J_{LT,h,t} \right) + \Gamma_{NW} \sum_h \gamma_{NW,h} K_{N,h,t} J_{NW,h,t} \qquad \forall t \quad (135)$$

The remaining transfers are calculated as a residual so that the government can meet its fiscal policy:

$$\Gamma_{\tau,t} = E_{G,t} - S_{G,t} - I_{G,t} - P_{C,K,t} D_{G,t} - \Gamma_{G,t} \qquad \forall t \quad (136)$$

They could be thought of as spending in other categories not included above that act like a cash payment to households.

3.4.5 Consumption

When the government spends money on investment or providing services, it is essentially purchasing consumption goods. The spending is first modified by its effectiveness, due to civic engagement and other (unspecified but constant) reasons:

$$\tilde{S}_{G,a,t} = A_{G,a} {K_{CG,S,t}}^{\eta_{CG,a}} S_{G,a,t} \qquad \forall a, t \quad (137)$$

Note that spending directed at different household types may have different levels of effectiveness due, for example, to lobbying:

$$\tilde{S}_{G,k,h,t} = A_{G,k,h} {K_{CG,h,t}}^{\eta_{CG,k,h}} S_{G,k,h,t} \qquad \forall t, \quad k \in \{ESF, HS, PS\} \quad (138)$$

The amount of consumption is defined by the effectiveness-adjusted amount of spending on that final good or service and the price of it. Adjusted spending on each type of consumption good is given by:

$$\tilde{S}_{G,k,t} = \begin{cases} \sum_h \tilde{S}_{G,k,h,t} & k \in \{ESF, HS, PS\} \\ \tilde{S}_{G,\Psi,t} + \tilde{S}_{G,EQ,t} + \tilde{S}_{G,other,t} & k \in \{other\} \\ \tilde{S}_{G,a=k,t} & k \in \{HOT, HOG, HOB\} \end{cases} \qquad \forall t \quad (139)$$

For investment, it is assumed the government gets "utility" from this investment that they try to maximise. This utility is given by a Cobb-Douglas function:

$$U_{C,G,t} = \prod_k {C_{G,I,k,t}}^{\alpha_{C,G,k}} \qquad \forall t \quad (140)$$

In maximising utility, the government faces a budget constraint based on the overall level of investment:

$$I_{G,t} = \sum_{k} P_{C,k,t} C_{G,I,k,t} \qquad \forall t \tag{141}$$

Note that the government does not pay consumption tax. This results in government consumption from investment given by:

$$C_{G,I,k,t} = \frac{\alpha_{C,G,k} I_{G,t}}{P_{C,k,t}} \qquad \forall k, t \tag{142}$$

The total consumption of final goods and services by the government for both investment and adjusted spending is therefore given by:

$$C_{G,k,t} = \frac{\tilde{S}_{G,k,t}}{P_{C,k,t}} + \frac{\alpha_{C,G,k} I_{G,t}}{P_{C,k,t}} \qquad \forall k, t \tag{143}$$

3.5 Aggregation for Society

While most variables in this model are intended primarily for use at a household level, there are a number of situation where it is necessary or desirable to aggregate to an all-of-society level. This includes when households interact with production or the government; when studying inequality and other interactions between household types; and when looking at wellbeing for society as a whole.

3.5.1 General Aggregation

Most societal variables are defined as the weighted average of the value for each household type:

$$\begin{gathered} V_{S,t} = \sum_{h} \frac{K_{N,h,t}}{K_{N,S,t}} V_{h,t} \qquad \forall t, \\ V \in \{C, C_{HO}, J_E, J_{LT}, K_{CG}, K_{ESF}, K_{ESS}, K_{HS}, K_{SC}, \\ P_{HO}, P_L, T_{HO}, \tilde{T}_{WL}, Y_h, y, \Lambda_{SW}, \Xi_{HO}, \Omega_{JE}\} \end{gathered} \tag{144}$$

To calculate this, we need to know the total number of households in the society:

$$K_{N,S,t} = \sum_{h} K_{N,h,t} \qquad \forall t \tag{145}$$

The total number of households is changed by natural growth $\delta_{N,h}$ and by net migration $I_{N,h,t}$:

$$K_{N,h,t+1} = (1 - \delta_{N,h}) K_{N,h,t} + I_{N,h,t} \qquad \forall h, t \tag{146}$$

3.5.2 Inequality

As this model has multiple household types, it is also possible to study the changes in inequality between household types. The household types that are modelled are determined by the modeller and the data they have available. In principle, each household in the country could be modelled as a separate household type in this framework. In practise, the data and computational power required for this degree of disaggregation is not readily available. More likely, the modeller would choose a small number of household types (say 5), divided along a dimension of interest. This could be a demographic dimension like ethnicity or family structure; or it could be a wellbeing dimension such as education level or income level. Note that if households within a household type are not identical (and this will always be the case if there are a small number of household types), the overall inequality will be under-represented when just looking at the inequality between household types.

It is then necessary to decide what inequality should be considered. Commonly, income inequality (and particularly gross income inequality) is studied, as the relevant statistical data is easiest to collect. Wealth inequality is also often considered when appropriate data is available. Less frequently considered is inequality based on other measures such as wellbeing, partly because there is little or no data available that can be used for this purpose. However, income, wealth, and wellbeing inequality are all relevant in understanding the wellbeing and living standards of a population. As such, the model calculates inequality measures for gross and net income, household wealth, and wellbeing by default. It can also calculate inequality based on any other variable that has different values for each household type.

There are many different ways of measuring inequality, and each show different things and take into account different values. Thus, it is useful to use more than one measure. Some measures, however, cannot generally be calculated in the model, such as the 80/20 ratio and other measures dependent on knowing value at certain population percentiles. Measures that incorporate the whole population can be calculated in this model, with the caveat that it only represents the inequality between household types and ignores the inequality within household types. This problem reduces as the number of household types increases, but increasing the number of household types causes other problems, particularly around data and computational complexity.

Perhaps the most famous whole-of-population measure is the Gini coefficient, given by (weighted unordered form):

$$\Upsilon_{G,m,t} = \frac{\sum\limits_{h_1} \sum\limits_{h_2>h_1} K_{N,h_1,t} K_{N,h_2,t} \left| V_{m,h_1,t} - V_{m,h_2,t} \right|}{K_{N,S,t} \sum\limits_{h_3} K_{N,h_3,t} V_{m,h_3,t}} \qquad (147)$$

$$\forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\}$$

Other inequality measures calculated in the model are the Theil measure:

$$\Upsilon_{T,m,t} = \sum_h \frac{K_{N,h,t}}{K_{N,S,t}} \frac{V_{m,h,t}}{V_{m,S,t}} \ln \left(\frac{V_{m,h,t}}{V_{m,S,t}} \right) \qquad \forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\} \qquad (148)$$

and the Atkinson index:

$$\Upsilon_{A,m,t} = \begin{cases} 1 - \frac{1}{V_{m,S,t}} \left(\frac{1}{K_{N,S,t}} \sum\limits_h K_{N,h,t} {V_{m,h,t}}^{1-\epsilon_\Upsilon} \right)^{\frac{1}{1-\epsilon_\Upsilon}} & \epsilon_\Upsilon = 1 \\ 1 - \frac{1}{V_{m,S,t}} \left(\prod\limits_h {V_{m,h,t}}^{K_{N,h,t}} \right)^{\frac{1}{K_{N,S,t}}} & \epsilon_\Upsilon \geq 0, \quad \epsilon_\Upsilon \neq 1 \end{cases} \qquad (149)$$

$$\forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\}$$

The Atkinson measure includes a parameter ϵ_Υ that indicates aversion to inequality, with larger values of ϵ_Υ indicating a greater aversion to inequality.

3.6 Calculation of Indexes

In this model, variables are kept in their “natural units”. These are the units that they are usually measured in, for example, New Zealand dollars for income, time-proportion for time (convertible to hours per day by multiplying by 24; or days per year by multiplying by 365), and proportion for level of employment. This means that each variable in the model has its own units, which may or may not be shared with other variables.

Using natural units makes it easier to interpret the results, and it avoids problems such as finding shadow prices to express all variables in dollar terms. Most of the results in this model, including the most meaningful results, are calculated using natural units. However, (although not important for the core model) we are left with the challenge of how we compare different aspects of wellbeing in a meaningful way. Which is bigger, if income has increased by $1,000 per year and leisure time by 1 hour per day?

To get around this problem, all variables that are used as indicators of wellbeing are calculated both in their natural units and as an index. The indexes can then be compared to one another and used in calculations that combine them. This then creates the next problem: How should the indexes be calculated?

There are many different ways indexes can be calculated, some of which are discussed in an OECD handbook (OECD, 2008). For the purposes of this model, there are some particular features that it would be desirable for the indexes to exhibit, and some that are unimportant. The important features are as follows:

- Shows changes consistently over time: The variables in this model evolve over time, and it is important to be able to tell how things are changing in time.
- Shows differences between household types consistently: Different types of households may respond to changes in different ways, and this needs to be visible to compare the effects of wellbeing on the different household types.
- Both good and bad effects move in the same direction: Indexes are easier to combine and interpret if up always means better. However, for some variables such as pollution, an increase is a bad thing. For such variables, the index should increase if the value of the variable decreases.
- Relative changes should only appear if there are also absolute changes: This means the relative values should be calculated by comparing to fixed values.
- Uses only data available in the model: Data from external sources can be hard to come by for this model.

International comparability is unimportant for this model, as the results in this model are only applied to one country, and no other country is calculating the same thing. To calculate the indexes in this model, they are calculated as the percentage improvement from a particular base value. Mathematically, this is represented by:

$$\hat{V}_t = \begin{cases} \frac{V_t}{V_B} & V_t \in \{C_{h_S}, C_{HO,h_S}, J_{E,h_S}, K_{CG,h_S}, K_{EQ,h_S}, K_{ESF,h_S}, K_{ESS,h_S}, K_{HS,h_S}, \\ & \quad K_{IW,h_S}, K_{PS,h_S}, K_{SC,h_S}, P_{L,h_S}, \tilde{T}_{WL,h_S}, y_{h_S}, \Lambda_{SW,h_S}, \Xi_{HO,h_S}, \Omega_{JE,h_S}\Big\} \\ \frac{V_B}{V_t} & V_t \in \{J_{LT,h_S}, K_{\Psi,h_S}, P_{HO,h_S}, T_{HO,h_S}\} \end{cases}$$
$$\forall h_S, t \tag{150}$$

The base V_B is given by the societal value of that variable at time $t = 0$. In the usual situation where the results of the model are given as the difference between a business-as-usual case and a scenario, the base value will always be the business-as-usual value at time $t = 0$. Using this method, an index value of

1 means the variable is the same as the base. An index value of 1.1 means the variable is 10% better (not necessarily larger!) than the base; and an index value of 0.9 means it is 10% worse than the base.

As a single base is used for all time periods, this method shows changes consistently over time. By using the same base for different household types, the differences between them can be distinguished and are treated consistently. Because the inverse calculation is used for variables where an increase is bad for wellbeing, an increase in the index always represents an increase in wellbeing. The choice of a fixed base means changes in the index will only appear if the value of the underlying variable changes. Finally, the choice of base described here is calculated as a fundamental part of the model. Thus this method satisfies the important features required for an index in this model.

It is worth noting however that using this method, all declines in wellbeing (relative to the base) will result in an index between 0 and 1, while improvements in wellbeing will have an index greater than 1 but not necessary smaller than 2, or 10, or 100 or more. An index representing something that is half as good as the base will have a value of 0.5, whereas something twice as good will have an index of 2. This runs the risk that improvements in wellbeing appear overstated compared to declines in wellbeing when using indexes, and is another reason why natural units should be emphasised over index values. However, as the indexes are not used in the core calculations of the model, this issues does not invalidate the model.

This method does have another significant weakness. No account is taken of the level of variability in the value of the variable. Some variables have a large amount of room to move and change, often with open-ended values e.g. income. Others are more constrained, such as the percentage of good quality housing, and as well as only being able to move between theoretical limits (e.g. 0% and 100%) may in practise be even more constrained. This could be improved by including standard deviation or some other measure of spread into the index calculation method, but this would require data not available in the model, violating one of the important criteria above.

3.7 Calculation of Wellbeing

Wellbeing is a heterogeneous concept, made up of many different aspects. Any person or household will put a unique weighting on each aspect that may change over time. Thus, many different indexes for wellbeing have been proposed (as described in the introduction), but there is no “best” index for all people, or even a large group of people. All such indexes require value judgements unique to any individual. For this reason, the emphasis on the model results should be on the indexes for each aspect of wellbeing and the values that are used to calculate them.

However, there is some value in calculating an overall value for wellbeing. In particular, it can provide a consistent way for assessing different policies which contain different trade-offs, provided the results are taken as approximate and the reasons for the results understood.

The choice of mathematical form for combining the aspects of wellbeing is in many respects arbitrary. However, it is desirable that it had diminishing increases in wellbeing from increases in an aspect of wellbeing, and that any aspect reaching zero is catastrophically bad. While a number of equational forms might meet these criteria, we will chose a weighted geometric mean (a standard Cobb-Douglas-style equation) for period wellbeing:

$$\hat{W}_{h_S,t} = \prod_i \hat{W}_{i,h_S,t}{}^{\alpha_{i,h_S}} \qquad \forall i, h_S, t \tag{151}$$

In addition to the overall wellbeing calculated for each time period, an intergenerational wellbeing value (i.e. wellbeing aggregated over time) can be calculated to aid comparisons. People may be willing to sacrifice some wellbeing now for greater wellbeing in the future, and consider their overall wellbeing throughout their life to be greater for that sacrifice. Note however that intergenerational wellbeing is even less meaningful than the overall period wellbeing calculated for equation (151), as it retains all the problems of that value, then adds in the problems of time discounting, which are also driven by value judgements unique to any individual.

Typically, good things in the future are rated less highly than good things now for a number of reasons including the future is uncertain. This implies that the future is discounted, and we assume it is discounted at a constant rate. We assume that we can write intergenerational wellbeing as the discounted sum of the wellbeing in each period. However, this does imply that the wellbeing of people in the distant future is discounted to almost zero and therefore unimportant relative to now—sometimes known as the dictatorship of the present. Chichilnisky (1997) proposes a modification to the standard discount equation that corrects for this effect, by adding a non-zero term in the limit $t \to \infty$. Using this, the intergenerational wellbeing (in terms of period wellbeing) is given by:

$$\hat{W}_{h_S} = \omega_{h_S} \sum_{t=0}^{\infty} \beta_{h_S}{}^t \hat{W}_{h_S,t} + (1 - \omega_{h_S}) \lim_{t \to \infty} \hat{W}_{h_S,t} \qquad \forall h_S \tag{152}$$

An alternative method that prevents the dictatorship of the present and may bear a closer relationship to reality is hyperbolic discounting, where the discount rate β_{hs} is not constant over time but instead depends on how far away a given time period is from the current time period.

4 Conclusions

In summary, this paper presents the first prototype of a model of wellbeing. This model is designed to support policy development and decisions in New Zealand by providing a tool for thinking about changes in aspects of wellbeing including trade-offs and synergies for policies. It does this by modelling the linkages between the eleven aspects of wellbeing used by the OECD in their How's Life? framework, in combination with a small general equilibrium model and stock-and-flow equations. A model of this nature is a new addition to the policy modelling toolkit.

It is clear that this model, perhaps more than most, has many flaws and unrealistic assumptions. However, it is the first attempt at such a model, and its existence has several uses. Firstly, it shows that such a model is possible, and with a level of complexity that, if not low, is low enough to be manageable and no higher than many CGE (computable general equilibrium) models used for policy purposes. Secondly, it provides a basis for criticism and improvement. It is far easier to criticise that which exists, than build a model from scratch; and from constructive criticism the model can evolve into one that is much better than the current model or another built from scratch. Third, despite its flaws, it is useful for policy analysis as it provides a tool for thinking about the trade-offs and synergies between different aspects of wellbeing and policy that currently does not exist.

Given the prototype nature of the model described in this paper, there is clearly room for significant further work. The model and the results it gives should undergo serious critical analysis to understand how the model behaves, what assumptions are critical, and where the model gives unrealistic and unhelpful results. These results will help inform where (and how) improvements to the model can be made and where they will be most important.

Some improvements and extensions to the model can easily be identified now. To bring the model into alignment with the current version of the Living Standards Framework a culture aspect of wellbeing will need to be added. Both employment and household consumption are exogenous in this version of the model, but in reality are driven largely by factors inside the model, such as income levels. Housing is treated as a consumption good, but house prices (particularly pertinent for Auckland) and the housing aspect of wellbeing in general will be better represented if it is treated as a durable good. Including demographics in the model will enlarge the range of policies it can be used for and allow a better understanding of sustainability in all its forms. Other improvements and extensions will be identified through disseminating this work widely and further study of the literature in the fields represented by each aspect of wellbeing and supporting parts of the model.

Data will be a challenge for this model, as it requires a large number of parameters and initial values. Many of these (especially the standard economic values) are available from Stats NZ and other appropriate sources; or can be calibrated from such data. Others may need to be proxied from international data, and some will be unavailable from reliable sources. All will need to be tested with sensitivity analyses, and for those that are completely unavailable this will be crucial. Note that as results will often be given qualitatively and relative to a baseline, the results might not be as sensitive to the parameters as might be expected at first. However, these data challenges need to be explored further before we can have confidence that we can use a computable version of the model.

As with all models, this model can be used and it can be abused. Models (and especially this one) are used well when they are used as a tool for thinking and for testing assumptions. They are abused when the results of a single model are used with spurious precision to produce the "right" answer without regard to the assumptions in the model. More specifically, if this model is used to create a chain of logic to explore the aspects of wellbeing that will be affected by a policy, it will be used appropriately. If it is used to make a precise numerical prediction of the effect on wellbeing of a given policy, it will be being abused. However, calculating values to represent the complex interactions between the different aspects of wellbeing makes it easier to determine the qualitative results of the model, and so are an important part of the process.

There is also much to learn about policy by applying the model. Without a model of this type, the second and third and higher order effects (particularly those in different policy areas to the original) are very difficult to anticipate or attribute to their original cause. This model allows all those effects to be recognised, as well as obtain some understanding of how significant they are. For example, if an increase in health spending by the government is modelled, the model results indicate a small negative effect on the natural environment could occur. This is not an intuitively obvious result, but tracing through the model logic and results shows it is not a spurious result: more health spending results in better health, better health results in higher productivity, higher productivity results in more consumption and production, more production results (all else equal) in more pollution and therefore a poorer natural environment. This is a chain of impacts that is too long for most people to perform mentally without the aid of a model, but the consequences could be important. Any of the parameters and exogenous variables in the model can be shocked by a policy change. Shocks can also be added to many other variables, especially the investment variables (for example, the destruction of government-owned physical capital by the 2016 Kaikoura earthquake).

The model can be used for both single policies and for policy packages. In the case of policy packages, it can be used to see how the effects of different policies balance each other out. It can also be used to help determine which policies should be included in a package. It helps this in two ways. Firstly, it provides a consistent framework for policies from different fields, e.g. education and health, to be compared with each other and in a broader way than tools such as cost-benefit analysis allow. Secondly, policies can be chosen on their ability to mitigate some of the undesirable trade-offs (such as the negative environmental effects of a health policy) inherent in the choice of policies that are the cornerstones of a package.

As mentioned at the beginning of this paper, Sen said “It is possible to be at once deeply appreciative and seriously critical of a theory” (2009, p.58). This paper describes a model that, as a prototype, has many flaws but provides a framework for improvement. The author expects, and invites, serious criticism of this model; but also hopes that the step forward it represents can be appreciated and is useful.

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A Notation

In this appendix are the definitions of every variable and parameter used in the model, as well as an explanation of some of the notation conventions used.

A.1 Subscripts and Generic Notation

The symbols in this tables refer to the given sets in all equations unless noted otherwise.

Table 1 – Sets

Symbol and Set	Description
$a \in \{\Psi, EQ, HOT, HOG, HOB, other\}$	Nationwide government spending sector
$e \in \{clean, dirty\}$	Production types
$f \in \{K, L_h\}$	Factors of production
$h \in \{HH1, HH2, ...HHn_H\}$	Household types
$h_S \in \{h, S\}$	Household types plus society
$i \in \{CG, SC, IW, ES, EQ, HS, HO, JE, SW, PS, WL\}$	Aspects of wellbeing
$j \in \{EQF, EQS\}$	Resource/extraction good types
$k \in \{HOT, ESF, HS, PS, HOG, HOB, other\}$	Final good types
$q \in \{j, k\}$	All production types

Table 2 – Subscripts

Symbol	Description
A	Atkinson inequality measure
$clean$	Lower pollution method of production
C	Consumption/consumer
CG	Civic engagement aspect of wellbeing
$dirty$	Higher pollution method of production
D	Domestic (i.e. not international)
EQ	Environmental quality aspect of wellbeing
EQF	Flow resource
EQS	Stock resource
ES	Education aspect of wellbeing
ESF	Formal education final production good
ESS	Skills
G	Government
G	Gini inequality measure
$HH1$	Household type 1
$HH2$	Household type 2
HHn_H	Household type n_H
HO	Housing aspect of wellbeing
HOB	Bad quality housing final production good
HOG	Good quality housing final production good
HOT	Transport final production good
HS	Health aspect of wellbeing/Healthcare services (final production good)
IW	Economic (income and wealth) aspect of wellbeing

J	Labour force
JE	Jobs aspect of wellbeing
K	Capital/stock
LT	Long term unemployment
M	Imports
NW	Not in the workforce
$other$	All other final production goods
PS	Personal safety aspect of wellbeing/personal safety services (final production good)
Q	Production
ROW	Rest of the world (international)
S	Society
SC	Social connection aspect of wellbeing
ST	Short term unemployment
SW	Subjective wellbeing aspect of wellbeing
T	Theil inequality measure
TQ	Home production/volunteering
WL	Work-life balance aspect of wellbeing
X	Exports
Y	Income

A.2 Exogenous Variables

Table 3 – Exogenous Model Variables

Variable	Full Description
$A_{Q,e,q,t} > 0$	Multi-factor productivity of production sector q using process e in time period t
$B_{h,t} > 0$	Consumption budget (i.e. amount for spending on market consumption) for household type h in time period t
$I_{j=EQF,t} \geq 0$	Investment in level of resource type $j = EQF$ (flow resource) by society in time period t
$0 \leq J_{E,h,t} \leq 1$	Proportion of household type h that are employed in time period t
$K_{EQ,ROW,h,t} \geq 0$	Level of biodiversity experienced by international household type h at the beginning of time period t
$K_{ESF,ROW,h,t} \geq 0$	Level of formal education remembered by international household type h at the beginning of time period t
$K_{PS,ROW,h,t} \geq 0$	Level of personal safety experienced by international household type h at the beginning of time period t

$K_{\Psi,ROW,h,t} \geq 0$	Level of pollution experienced by international household type h at the beginning of time period t
$L_{h,t} \geq 0$	Proportion of time spend on paid labour by working households of type h in time period t
$n_{h,t} > 0$	Number of people living in household type h in time period t
$P_{L,ROW,h,t} > 0$	International wage rate for labour paid to household type h in time period t in international currency
$P_{ROWM,q,t} > 0$	Production price paid for imported good q in time period t in international currency
$P_{ROWM,K,t} > 0$	Price of capital (use of financial wealth) in time period t paid for imports of capital into the country in international currency
$P_{ROWX,q,t} > 0$	Production price received for exported good q in time period t in international currency
$P_{ROWX,K,t} > 0$	Price of capital (use of financial wealth) in time period t paid for exports of capital out of the country in international currency
$R_{ROW,t} > 0$	Exchange rate between the international currency and the local currency in time period t
$S_{G,a,t} \geq 0$	Government spending on expenditure type a in time period t
$S_{G,k,h,t} \geq 0$	Total government spending (excluding investment spending) on final good $k \in \{ESF, HS, PS\}$ for household type h in time period t
$0 \leq T_{HO,h,t} \leq 1$	Proportion of time spent on transport by household h in time period t
$0 \leq T_{Q,h_1,h_2,k,t} \leq 1$	Proportion of time spent on home production of production good or service k by household h_1 for household h_2 (which may or may not be the same as h_1) in time period t. Examples of home production include cooking, cleaning, childcare, and home maintenance
$\tilde{T}_{WL,ROW,h,t} \geq 0$	Proportion of time spent on leisure and personal care (including sleeping) by international household type h in time period t, adjusted for health and social connection
$\Lambda_{SW,ROW,h,t} \geq 0$	Level of life satisfaction for international household type h at the beginning of time period t
$\Omega_{JE,h,t} \geq 0$	Job security for household type h in time period t

A.3 Endogenous Variables

A hat on a variable indicates the index of that variable, for example, $\hat{J}_{E,h,t}$ is the index of the employment variable $J_{E,h,t}$.

Table 4 – Endogenous Model Variables Model Variables

Variables	Full Description
$C_{G,k,t} \geq 0$	Total consumption of market goods and services of type k by the government in time period t
$C_{h,t} \geq 0$	Total consumption (both market and home production) of all final goods except transport and safety (which are not considered to enhance wellbeing directly) by household type h in time period t
$C_{h,k,t} \geq 0$	Total consumption of goods and services of type k by households of type h in time period t
$C_{HO,h,t} > 0$	Total consumption of housing (both good and bad quality) per person in household type h in time period t
$C_{HS,h,t} > 0$	Demand for healthcare services from all sources by household type h in time period t
$C_{PS,h,t} > 0$	Demand for policing and security services from all sources by household type h in time period t
$C_{S,k,t} \geq 0$	Average consumption of goods and services of type k in society in time period t
$D_{G,t} > 0$	Level of debt owed by the government at the beginning of time period t
$E_{G,t} \geq 0$	Total government expenditure from all sources (debt servicing, transfers, spending, investment) in time period t
$I_{CG,h,t}$	Investment in increasing civic engagement levels of household type h in time period t
$I_{EQ,S,t}$	Investment in increasing biodiversity of society in time period t
$I_{ESF,h,t} \geq 0$	Investment in increasing the formal education level of household h in time period t
$I_{ESS,h,t} \geq 0$	Investment in increasing the skill level of household h in time period t
$I_{G,t}$	Investment in the infrastructure (physical wealth) of the society by the government in time period t
$I_{HS,h,t}$	Investment in the health level of household type h in time period t

$I_{IW,h,t}$	Investment in the financial wealth of household type h in time period t
$I_{j=EQS} \leq 0$	Investment in increasing the level of resource type $j = EQS$ (stock resource) by society in time period t
$I_{N,h,t}$	Net increase in population due to migration for household type h in time period t
$I_{PS,h,t}$	Investment in the personal safety of household type h in time period t
$I_{ROW,t}$	Balance of payments (M-X) in time period t
$I_{SC,h,t}$	Investment in the social connections of household type h in the time period t
$I_{\Psi,S,t}$	Investment in increasing pollution stock experienced by all society in time period t
$0 \leq J_{LT,h,t} \leq 1$	Proportion of household type h that are long-term (greater than 1 year) unemployed in time period t
$0 \leq J_{NW,h,t} \leq 1$	Proportion of household type h that are not-in-the-workforce in time period t
$0 \leq J_{ST,h,t} \leq 1$	Proportion of household type h that are short-term (less than 1 year) unemployed in time period t
$K_{CG,h,t} > 0$	Level of civic engagement by household type h at the beginning of time period t
$K_{CG,S,t} > 0$	Level of civic engagement by society at the beginning of time period t
$K_{EQ,h,t} > 0$	Level of biodiversity experienced by household h at the beginning of time period t
$K_{EQ,S,t} > 0$	Level of biodiversity experienced by society at the beginning of time period t
$K_{ESF,h,t} > 0$	Level of formal education remembered by household h at the beginning of time period t
$K_{ESS,h,t} > 0$	Level of skills maintained by household h at the beginning of time period t
$K_{G,t} > 0$	Level of physical infrastructure (capital, wealth) owned by government at the beginning of time period t
$K_{HS,h,t} > 0$	Level of health experienced by household h at the beginning of time period t
$K_{IW,C,t} > 0$	Total capital demanded (for production and government debt) at the beginning of time period t
$K_{IW,D,t} > 0$	Total capital domestically supplied and used at the beginning of time period t

$K_{IW,h,t} > 0$	Financial wealth owned by household type h at the beginning of time period t
$K_{IW,M,t} \geq 0$	Total capital imported into the economy at the beginning of time period t
$K_{IW,X,t} \geq 0$	Total capital exported from the economy at the beginning of time period t
$K_{IW,S,t} \geq 0$	Average capital supply from households available for production at the beginning of time period t
$K_{j=EQF,t} \geq 0$	"Stock" of flow resource at the beginning of time period t. Unlike most stocks, the value of this variable has no persistence between time periods and exists primarily for notational convenience
$K_{j=EQS,t} \geq 0$	Level of stock resource at the beginning of time period t available for extraction
$K_{N,h,t} > 0$	Number of households of type h in time period t
$K_{N,S,t} > 0$	Number of households in society in time period t
$K_{PS,h,t} \geq 0$	Level of personal safety experienced by household h at the beginning of time period t
$K_{PS,S,t} > 0$	Average level of personal safety in the society at the beginning of time period t
$K_{SC,h,t} > 0$	Level of social connection for household type h at the beginning of time period t
$K_{SC,S,t} > 0$	Level of social connection for society at the beginning of time period t
$K_{\Psi,h,t} > 0$	Level of pollution experienced by household h at the beginning of time period t
$K_{\Psi,S,t} > 0$	Level of pollution experienced by society at the beginning of time period t
$\tilde{L}_{ESS,h,e,q,t} \geq 0$	Time dedicated to skills training by labour from household type h in production q by process e in time period t, including adjustments for health and education productivity
$\tilde{L}_{Q,h,e,q,t} \geq 0$	Labour from household type h demanded for production of goods q by process e in time period t for both production and skills training, including adjustments for health and education productivity
$\tilde{L}_{S,h,t} \geq 0$	Total labour from household type h available for production in time period t, including adjustments for health and education productivity
$N_{M,h,t} \geq 0$	Number of immigrants to household type h in time period t

$N_{X,h,t} \geq 0$	Number of emigrants from household type h in time period t
$P_{C,q,t} > 0$	Price paid by consumers (but excluding consumption tax) for goods and services q in time period t
$P_{C,K,e,q,t} > 0$	Price of capital used for production of good q by process e in time period t. Included for notational consistency with labour
$P_{C,K,t} > 0$	Price of capital (use of financial wealth) use in time period t
$P_{D,K,t} > 0$	Price of capital (use of financial wealth) domestically supplied and used in time period t
$P_{D,q,t} > 0$	Price received by domestic producers of domestically supplied good q in time period t
$P_{f,e,q,t} > 0$	Effective price of factor of production f used for production of good q by process e in time period t
$P_{f,t} > 0$	Price of factor of production f in time period t
$P_{HO,h,t} > 0$	Weighted average price of housing (good and bad quality) relative to net household income for household type h in time period t
$P_{K,t} > 0$	Price of capital (use of financial wealth) in time period t
$P_{L,h,t} > 0$	Wage rate for labour paid to household type h in time period t
$P_{\tilde{L},Q,h,e,q,t} > 0$	Effective price of labour from household h used for production of good q by process e in time period t, after adjusting for time spent developing skills (paid for by the employer) and where the labour supply includes adjustments for health and education productivity
$P_{\tilde{L},S,h,t} > 0$	Price of labour from household type h in time period t, where the labour supply includes adjustments for health and education productivity
$P_{M,K,t} > 0$	Price of capital (use of financial wealth) in time period t paid for imports of capital into the country in local currency
$P_{M,q,t} > 0$	Production price paid for imported extracted good q in time period t in local currency
$P_{Q,K,t} > 0$	Price of capital (use of financial wealth) supply in time period t
$P_{Q,q,t} > 0$	Production price of good q in time period t
$P_{X,K,t} > 0$	Price of capital (use of financial wealth) in time period t paid for exports of capital out of the country in local currency

$P_{X,q,t} > 0$	Production price received for exported good q in time period t in local currency
$Q_{C,q,t} > 0$	Amount of goods of type q demanded by all consumers in time period t
$Q_{D,q,t} > 0$	Amount of goods of type q produced and sold domestically in time period t
$Q_{e,q,t} \geq 0$	Amount of goods of type q produced by process e in time period t
$Q_{h_1,h_2,k,t} \geq 0$	Level of home production of production good or service k (e.g. cooking, cleaning, childcare, etc) by household h_1 for household h_2 (which may or may not be the same as h_1) in time period t
$Q_{q,t} \geq 0$	Amount of goods of type q produced domestically by any process in time period
$Q_{k,h,t} \geq 0$	Total supply of goods and services of type $k \in \{ESF, HS, PS\}$ from all sources (i.e. home production, household demand, and government demand) for household type h in time period t
$Q_{M,q,t} \geq 0$	Amount of goods of type q imported from overseas in time period t
$Q_{X,q,t} \geq 0$	Amount of goods of type q exported in time period t
$r_{K,G,t}$	Rate of extra investment (beyond replacing depreciation) in infrastructure by government in time period t
$S_{G,t} \geq 0$	Total government spending in time period t
$\tilde{S}_{G,a,t} \geq 0$	Government spending on expenditure type a in time period t, adjusted for civic engagement
$\tilde{S}_{G,k,t} \geq 0$	Total government spending (excluding investment spending) on final good k in time period t adjusted for civic engagement
$\tilde{S}_{G,k,h,t} \geq 0$	Total government spending (excluding investment spending) on final good $k \in \{ESF, HS, PS\}$ for household type h in time period t, adjusted for civic engagement
$0 \leq T_{ES,h,t} \leq 1$	Proportion of time spent on education by household h in time period t
$\tilde{T}_{ES,h,t} \geq 0$	Proportion of time spent on education by household h in time period t, adjusted for health-productivity
$0 \leq T_{IW,h,t} \leq 1$	Proportion of time spent on in paid employment by household h in time period t

$0 \leq T_{Q,h,t} \leq 1$	Proportion of time spent volunteering by creating home production for other households, particularly other household types, by household type h in time period t
$\tilde{T}_{Q,h_1,h_2,k,t} \geq 0$	Proportion of time spent on home production of production good or service k by household h_1 for household h_2 (which may or may not be the same as h_1) in time period t, adjusted for health-productivity and education-productivity
$0 \leq T_{WL,h,t} \leq 1$	Proportion of time spent on leisure and personal care (including sleeping) by household h in time period t
$\tilde{T}_{WL,h,t} \geq 0$	Proportion of time spent on leisure and personal care (including sleeping) by household h in time period , adjusted for health and social connection
V_B	Base for index for index of variable V. Usually the value of $V_{S,t=0}$ from the business-as-usual section of the model results
$V_{f,e,q,t} \geq 0$	Amount of factor of production f used for production of good q by process e in time period t
$V_{g,t} > 0$	Amount of enabler of production g available in time period t
$V_{h,t}$	Value of a variable for household type h in time period t
$V_{j,e,k,t} \geq 0$	Amount of extracted good j used for production of final good k by process e in time period t
$V_{S,t}$	Societal value of a variable in time period t
$V_{S,f,t} \geq 0$	Supply of factor of production f from society in time period t
V_t	Value of the variable V at time t
$\hat{W}_{CG,h_S,t} > 0$	Index for the civic engagement aspect of wellbeing for household h_S in time period t
$\hat{W}_{EQ,h_S,t} > 0$	Index for the environment aspect of wellbeing for household h_S in time period t
$\hat{W}_{ES,h_S,t} > 0$	Index for the education aspect of wellbeing for household h_S in time period t
$\hat{W}_{h_S} > 0$	Index for overall intergenerational wellbeing for household h_S
$\hat{W}_{h_S,t} > 0$	Index for overall wellbeing for household h_S in time period t
$\hat{W}_{HO,h_S,t} > 0$	Index for the housing aspect of wellbeing for household h_S in time period t
$\hat{W}_{HS,h_S,t} > 0$	Index for the health aspect of wellbeing for household h_S in time period t

$\hat{W}_{JE,h_S,t} > 0$	Index for the jobs and employment aspect of wellbeing for household h_S in time period t
$\hat{W}_{IW,h_S,t} > 0$	Index for the economic aspect of wellbeing for household h_S in time period t
$\hat{W}_{PS,h_S,t} > 0$	Index for the personal safety aspect of wellbeing for household h_S in time period t
$\hat{W}_{SC,h_S,t} > 0$	Index for the social connection aspect of wellbeing for household h_S in time period t
$\hat{W}_{SW,h_S,t} > 0$	Index for the life satisfaction aspect of wellbeing for household h_S in time period t
$\hat{W}_{WL,h_S,t} > 0$	Index for the work-life balance aspect of wellbeing for household h_S in time period t
$X_{e,q,t} > 0$	Price factor for goods q produced by process e in time period t, defined for notational convenience
$Y_{G,t} > 0$	Government income from taxes in time period t
$Y_{GDP} > 0$	Gross Domestic Product of the economy (calculated from production) in time period t
$Y_{h,t} \geq 0$	Gross (before taxes and transfers) income for household type h in time period t
$y_{h,t} \geq 0$	Net (after taxes and transfers) income for household type h in time period t
$\Gamma_{G,t} \geq 0$	Total social welfare transfers (unemployment and other benefits) from government to households in time period t
$\Gamma_{\tau,t}$	Other transfers (excluding social welfare benefits) from government to households in time period t
$\Lambda_{SW,h,t} \geq 0$	Level of life satisfaction for household type h at the beginning of time period t
$\Psi_{e,q,t}$	Total amount of pollution emitted producing good q using process e in time period t
$\Theta_{IWK,h,t}$	Difference between societal (average) wealth and the wealth of household type h at the beginning of time period t
$0 \leq \Upsilon_{A,IWK,t} \leq 1$	Level of wealth inequality in the society as measured by the Atkinson index in time period t
$0 \leq \Upsilon_{A,m,t} \leq 1$	Level of inequality in the society as measured by the Atkinson index in time period t
$0 \leq \Upsilon_{G,m,t} \leq 1$	Level of inequality in the society as measured by the Gini index in time period t
$\Upsilon_{T,m,t} > 0$	Level of inequality in the society as measured by the Theil index in time period t

$0 \leq \Xi_{HO,h,t} \leq 1$	Proportion of good quality housing to total housing for households of type h in time period t

A.4 Parameters

Table 5 – Model Parameters

Parameter	Full Description
$A_{ES,T,h} > 0$	Health-adjusted time required for investment in formal education for household type h
$A_{ESF,h} > 0$	Formal education-adjusted level of formal education services required to increase education for household type h
$A_{ESS,h} > 0$	Skills-adjusted level of time spent on skills training require to increase skill level for household type h
$A_{G,a} > 0$	Civic engagement-adjusted level of government spending on spending category a
$A_{G,k,h} > 0$	Civic-engagement-adjusted level of government spending on final good k for household type h
$A_{HS,h} > 0$	Effect of health level on demand for healthcare for household type h
$A_{HOT,h} > 0$	Consumption of transport goods and services per time spent on transportation by household type h
$A_{HS,ES,h} > 0$	Effect of health level on the productivity of time spent in formal education for household type h
$A_{HS,T,h} > 0$	Minimum level of leisure and personal care time required for avoiding health problems (including amount of sleep) for household type h
$A_{HS,TQ,k} > 0$	Effect of health, education, and skill level on the productivity of time spent in home production of production good or service k
$A_{HS,WL} > 0$	Effect of health level on the value of time spent on leisure and personal care
$A_{h,k} > 0$	Skill level of household at producing production good or service k
$A_{J,LT,h} > 0$	Scaling factor for long-term unemployment rate of household type h
$A_{J,NW,h} > 0$	Scaling factor for rate of not-in-the-workforce for household type h
$A_{J,ST,h} > 0$	Effect of employment level on the short-term unemployment rate of household type h

$A_{\tilde{L},h} > 0$	Effect of health, education, and skill level on the productivity of time spent in paid labour for household type h
$A_{MK} > 0$	Scaling factor for combining domestically supplied financial capital with imported financial capital
$A_{M,q} > 0$	Scaling factor for combining domestically produced goods q with imported goods
$A_{PS,h} > 0$	Effect of personal safety level on demand for crime prevention for household type h
$A_{Q,q} > 0$	Scaling factor for combining production of goods q from different processes
$A_{XK} > 0$	Scaling factor for splitting domestic supply of financial capital into exported capital and domestically used financial capital
$A_{X,q} > 0$	Scaling factor for splitting domestic production into exports and domestic market goods q
$P_{\Psi} \geq 0$	Price (tax) to emit pollution, paid to the government
$\Gamma_J \geq 0$	Unemployment benefit payment to each unemployed household of type h
$\Gamma_{NW} \geq 0$	Benefit paid to households not-in-the-workforce (e.g. disability, sole parent)
$0 \leq \alpha_{C,G,k} \leq 1$	Share of consumption for final goods and services k in government investment in infrastructure (physical wealth)
$0 \leq \alpha_{C,h,k} \leq 1$	Share of consumption for final goods and services k by household type h
$0 \leq \alpha_{f,e,q} \leq 1$	Amount of input (factor of production) f as a proportion of all inputs into production process e for good q
$0 \leq \alpha_{i,h_S} \leq 1$	Geometric average weights for the importance of wellbeing aspect i to overall wellbeing for household type h_S
$0 \leq \alpha_{j,e,k} \leq 1$	Amount of input (extraction good) j as a proportion of all inputs into production process e for final good k
$0 < \beta_{h_S} \leq 1$	Time discount rate for household type h_S. Expressed as the ratio of the importance of the next time period relative to the current time period
$0 < \beta_{SW,ESF,h} \leq 1$	Time discount rate for the effect of a change of formal education on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period

$0 < \beta_{SW,EQ,h} \leq 1$	Time discount rate for the effect of a change of biodiversity on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,HOT,h} \leq 1$	Time discount rate for the effect of a change of transport time (commuting) on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,HS,h} \leq 1$	Time discount rate for the effect of a change of health on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,IW,h} \leq 1$	Time discount rate for the effect of a change of wealth on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,JLT,h} \leq 1$	Time discount rate for the effect of a change of long-term unemployment on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,JST,h} \leq 1$	Time discount rate for the effect of a change of short-term unemployment on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,PS,h} \leq 1$	Time discount rate for the effect of a change of personal safety on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,Q,h} \leq 1$	Time discount rate for the effect of a change of volunteering on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,SC,h} \leq 1$	Time discount rate for the effect of a change of social connection on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,y,h} \leq 1$	Time discount rate for the effect of a change of net income on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period

$0 < \beta_{SW,\Psi,h} \leq 1$	Time discount rate for the effect of a change of pollution levels on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,\Theta,h} \leq 1$	Time discount rate for the effect of a change of relative wealth on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 < \beta_{SW,\Omega,h} \leq 1$	Time discount rate for the effect of a change of job security on life satisfaction for household type h. Expressed as the ratio of the importance of the previous time period relative to the current time period
$0 \leq \delta_{CG,h} \leq 1$	Rate of natural decline of civic engagement for household type h
$\delta_{EQ,j=EQF} = 1$	Natural regeneration rate of resource type $j = EQF$ (flow resource)
$-1 \leq \delta_{EQ,j=EQS} \leq 0$	Natural regeneration rate of resource type $j = EQS$ (stock resource)
$-1 \leq \delta_{EQ,S} \leq 0$	Regeneration rate of biodiversity
$0 \leq \delta_{ESF,h} < 1$	Rate of natural decline of knowledge from formal education (i.e. forgetting unused knowledge) for household h
$0 \leq \delta_{ESS,h} < 1$	Rate of natural decline of skills (i.e. forgetting unused skills) for household h
$0 \leq \delta_{G} < 1$	Depreciation of infrastructure (physical wealth) owned by government
$0 \leq \delta_{HS,h} < 1$	Rate of natural decline of health for household h. Likely to be age-dependent
$0 \leq \delta_{IW,h} \leq 1$	Rate of inflation (causing decline in value of financial wealth) for household h
$\delta_{N,h}$	Rate of decrease (negative gives increase) in population excluding migration for household type h
$0 \leq \delta_{PS,h} \leq 1$	Rate of natural decline of personal safety for household type h
$0 \leq \delta_{SC,h} \leq 1$	Rate of natural decline of social connection for household type h
$0 \leq \delta_{\Psi,S} \leq 1$	Rate of neutralisation of pollution be the environment
$\epsilon_{\Upsilon A} \geq 0$	Aversion to inequality as used in the Atkinson index
$0 \leq \gamma_{e,q} \leq 1$	Proportion of goods of type q produced by process e
$0 \geq \gamma_{MK} \geq 1$	Proportion of domestically supplied capital exported

$0 \geq \gamma_{M,q} \geq 1$	Proportion of domestic production of goods q exported
$0 \leq \gamma_{NW,h} \leq 1$	Proportion of households of type h that are not-in-the-workforce and are eligible for a benefit
$0 \leq \gamma_{TQ,h} \leq 1$	Proportion of time spent in home production for same household type h that is volunteering (i.e. for other households within the same household type)
$0 \geq \gamma_{XK} \geq 1$	Proportion of domestically consumed capital imported
$0 \geq \gamma_{X,q} \geq 1$	Proportion of domestic consumption of goods q imported
$\gamma_{\tau,h} \geq 0$	Proportion of other transfers (i.e. excluding standard benefit payments to unemployed and not-in-the-workforce households) paid to households of type h
$\eta_{CG,a} \geq 0$	Elasticity of civic engagement on the effectiveness of government spending for spending type a
$\eta_{CG,k,h} \geq 0$	Elasticity of civic engagement on the effectiveness of government spending for final good $k \in \{ESF, HS, PS\}$ for household type h
$\eta_{ESF,h} \geq 0$	Elasticity of formal education level on ability to make use of formal education services by household type h
$\eta_{HS,h} \geq 0$	Elasticity of health level on demand for healthcare services by household type h
$\eta_{HS,ES,h} \geq 0$	Elasticity of health level on time required for formal education in household type h
$\eta_{g,e,q} \geq 0$	Elasticity of enabler of production g with respect to production of good q by process e
$\eta_{L,ESF,h} \geq 0$	Elasticity of labour productivity to formal education of household type h
$\eta_{L,ESS,h} \geq 0$	Elasticity of labour productivity to skills if household type h
$\eta_{L,HS,h} \geq 0$	Elasticity of labour productivity to health of household type h
$\eta_{PS,h} \geq 0$	Elasticity of personal safety level on demand for crime prevention services by household type h
$\eta_{Q,T,h,k} \geq 0$	Elasticity of home production of production good or service k with respect to the health-adjusted time input of household h
$\eta_{ST,h} > 0$	Elasticity of substitution between employment and short-term unemployment for household type h
$\eta_{TQ,ESF,h,k} \geq 0$	Elasticity of formal education level on home production by household h for good k

$\eta_{TQ,ESS,h,k} \geq 0$	Elasticity of skills level on home production by household h for good k
$\eta_{TQ,HS,h,k} \geq 0$	Elasticity of health level on home production by household h for good k
$\eta_{WL,HS,h} \geq 0$	Elasticity of health level on work-life balance for household h
$\eta_{WL,SC,h} \geq 0$	Elasticity of social connection level on work-life balance for household h
$\kappa \geq 0$	Government debt as a proportion of GDP
$\phi_{J,h} > 0$	Ratio of change in long-term unemployment to change in people not in the labour force for household type h
$\varpi_{\Delta,EQ,h} \geq 1$	Loss aversion effect of changes in biodiversity over time on life satisfaction for household type h. $\varpi_{\Delta,EQ,h} = 1$ if biodiversity improves, and if biodiversity declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,ESF,h} \geq 1$	Loss aversion effect of changes in formal education over time on life satisfaction for household type h. $\varpi_{\Delta,ESF,h} = 1$ if education improves, and if education declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,HO,h} \geq 1$	Loss aversion effect of changes in transport/commuting over time on life satisfaction for household type h. $\varpi_{\Delta,HO,h} = 1$ if transport time decreases, and if transport time increases a larger value indicating the level of loss aversion
$\varpi_{\Delta,HS,h} \geq 1$	Loss aversion effect of changes in health over time on life satisfaction for household type h. $\varpi_{\Delta,HS,h} = 1$ if health improves, and if health declines, a larger value indicating the level of loss aversion
$\varpi_{\Delta,IW,h} \geq 1$	Loss aversion effect of changes in household wealth over time on life satisfaction for household type h. $\varpi_{\Delta,IW,h} = 1$ if wealth increases, and if wealth declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,JLT,h} \geq 1$	Loss aversion effect of changes in long term unemployment over time on life satisfaction for household type h. $\varpi_{\Delta,JLT,h} = 1$ if unemployment decreases, and if unemployment increases a larger value indicating the level of loss aversion

$\varpi_{\Delta,JST,h} \geq 1$	Loss aversion effect of changes in short term unemployment over time on life satisfaction for household type h. $\varpi_{\Delta,JST,h} = 1$ if unemployment decreases, and if unemployment increases a larger value indicating the level of loss aversion
$\varpi_{\Delta,PS,h} \geq 1$	Loss aversion effect of changes in personal safety over time on life satisfaction for household type h. $\varpi_{\Delta,PS,h} = 1$ if safety improves, and if safety declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,Q,h} \geq 1$	Loss aversion effect of changes in volunteering/home production over time on life satisfaction for household type h. $\varpi_{\Delta,Q,h} = 1$ if volunteering increases, and if volunteering declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,SC,h} \geq 1$	Loss aversion effect of changes in social connection over time on life satisfaction for household type h. $\varpi_{\Delta,SC,h} = 1$ if social connection increases, and if social connection decline a larger value indicating the level of loss aversion
$\varpi_{\Delta,y,h} \geq 1$	Loss aversion effect of changes in net household income over time on life satisfaction for household type h. $\varpi_{\Delta,y,h} = 1$ if income increases, and if income declines a larger value indicating the level of loss aversion
$\varpi_{\Delta,\Psi,h} \geq 1$	Loss aversion effect of changes in pollution level over time on life satisfaction for household type h. $\varpi_{\Delta,\Psi,h} = 1$ if pollution decreases, and if pollution increases a larger value indicating the level of loss aversion
$\varpi_{\Delta,\Theta,h} \geq 1$	Loss aversion effect of changes in relative wealth over time on life satisfaction for household type h. $\varpi_{\Delta,\Theta,h} = 1$ if relative wealth decreases, and if relative wealth increases a larger value indicating the level of loss aversion
$\varpi_{\Delta,\Omega,h} \geq 1$	Loss aversion effect of changes in job security over time on life satisfaction for household type h. $\varpi_{\Delta,\Omega,h} = 1$ if job security improves, and if job security declines a larger value indicating the level of loss aversion
σ_{MK}	Elasticity of substitution between imported financial capital and domestically supplied financial capital for domestic consumption

$\sigma_{M,q}$	Elasticity of substitution between imports and domestically produced goods q for domestic consumption
σ_q	Elasticity of substitution between different processes for creating goods q
σ_{XK}	Elasticity of transformation between exported financial capital and domestically used financial capital from domestic supply
$\sigma_{X,q}$	Elasticity of transformation between exports and domestically used goods q from domestic production
$\tau_{C,k}$	Tax rate on consumption good k
$\tau_{Y,h}$	Tax rate on income of household h
$\xi_{CG,ESF,h} > 0$	Effect of level of formal education on civic engagement of household type h
$\xi_{CG,J,h} > 0$	Effect of unemployment on civic engagement of household type h
$\xi_{CG,NM,h} > 0$	Effect of immigration into household type h on civic engagement
$\xi_{CG,SC,h} > 0$	Effect of social connection on civic engagement of household type h
$\xi_{CG,WL,h} > 0$	Effect of leisure time on civic engagement of household type h
$\xi_{CG,y,h} > 0$	Effect of net household income on civic engagement of household type h
$\xi_{EQ,S} > 0$	Effect of government spending on biodiversity
$\xi_{HS,EQ,h} > 0$	Effect of biodiversity on health of household type h
$\xi_{HS,ESF,h} > 0$	Effect of formal education on health of household type h
$\xi_{HS,HO,h} > 0$	Effect of housing quantity on health of household type h
$\xi_{HS,IW,h} > 0$	Effect of household wealth on health of household type h
$\xi_{HS,J,h} > 0$	Effect of unemployment level on health of household type h
$\xi_{HS,PS,h} > 0$	Effect of personal safety level on health of household type h
$\xi_{HS,SC,h} > 0$	Effect of social connection on health of household type h
$\xi_{HS,service,h} > 0$	Effect of difference between supply of health services and demand for health services on health of household type h
$\xi_{HS,LS,h} > 0$	Effect of life satisfaction level on health of household type h

$\xi_{HS,WL,h} > 0$	Effect of health-adjusted leisure and personal care time on health of household type h
$\xi_{HS,\Psi,h} > 0$	Effect of pollution on health of household type h
$\xi_{HS,\Xi,h} > 0$	Effect of housing quality on health of household type h
$\xi_{NM,EQ,h} > 0$	Effect of biodiversity on immigration into household type h
$\xi_{NM,ESF,h} > 0$	Effect of formal education on immigration into household type h
$\xi_{NM,PS,h} > 0$	Effect of personal safety on immigration into household type h
$\xi_{NM,SW,h} > 0$	Effect of subjective wellbeing on immigration into household type h
$\xi_{NM,WL,h} > 0$	Effect of work-life balance on immigration into household type h
$\xi_{NM,\Psi,h} > 0$	Effect of pollution on immigration into household type h
$\xi_{NX,PL,h} > 0$	Effect of wages on emigration for household type h
$\xi_{PS,ESF,h} > 0$	Effect of formal education on personal safety of household type h
$\xi_{PS,HS,h} > 0$	Effect of health on personal safety of household type h
$\xi_{PS,IW,h} > 0$	Effect of household wealth on personal safety of household type h
$\xi_{PS,JLT,h} > 0$	Effect of long term unemployment on personal safety of household type h
$\xi_{PS,JST,h} > 0$	Effect of short term unemployment on personal safety of household type h
$\xi_{PS,NM,h} > 0$	Effect of immigration into household type h on personal safety
$\xi_{PS,SC,h} > 0$	Effect of social connections on personal safety of household type h
$\xi_{PS,service,h} > 0$	Effect of difference between supply of policing and security services and demand for policing and security services on the personal safety of household type h
$\xi_{PS,\Upsilon,h} > 0$	Effect of societal inequality on personal safety of household type h
$0 \leq \xi_{Q,ESS,h,e,q} \leq 1$	Proportion of time paid for by producers q of process e for development of skills for household type h instead of directly on production
$\xi_{SC,CG,h} > 0$	Effect of an improvement in civic engagement on social connection of household type h

$\xi_{SC,ESF,h} > 0$	Effect of an improvement in formal education on social connection of household type h
$\xi_{SC,HS,h} > 0$	Effect of an improvement in health on social connection of household type h
$\xi_{SC,J,h} > 0$	Effect of an increase in employment on social connection of household type h
$\xi_{SC,N,h} > 0$	Effect of a decrease in population on social connection of household type h
$\xi_{SC,NM,h} > 0$	Effect of immigration into household type h on social connection
$\xi_{SC,PS,h} > 0$	Effect of an improvement in personal safety on social connection of household type h
$\xi_{SC,TQ,h} > 0$	Effect of an increase in volunteering on social connection of household type h
$\xi_{SC,WL,h} > 0$	Effect of an improvement in adjusted leisure time on social connection of household type h
$\xi_{SC,\Upsilon,h} > 0$	Effect of a decrease in inequality on social connection of household type h
$\xi_{SW,EQ,h} > 0$	Effect of an improvement in biodiversity on life satisfaction of household type h
$\xi_{SW,ESF,h} > 0$	Effect of an improvement in formal education on life satisfaction of household type h
$\xi_{SW,HOT,h} > 0$	Effect of a decrease in commuting/transport time on life satisfaction of household type h
$\xi_{SW,HS,h} > 0$	Effect of an improvement in health on life satisfaction of household type h
$\xi_{SW,IW,h} > 0$	Effect of an increase in wealth on life satisfaction of household type h
$\xi_{SW,JLT,h} > 0$	Effect of a decrease in long term unemployment on life satisfaction of household type h
$\xi_{SW,JST,h} > 0$	Effect of a decrease in short term unemployment on life satisfaction of household type h
$\xi_{SW,PS,h} > 0$	Effect of an improvement in personal safety on life satisfaction of household type h
$\xi_{SW,Q,h} > 0$	Effect of an increase in volunteering/home production on life satisfaction of household type h
$\xi_{SW,SC,h} > 0$	Effect of an increase in social connection on life satisfaction of household type h
$\xi_{SW,y,h} > 0$	Effect of an increase in net income on life satisfaction of household type h

$\xi_{SW,\Psi,h} > 0$	Effect of a decrease in pollution on life satisfaction of household type h
$\xi_{SW,\Theta,h} > 0$	Effect of an decrease in relative wealth on life satisfaction of household type h
$\xi_{SW,\Omega,h} > 0$	Effect of an improvement in job security on life satisfaction of household type h
$\xi_{\Psi,e,q}$	Rate of emission of pollution from producing good q using process e
$\xi_{\Psi,S} > 0$	Effect of government spending on decreasing pollution
$\zeta_{EQ,S} > 0$	Effect of pollution on biodiversity
$0 \leq \omega_{h_S} \leq 1$	Importance of current generation relative to the future generations for household type h_S

B Model Equations

The model equations are summarised in this appendix. They are given in the order and arrangement that they need to be calculated in, and generally each equation only depends on variables calculated in earlier equations. Variable and parameter definitions are given in the previous appendix, derivations in the next appendix, and explanations in the main body of this document.

B.1 Exogenous Variables

All exogenous variables in the model are defined to be constant over time.

$$\begin{aligned} V_t =& V_{t=0} \qquad \forall t, \\ V \in& \{A_{Q,e,q}, B_h, I_{j=EQF}, J_{E,h}, K_{EQ,ROW,h}, K_{ESF,ROW,h}, K_{PS,ROW,h}, \\ & K_{\Psi,ROW,h}, L_h, n_h, P_{L,ROW,h}, P_{ROW\,M,K}, P_{ROW\,M,q}, P_{ROW\,X,K}, P_{ROW\,X,q}, \\ & R_{ROW}, S_{G,a}, S_{G,k,h}, T_{HO,h}, T_{Q,h_1,h_2,k}, \tilde{T}_{WL,ROW,h}, \Lambda_{SW,ROW,h}, \Omega_{JE,h,t}\} \end{aligned} \tag{153}$$

B.2 Variables Derived Exclusively From Exogenous Variables

As all exogenous variables are constant over time, these variables will also be constant over time as they are derived exclusively from the exogenous variables.

Volunteering:

$$T_{Q,h,t} = \gamma_{TQ,h} \sum_{k} T_{Q,h,h_2=h,k,t} + \sum_{h_2 \neq h} \sum_{k} T_{Q,h,h_2,k,t} \qquad \forall h,t \tag{154}$$

Time spent in paid work:

$$T_{IW,h,t} = L_{h,t} J_{E,h,t} \qquad \forall h,t \tag{155}$$

Short-term unemployment:

$$J_{ST,h,t} = A_{J,ST,h} {J_{E,h,t}}^{-1/\eta_{ST,h}} \qquad \forall h,t \tag{156}$$

Long-term unemployment:

$$J_{LT,h,t} = (1 + \phi_{J,h})^{-1} (A_{J,LT,h} - J_{E,h,t} - J_{ST,h,t}) \qquad \forall h,t \tag{157}$$

Not in labour force:

$$J_{NW,h,t} = \left(1 + \phi_{J,h}^{-1}\right)^{-1} (A_{J,NW,h} - J_{E,h,t} - J_{ST,h,t}) \qquad \forall h,t \tag{158}$$

Total government spending:

$$S_{G,t} = \sum_{a} S_{G,a,t} + \sum_{k \in \{ESF,HS,PS\}} \sum_{h} S_{G,k,h,t} \qquad \forall t \tag{159}$$

Overseas price of imports:

$$P_{M,b,t} = R_{ROW,t} P_{ROWM,b,t} \qquad \forall t, \quad b \in \{j,k,K\} \tag{160}$$

Overseas price of exports:

$$P_{X,b,t} = R_{ROW,t} P_{ROWX,b,t} \qquad \forall t, \quad b \in \{j,k,K\} \tag{161}$$

B.3 Evolving Variables

These variables change over time, and a subset of them must be solved, rather than simply being calculated from values from the current and previous timesteps.

B.3.1 Evolving Variables Needed by Solver

These values depend upon those calculated in the previous timestep and are needed for the solver.

Societal population:

$$K_{N,S,t} = \sum_h K_{N,h,t} \qquad \forall t \tag{162}$$

Average labour supply from households:

$$\tilde{L}_{S,h,t} = A_{\tilde{L},h} K_{ESF,h,t}{}^{\eta_{L,ESF,h}} K_{ESS,h,t}{}^{\eta_{L,ESS,h}} K_{HS,h,t}{}^{\eta_{L,HS,h}} T_{IW,h,t} \frac{K_{N,h,t}}{K_{N,S,t}} \qquad \forall h, t \tag{163}$$

Average stocks from households:

$$K_{l,S,t} = \sum_h \frac{K_{N,h,t}}{K_{N,S,t}} K_{l,h,t} \qquad \forall t, \quad l \in \{CG, IW, PS, SC\} \tag{164}$$

Rate of extra government investment:

$$r_{K,G,t} = \frac{K_{N,S,t} - K_{N,S,t-1}}{K_{N,S,t-1}} \qquad \forall t \tag{165}$$

Government social welfare transfers:

$$\Gamma_{G,t} = \Gamma_J \sum_h K_{N,h,t} \left(J_{ST,h,t} + J_{LT,h,t}\right) + \Gamma_{NW} \sum_h \gamma_{NW,h} K_{N,h,t} J_{NW,h,t} \qquad \forall t \tag{166}$$

Government investment:

$$I_{G,t} = \left(\delta_G + r_{K,G,t}\right) K_{G,t} \qquad \forall t \tag{167}$$

Adjusted government spending on consumption:

$$\tilde{S}_{G,a,t} = A_{G,a} K_{CG,S,t}{}^{\eta_{CG,a}} S_{G,a,t} \qquad \forall a, t \tag{168}$$

Adjusted government spending on consumption:

$$\tilde{S}_{G,k,h,t} = A_{G,k,h} K_{CG,h,t}{}^{\eta_{CG,k,h}} S_{G,k,h,t} \qquad \forall t, \quad k \in \{ESF, HS, PS\} \tag{169}$$

Government spending on consumption:

$$\tilde{S}_{G,k,t} = \begin{cases} \sum_h \tilde{S}_{G,k,h,t} & k \in \{ESF, HS, PS\} \\ \tilde{S}_{G,\Psi,t} + \tilde{S}_{G,EQ,t} + \tilde{S}_{G,other,t} & k \in \{other\} \\ \tilde{S}_{G,a=k,t} & k \in \{HOT, HOG, HOB\} \end{cases} \qquad \forall t \tag{170}$$

Adjustment to home production:

$$\begin{aligned} \tilde{T}_{Q,h_1,h_2,k,t} = & K_{HS,h_1,t}{}^{\eta_{TQ,HS,h_1,k}} K_{ESF,h_1,t}{}^{\eta_{TQ,ESF,h_1,k}} K_{ESS,h_1,t}{}^{\eta_{TQ,ESS,h_1,k}} \\ & \times A_{HS,TQ,k} T_{Q,h_1,h_2,k,t} \qquad \forall h_1, h_2, k, t \end{aligned} \tag{171}$$

Home production:

$$Q_{h_1,h_2,k,t} = A_{h_1,k} \tilde{T}_{Q,h_1,h_2,k,t}{}^{\eta_{Q,T,h_1,k}} \qquad \forall h_1, h_2, k, t \tag{172}$$

B.3.2 Solved System

The equations in this section form a loop: Once the prices of the factors of production are known, the remaining variables in this section can be calculated (in the order given), including (finally) the prices of the factors of production and the exchange rate.

Effective price of labour:

$$P_{\tilde{L},Q,h,e,q,t} = \frac{P_{\tilde{L},S,h,t}}{1 - \xi_{Q,ESS,h,e,q}} \qquad \forall h, e, q, t \tag{173}$$

Effective price of factors of production:

$$P_{C,K,e,q,t} = P_{C,K,t} \qquad \forall e, q, t \tag{174}$$

Price factor (extraction goods):

$$\begin{gathered} X_{e,j,t} = P_\Psi \xi_{\Psi,e,j} + A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}} \\ \forall e, j, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\} \end{gathered} \tag{175}$$

Price of extracted (intermediate) goods:

$$P_{Q,j,t} = A_{Q,j}{}^{-1} \left(\sum_e \gamma_{e,j}{}^{\sigma_j} X_{e,j,t}{}^{1-\sigma_j} \right)^{\frac{-1}{\sigma_j - 1}} \qquad \forall j, t \tag{176}$$

Price factor (final goods):

$$\begin{gathered} X_{e,k,t} = P_\Psi \xi_{\Psi,e,k} + A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \\ \times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \\ \forall e, k, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \left\{P_{\tilde{L},Q,h,e}, P_{C,K,e}\right\} \end{gathered} \tag{177}$$

Price of produced final goods and services:

$$P_{Q,k,t} = A_{Q,k}{}^{-1} \left(\sum_e \gamma_{e,k}{}^{\sigma_k} X_{e,k,t}{}^{1-\sigma_k} \right)^{\frac{-1}{\sigma_k - 1}} \qquad \forall k, t \tag{178}$$

Price of domestically produced and sold goods:

$$P_{D,q,t} = (1-\gamma_{X,q})^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left((A_{X,q} P_{Q,q,t})^{-(\sigma_{X,q}-1)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-(\sigma_{X,q}-1)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \quad \forall q,t \tag{179}$$

Price of consumption:

$$P_{C,q,t} = A_{M,q}{}^{-1} \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-(\sigma_{M,q}-1)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-(\sigma_{M,q}-1)} \right)^{\frac{-1}{\sigma_{M,q}-1}} \quad \forall q,t \tag{180}$$

Household transport consumption:

$$C_{h,k=HOT,t} = A_{HOT,h} T_{HO,h,t} - \sum_{h_1} Q_{h_1,h,k=HOT,t} \qquad \forall h,t \tag{181}$$

Household consumption:

$$C_{h,k,t} = \frac{\alpha_{C,h,k} \left(B_{h,t} - (1+\tau_{C,k_2=HOT}) P_{C,k_2=HOT,t} C_{h,k_2=HOT,t} \right)}{(1+\tau_{C,k}) P_{C,k,t}} \quad \forall h, k \neq HOT, t \tag{182}$$

Government consumption:

$$C_{G,k,t} = \frac{\tilde{S}_{G,k,t}}{P_{C,k,t}} + \frac{\alpha_{C,G,k} I_{G,t}}{P_{C,k,t}} \qquad \forall k,t \tag{183}$$

Market-clearing demand for final production:

$$Q_{C,k,t} = \sum_h K_{N,h,t} C_{h,k,t} + C_{G,k,t} \qquad \forall k,t \tag{184}$$

Quantity of domestically produced and sold final goods:

$$Q_{D,k,t} = (1-\gamma_{M,k})^{\sigma_{M,k}} P_{D,k,t}{}^{-\sigma_{M,k}} A_{M,k}{}^{-1} Q_{C,k,t} \times \left(\gamma_{M,k}{}^{\sigma_{M,k}} P_{M,k,t}{}^{-(\sigma_{M,k}-1)} + (1-\gamma_{M,k})^{\sigma_{M,k}} P_{D,k,t}{}^{-(\sigma_{M,k}-1)} \right)^{-\frac{\sigma_{M,k}}{\sigma_{M,k}-1}} \quad \forall k,t \tag{185}$$

Quantity of domestically-produced final goods:

$$Q_{k,t} = \left(\gamma_{X,k}{}^{\sigma_{X,k}} P_{X,k,t}{}^{-(\sigma_{X,k}-1)} + (1-\gamma_{X,k})^{\sigma_{X,k}} P_{D,k,t}{}^{-(\sigma_{X,k}-1)} \right)^{\frac{\sigma_{X,k}}{\sigma_{X,k}-1}} \times (1-\gamma_{X,k})^{-\sigma_{X,k}} P_{D,k,t}{}^{\sigma_{X,k}} A_{X,k} Q_{D,k,t} \qquad \forall k,t \tag{186}$$

Production process (final products):

$$Q_{e,k,t} = Q_{k,t}{\gamma_{e,k}}^{\sigma_k}{P_{Q,k,t}}^{\sigma_k}{A_{Q,k}}^{\sigma_k-1}{X_{e,k,t}}^{-\sigma_k} \qquad \forall e,k,t \tag{187}$$

Pollution (final products):

$$\Psi_{e,k,t} = \xi_{\Psi,e,k}Q_{e,k,t} \qquad \forall e,k,t \tag{188}$$

Extraction products demanded for final production:

$$V_{j,e,k,t} = \alpha_{j,e,k}{P_{C,j,t}}^{-1}\left(\gamma_{e,k}P_{Q,k,t}Q_{k,t}{A_{Q,k}}^{\frac{\sigma_k-1}{\sigma_k}}\left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} - P_\Psi\Psi_{e,k,t}\right) \tag{189}$$
$$\forall e,j,k,t$$

Market-clearing demand for extraction goods:

$$Q_{C,j,t} = \sum_k\sum_e V_{j,e,k,t} \qquad \forall j,t \tag{190}$$

Quantity of domestically produced and sold extraction goods:

$$Q_{D,j,t} = \left({\gamma_{M,j}}^{\sigma_{M,j}}{P_{M,j,t}}^{-(\sigma_{M,j}-1)} + (1-\gamma_{M,j})^{\sigma_{M,j}}{P_{D,j,t}}^{-(\sigma_{M,j}-1)}\right)^{-\frac{\sigma_{M,j}}{\sigma_{M,j}-1}}$$
$$\times(1-\gamma_{M,j})^{\sigma_{M,j}}{P_{D,j,t}}^{-\sigma_{M,j}}{A_{M,j}}^{-1}Q_{C,j,t} \qquad \forall j,t \tag{191}$$

Quantity of domestically-produced extraction goods:

$$Q_{j,t} = \left({\gamma_{X,j}}^{\sigma_{X,j}}{P_{X,j,t}}^{-(\sigma_{X,j}-1)} + (1-\gamma_{X,j})^{\sigma_{X,j}}{P_{D,j,t}}^{-(\sigma_{X,j}-1)}\right)^{\frac{\sigma_{X,j}}{\sigma_{X,j}-1}}$$
$$\times(1-\gamma_{X,j})^{-\sigma_{X,j}}{P_{D,j,t}}^{\sigma_{X,j}}A_{X,j}Q_{D,j,t} \qquad \forall j,t \tag{192}$$

Production of extraction goods:

$$Q_{e,j,t} = Q_{j,t}{\gamma_{e,j}}^{\sigma_j}{P_{Q,j,t}}^{\sigma_j}{A_{Q,j}}^{\sigma_j-1}{X_{e,j,t}}^{-\sigma_j} \qquad \forall e,j,t \tag{193}$$

Pollution (extraction goods):

$$\Psi_{e,j,t} = \xi_{\Psi,e,j}Q_{e,j,t} \qquad \forall e,j,t \tag{194}$$

Price of domestically produced and sold capital:

$$P_{D,K,t} = (1-\gamma_{MK})^{\frac{\sigma_{MK}}{\sigma_{MK}-1}}\left((A_{MK}P_{C,K,t})^{-(\sigma_{MK}-1)} - {\gamma_{MK}}^{\sigma_{MK}}{P_{M,K,t}}^{-(\sigma_{MK}-1)}\right)^{\frac{-1}{\sigma_{MK}-1}}$$
$$\forall t \tag{195}$$

Factors demanded for production:

$$V_{f,e,q,t} = \alpha_{f,e,q} P_{f,e,q,t}{}^{-1} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_{\Psi} \Psi_{e,q,t} \right) \tag{196}$$
$$\forall e, q, t, \quad P_f \in \left\{ P_{\tilde{L},Q,h}, P_{C,K} \right\}, \quad V_f \in \left\{ \tilde{L}_{Q,h}, K_Q \right\}$$

Quantity of domestically-demanded capital:

$$K_{IW,C,t} = \sum_e \sum_q V_{f=K,e,q,t} + D_{G,t} \qquad \forall t \tag{197}$$

Quantity of domestically produced and sold capital:

$$K_{IW,D,t} = \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-(\sigma_{XK}-1)} + (1 - \gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}}$$
$$\times (1 - \gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-\sigma_{XK}} A_{XK}{}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \tag{198}$$

Price of factors of production (labour):

$$P_{\tilde{L},S,h,t} = \sum_q \sum_e \alpha_{\tilde{L},e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_{\Psi} \Psi_{e,q,t} \right)$$
$$\times \left(K_{N,S,t} \tilde{L}_{S,h,t} \right)^{-1} \qquad \forall t \tag{199}$$

Price of consumption of capital:

$$P_{C,K,t} = (1 - \gamma_{MK})^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} P_{D,K,t} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}} \right)^{\frac{1}{\sigma_{MK}}} \qquad \forall t \tag{200}$$

B.3.3 Evolving Variables That Need Solved Results

These variables require the results of the solved system and are used in calculating the next timestep.

Price for supply of capital:

$$P_{Q,K,t} = A_{XK}{}^{-1} \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-(\sigma_{XK}-1)} + (1 - \gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-(\sigma_{XK}-1)} \right)^{\frac{-1}{\sigma_{XK}-1}}$$
$$\forall t \tag{201}$$

Labour demanded by producers for skills training and production:

$$\tilde{L}_{Q,h,e,q,t} = \frac{V_{f=\tilde{L},h,e,q,t}}{1 - \xi_{Q,ESS,h,e,q}} \qquad \forall h, e, q, t \tag{202}$$

Labour for training by producers:

$$\tilde{L}_{ESS,h,e,q,t} = \xi_{Q,ESS,h,e,q} \tilde{L}_{Q,h,e,q,t} \qquad \forall h, e, q, t \tag{203}$$

Quantity of imported goods:

$$Q_{M,q,t} = \left({\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-(\sigma_{M,q}-1)} + (1-\gamma_{M,q})^{\sigma_{M,q}} {P_{D,q,t}}^{-(\sigma_{M,q}-1)} \right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \times {\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-\sigma_{M,q}} {A_{M,q}}^{-1} Q_{C,q,t} \qquad \forall q, t \tag{204}$$

Quantity of exported extraction goods:

$$Q_{X,q,t} = \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-(\sigma_{X,q}-1)} + (1-\gamma_{X,q})^{\sigma_{X,q}} {P_{D,q,t}}^{-(\sigma_{X,q}-1)} \right)^{-\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \times {\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} Q_{q,t} \qquad \forall q, t \tag{205}$$

Wages for households:

$$P_{L,h,t} = A_{\tilde{L},h} {K_{ESF,h,t}}^{\eta_{L,ESF,h}} {K_{ESS,h,t}}^{\eta_{L,ESS,h}} {K_{HS,h,t}}^{\eta_{L,HS,h}} P_{\tilde{L},S,h,t} \qquad \forall h, t \tag{206}$$

Gross household income:

$$Y_{h,t} = P_{Q,K,t} K_{IW,h,t} + P_{L,h,t} T_{IW,h,t} \qquad \forall h, t \tag{207}$$

Average societal market consumption:

$$C_{S,k,t} = \sum_h \frac{K_{N,h,t}}{K_{N,S,t}} C_{h,k,t} \qquad \forall t \tag{208}$$

Atkinson wealth inequality measure:

$$\Upsilon_{A,IWK,t} = \begin{cases} 1 - \frac{1}{K_{IW,S,t}} \left(\frac{1}{K_{N,S,t}} \sum_h K_{N,h,t} {K_{IW,h,t}}^{1-\epsilon_\Upsilon} \right)^{\frac{1}{1-\epsilon_\Upsilon}} & \epsilon_\Upsilon = 1 \\ 1 - \frac{1}{K_{IW,S,t}} \left(\prod_h {K_{IW,h,t}}^{K_{N,h,t}} \right)^{\frac{1}{K_{N,S,t}}} & \epsilon_\Upsilon \geq 0, \quad \epsilon_\Upsilon \neq 1 \end{cases} \qquad \forall t \tag{209}$$

Investment in biodiversity:

$$I_{EQ,S,t} = \xi_{EQ,S} \tilde{S}_{G,EQ,t} - \zeta_{EQ,S} K_{\Psi,S,t} \qquad \forall t \tag{210}$$

Investment in (increasing) pollution:

$$I_{\Psi,S,t} = \sum_q \sum_e \Psi_{e,q,t} - \xi_{\Psi,S} \tilde{S}_{G,\Psi,t} \qquad \forall t \tag{211}$$

Investment in stock resource:

$$I_{j=EQS,t} = -Q_{j=EQS,t} \qquad \forall t \tag{212}$$

GDP:

$$Y_{GDP,t} = \sum_k P_{C,k,t}Q_{C,k,t} + \sum_k P_{X,k,t}Q_{X,k,t} - \sum_k P_{M,k,t}Q_{M,k,t} \qquad \forall t \tag{213}$$

Government debt level:

$$D_{G,t+1} = \kappa Y_{GDP,t} \qquad \forall t \tag{214}$$

Government income:

$$Y_{G,t} = \sum_h \tau_{Y,h}K_{N,h,t}Y_{h,t} + K_{N,S,t}\sum_k \tau_{C,k}P_{C,k,t}C_{S,k,t} + P_\Psi \sum_e \sum_q \Psi_{e,q,t} \qquad \forall t \tag{215}$$

Government expenditure (budget):

$$E_{G,t} = Y_{G,t} + (D_{G,t+1} - D_{G,t}) \qquad \forall t \tag{216}$$

Government other transfers:

$$\Gamma_{\tau,t} = E_{G,t} - S_{G,t} - I_{G,t} - P_{C,K,t}D_{G,t} - \Gamma_{G,t} \qquad \forall t \tag{217}$$

Net household income:

$$y_{h,t} = (1 - \tau_{Y,h})\, Y_{h,t} + \Gamma_J\,(J_{ST,h,t} + J_{LT,h,t}) + \gamma_{NW,h}\Gamma_{NW}J_{NW,h,t} + \gamma_{\tau,h}\Gamma_{\tau,t}{K_{N,S,t}}^{-1} \qquad \forall h,t \tag{218}$$

Household investment:

$$I_{IW,h,t} = y_{h,t} - B_{h,t} \qquad \forall h,t \tag{219}$$

Housing quality indicator:

$$\Xi_{HO,h,t} = \frac{C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t}}{C_{h,k=HOB,t} + C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOB,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t}} \qquad \forall h,t \tag{220}$$

Housing quantity indicator:

$$C_{HO,h,t} = {n_{h,t}}^{-1}\left(C_{h,k=HOB,t} + C_{h,k=HOG,t} + \sum_{h_1} Q_{h_1,h,k=HOB,t} + \sum_{h_1} Q_{h_1,h,k=HOG,t}\right) \qquad \forall h,t \tag{221}$$

Household biodiversity indicator:

$$K_{EQ,h,t} = K_{EQ,S,t} \qquad \forall h,t \tag{222}$$

Household pollution indicator:

$$K_{\Psi,h,t} = K_{\Psi,S,t} \qquad \forall h,t \tag{223}$$

Supply of formal education, health services, policing:

$$Q_{k,h,t} = \sum_{h_1} Q_{h_1,h,k,t} + C_{h,k,t} + \frac{C_{G,k,t}}{K_{N,h,t}} \frac{\tilde{S}_{G,k,h,t}}{\sum_{h} \tilde{S}_{G,k,h,t}} \qquad \forall h,t, \quad k \in \{ESF, HS, PS\} \tag{224}$$

Demand for health services and crime prevention:

$$C_{l,h,t} = A_{l,h} K_{l,h,t}{}^{-\eta_{l,h}} \qquad \forall h,t \quad l \in \{HS, PS\} \tag{225}$$

Wealth comparison:

$$\Theta_{IWK,h,t} = K_{IW,S,t} - K_{IW,h,t} \qquad \forall h,t \tag{226}$$

Investment in life satisfaction:

$$\begin{aligned} \Lambda_{SW,h,t} =& \Lambda_{SW,h,t=0} + \sum_{\Lambda_p} \sum_{s=0}^{s \le t} \beta_{SW,\Lambda_p,h}{}^{t-s} \varpi_{\Delta,\Lambda_p,h} \xi_{SW,\Lambda_p,h} \left(V_{HS,\Lambda_p,s} - V_{HS,\Lambda_p,s-1} \right) \\ & - \sum_{\Lambda_m} \sum_{s=0}^{s \le t} \beta_{SW,\Lambda_m,h}{}^{t-s} \varpi_{\Delta,\Lambda_m,h} \xi_{SW,\Lambda_m,h} \left(V_{HS,\Lambda_m,s} - V_{HS,\Lambda_m,s-1} \right) \\ & \forall h, \quad t > 0, \quad V_{\Lambda_p} \in \{K_{EQ}, K_{ESF}, K_{HS}, K_{IW}, K_{PS}, K_{SC}, T_Q, y, \Omega_{JE}\}, \\ & V_{\Lambda_m} \in \{J_{LT}, J_{ST}, K_{\Psi}, T_{HO}, \Theta_{IWK}\} \end{aligned} \tag{227}$$

Investment in skills:

$$I_{ESS,h,t} = A_{ESS,h} \sum_{e} \sum_{q} \tilde{L}_{ESS,h,e,q,t} \qquad \forall h,t \tag{228}$$

Investment in formal education:

$$I_{ESF,h,t} = A_{ESF,h} K_{ESF,h,t}{}^{\eta_{ESF,h}} Q_{ESF,h,t} \qquad \forall h,t \tag{229}$$

Health-adjusted time spent on formal education:

$$\tilde{T}_{ES,h,t} = A_{ES,T,h} I_{ESF,h,t} \qquad \forall h,t \tag{230}$$

Education time:

$$T_{ES,h,t} = A_{HS,ES,h} K_{HS,h,t}{}^{-\eta_{HS,ES,h}} \tilde{T}_{ES,h,t} \qquad \forall h, t \tag{231}$$

Household leisure (time budget):

$$T_{WL,h,t} = 1 - T_{IW,h,t} - \sum_{h_2} \sum_{k} T_{Q,h,h_2,k,t} - T_{ES,h,t} - T_{HO,h,t} \qquad \forall h, t \tag{232}$$

Adjusted household leisure:

$$\tilde{T}_{WL,h,t} = A_{HS,WL} K_{HS,h,t}{}^{\eta_{WL,HS,h}} K_{SC,h,t}{}^{\eta_{WL,SC,h}} T_{WL,h,t} \qquad \forall h, t \tag{233}$$

Emigrants:

$$N_{X,h,t} = \xi_{NX,PL,h} \left(R_{ROW,t} P_{L,ROW,h,t} - P_{L,h,t} \right) \qquad \forall h, t \tag{234}$$

Immigrants:

$$\begin{aligned} N_{M,h,t} =& \xi_{NM,PS,h} \left(K_{PS,h,t} - K_{PS,ROW,h,t} \right) + \xi_{NM,EQ,h} \left(K_{EQ,h,t} - K_{EQ,ROW,h,t} \right) \\ & + \xi_{NM,ESF,h} \left(K_{ESF,h,t} - K_{ESF,ROW,h,t} \right) + \xi_{NM,SW,h} \left(\Lambda_{SW,h,t} - \Lambda_{SW,ROW,h,t} \right) \\ & + \xi_{NM,WL,h} \left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,ROW,h,t} \right) - \xi_{NM,\Psi,h} \left(K_{\Psi,h,t} - K_{\Psi,ROW,h,t} \right) \\ & \forall h, t \end{aligned} \tag{235}$$

Net migration:

$$I_{N,h,t} = N_{M,h,t} - N_{X,h,t} \qquad \forall h, t \tag{236}$$

Investment in crime prevention:

$$\begin{aligned} I_{PS,h,t} =& \xi_{PS,service,h} \left(Q_{PS,h,t} - C_{PS,h,t} \right) + \xi_{PS,IW,h} \left(y_{h,t} - y_{h,t-1} \right) \\ & + \xi_{PS,ESF,h} \left(K_{ESF,h,t} - K_{ESF,h,t-1} \right) + \xi_{PS,HS,h} \left(K_{HS,h,t} - K_{HS,h,t-1} \right) \\ & + \xi_{PS,SC,h} \left(K_{SC,h,t} - K_{SC,h,t-1} \right) - \xi_{PS,JLT,h} \left(J_{LT,h,t} - J_{LT,h,t-1} \right) \\ & - \xi_{PS,JST,h} \left(J_{ST,h,t} - J_{ST,h,t-1} \right) - \xi_{PS,\Upsilon,t} \left(\Upsilon_{A,IWK,t} - \Upsilon_{A,IWK,t-1} \right) \\ & \forall h, t \end{aligned} \tag{237}$$

Household investment in health:

$$\begin{aligned}I_{HS,h,t} =& \xi_{HS,service,h}\left(Q_{HS,h,t} - C_{HS,h,t}\right) + \xi_{HS,EQ,h}\left(K_{EQ,h,t} - K_{EQ,h,t-1}\right)\\ &- \xi_{HS,\Psi,h} K_{\Psi,h,t} - \xi_{HS,J,h}\left(J_{ST,h,t} + J_{LT,h,t} - J_{ST,h,t-1} - J_{LT,h,t-1}\right)\\ &+ \xi_{HS,IW,h}\left(K_{IW,h,t} - K_{IW,h,t-1}\right) + \xi_{HS,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right)\\ &+ \xi_{HS,SW,h}\left(\Lambda_{SW,h,t} - \Lambda_{SW,h,t-1}\right) + \xi_{HS,PS,h}\left(K_{PS,h,t} - K_{PS,h,t-1}\right)\\ &+ \xi_{HS,SC,h}\left(K_{SC,h,t} - K_{SC,h,t-1}\right) + \xi_{HS,HO,h}\left(C_{HO,h,t} - C_{HO,h,t-1}\right)\\ &- \xi_{HS,\Xi,h}\left(1 - \Xi_{HO,h,t}\right) + \xi_{HS,WL,h}\left(\tilde{T}_{WL,h,t} - A_{HS,T,h}\right) \qquad \forall h,t \end{aligned} \tag{238}$$

Household investment in social connection:

$$\begin{aligned}I_{SC,h,t} =& \xi_{SC,WL,h}\left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,h,t-1}\right) + \xi_{SC,TQ,h}\left(T_{Q,h,t} - T_{Q,h,t-1}\right)\\ &+ \xi_{SC,J,h}\left(J_{E,h,t} - J_{E,h,t-1}\right) + \xi_{SC,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right)\\ &+ \xi_{SC,PS,h}\left(K_{PS,h,t} - K_{PS,h,t-1}\right) + \xi_{SC,HS,h}\left(K_{HS,h,t} - K_{HS,h,t-1}\right)\\ &+ \xi_{SC,CG,h}\left(K_{CG,h,t} - K_{CG,h,t-1}\right) - \xi_{SC,N,h}\left(K_{N,h,t} - K_{N,h,t-1}\right)\\ &- \xi_{SC,NM,h} N_{M,h,t} - \xi_{SC,\Upsilon,h}\left(\Upsilon_{A,IWK,t} - \Upsilon_{A,IWK,t-1}\right) \qquad \forall h,t \end{aligned} \tag{239}$$

Household investment in civic engagement:

$$\begin{aligned}I_{CG,h,t} =& \xi_{CG,ESF,h}\left(K_{ESF,h,t} - K_{ESF,h,t-1}\right) + \xi_{CG,y,h}\left(y_{h,t} - y_{h,t-1}\right)\\ &+ \xi_{CG,WL,h}\left(\tilde{T}_{WL,h,t} - \tilde{T}_{WL,h,t-1}\right) + \xi_{CG,SC,h}\left(K_{SC,h,t} - K_{SC,h,t-1}\right)\\ &- \xi_{CG,J,h}\left(J_{ST,h,t} + J_{LT,h,t} - J_{ST,h,t-1} - J_{LT,h,t-1}\right)\\ &- \xi_{CG,NM,h}\left(N_{M,h,t} - N_{M,h,t-1}\right) \qquad \forall h,t \end{aligned} \tag{240}$$

Evolution of Stocks:

$$\begin{aligned} K_{l,h,t+1} = \left(1 - \delta_{l,h}\right) K_{l,h,t} + I_{l,h,t}\\ \forall h,t, \quad l \in \{CG, ESF, ESS, HS, IW, N, PS, SC\} \end{aligned} \tag{241}$$

$$K_{EQ,S,t+1} = \left(1 - \delta_{EQ,S}\right) K_{EQ,S,t} + I_{EQ,S,t} \qquad \forall t \tag{242}$$

$$K_{\Psi,S,t+1} = \left(1 - \delta_{\Psi,S}\right) K_{\Psi,S,t} + I_{\Psi,S,t} \qquad \forall t \tag{243}$$

$$K_{G,t+1} = \left(1 - \delta_{G}\right) K_{G,t} + I_{G,t} \qquad \forall t \tag{244}$$

$$K_{j,t+1} = \left(1 - \delta_{j}\right) K_{j,t} + I_{j,t} \qquad \forall j,t \tag{245}$$

B.4 Non-Evolved Variables

These variables are not used for calculating any of the evolved variables and so can be calculated for all time periods at once after the evolved variables have been calculated.

Total household consumption:

$$C_{h,t} = \sum_{k \notin \{PS,HOT\}} C_{h,k,t} + \sum_{h_1} \sum_{k \notin \{PS,HOT\}} Q_{h_1,h,k,t} \qquad \forall h,t \tag{246}$$

Housing price indicator:

$$P_{HO,h,t} = {y_{h,t}}^{-1} \left(P_{C,k=HOB,t} C_{h,k=HOB,t} + P_{C,k=HOG,t} C_{h,k=HOG,t} \right) \qquad \forall h,t \tag{247}$$

Quantity of exported capital:

$$\begin{aligned} K_{IW,X,t} = &\left({\gamma_{XK}}^{\sigma_{XK}} {P_{X,K,t}}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \\ &\times {\gamma_{XK}}^{\sigma_{XK}} {P_{X,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \end{aligned} \tag{248}$$

Quantity of imported capital:

$$\begin{aligned} K_{IW,M,t} = &\left({\gamma_{MK}}^{\sigma_{MK}} {P_{M,K,t}}^{-(\sigma_{MK}-1)} + (1-\gamma_{MK})^{\sigma_{MK}} {P_{D,K,t}}^{-(\sigma_{MK}-1)} \right)^{-\frac{\sigma_{MK}}{\sigma_{MK}-1}} \\ &\times {\gamma_{MK}}^{\sigma_{MK}} {P_{M,K,t}}^{-\sigma_{MK}} {A_{MK}}^{-1} K_{IW,C,t} \qquad \forall t \end{aligned} \tag{249}$$

Balance of Payments:

$$\begin{aligned} I_{ROW,t} = &{R_{ROW,t}}^{-1} \left(\sum_q P_{X,q,t} Q_{X,q,t} + P_{X,K,t} K_{IW,X,t} \right. \\ &\left. - \sum_q P_{M,q,t} Q_{M,q,t} - P_{M,K,t} K_{IW,M,t} \right) \qquad \forall t \end{aligned} \tag{250}$$

Societal averages:

$$\begin{gathered} V_{S,t} = \sum_h \frac{K_{N,h,t}}{K_{N,S,t}} V_{h,t} \qquad \forall t, \\ V \in \{C, C_{HO}, J_E, J_{LT}, K_{ESF}, K_{ESS}, K_{HS}, P_{HO}, \\ P_L, T_{HO}, \tilde{T}_{WL}, Y_h, y, \Lambda_{SW}, \Xi_{HO}, \Omega_{JE}\} \end{gathered} \tag{251}$$

Index calculation:

$$\hat{V}_t = \begin{cases} \frac{V_t}{V_B} & V_t \in \{C_{h_S}, C_{HO,h_S}, J_{E,h_S}, K_{CG,h_S}, K_{EQ,h_S}, K_{ESF,h_S}, K_{ESS,h_S}, K_{HS,h_S}, \\ & K_{IW,h_S}, K_{PS,h_S}, K_{SC,h_S}, P_{L,h_S}, \tilde{T}_{WL,h_S}, y_{h_S}, \Lambda_{SW,h_S}, \Xi_{HO,h_S}, \Omega_{JE,h_S}\} \\ \frac{V_B}{V_t} & V_t \in \{J_{LT,h_S}, K_{\Psi,h_S}, P_{HO,h_S}, T_{HO,h_S}\} \end{cases}$$
$$\forall h_S, t \tag{252}$$

B.4.1 Wellbeing

Civic engagement index:

$$\hat{W}_{CG,h_S,t} = \hat{K}_{CG,h_S,t} \qquad \forall h_S, t \tag{253}$$

Economic index:

$$\hat{W}_{IW,h_S,t} = \frac{1}{3}\left(\hat{y}_{h_S,t} + \hat{C}_{h_S,t} + \hat{K}_{IW,h_S,t}\right) \qquad \forall h_S, t \tag{254}$$

Education index:

$$\hat{W}_{ES,h_S,t} = \frac{1}{2}\left(\hat{K}_{ESF,h_S,t} + \hat{K}_{ESF,h_S,t}\right) \qquad \forall h_S, t \tag{255}$$

Environment index:

$$\hat{W}_{EQ,h_S,t} = \frac{1}{2}\left(\hat{K}_{EQ,h_S,t} + \hat{K}_{\Psi,h_S,t}\right) \qquad \forall h_S, t \tag{256}$$

Health index:

$$\hat{W}_{HS,h_S,t} = \hat{K}_{HS,h_S,t} \qquad \forall h_S, t \tag{257}$$

Housing index:

$$\hat{W}_{HO,h_S,t} = \frac{1}{4}\left(\hat{\Xi}_{HO,h_S,t} + \hat{C}_{HO,h_S,t} + \hat{P}_{HO,h_S,t} + \hat{T}_{HO,h_S,t}\right) \qquad \forall h_S, t \tag{258}$$

Jobs index:

$$\hat{W}_{JE,h_S,t} = \frac{1}{4}\left(\hat{J}_{E,h_S,t} + \hat{J}_{LT,h_S,t} + \hat{P}_{L,h_S,t} + \hat{\Omega}_{JE,h_S,t}\right) \qquad \forall h_S, t \tag{259}$$

Personal safety index:

$$\hat{W}_{PS,h_S,t} = \hat{K}_{PS,h_S,t} \qquad \forall h_S, t \tag{260}$$

Social connection index:

$$\hat{W}_{SC,h_S,t} = \hat{K}_{SC,h_S,t} \qquad \forall h_S, t \tag{261}$$

Subjective wellbeing index:

$$\hat{W}_{SW,h_S,t} = \hat{\Lambda}_{SW,h_S,t} \qquad \forall h_S, t \tag{262}$$

Work-life balance index:

$$\hat{W}_{WL,h_S,t} = \hat{\hat{T}}_{WL,h_S,t} \qquad \forall h_S, t \tag{263}$$

Period wellbeing:

$$\hat{W}_{h_S,t} = \prod_i \hat{W}_{i,h_S,t}{}^{\alpha_{i,h_S}} \qquad \forall i, h_S, t \tag{264}$$

Intergenerational wellbeing (Chichilnisky equation):

$$\hat{W}_{h_S} = \omega_{h_S} \sum_{t=0}^{\infty} \beta_{h_S}{}^t \hat{W}_{h_S,t} + (1 - \omega_{h_S}) \lim_{t \to \infty} \hat{W}_{h_S,t} \qquad \forall h_S \tag{265}$$

B.4.2 Inequality

Atkinson inequality measure:

$$\Upsilon_{A,m,t} = \begin{cases} 1 - \dfrac{1}{V_{m,S,t}} \left(\frac{1}{K_{N,S,t}} \sum\limits_h K_{N,h,t} V_{m,h,t}{}^{1-\epsilon_\Upsilon} \right)^{\frac{1}{1-\epsilon_\Upsilon}} & \epsilon_\Upsilon = 1 \\ 1 - \dfrac{1}{V_{m,S,t}} \left(\prod\limits_h V_{m,h,t}{}^{K_{N,h,t}} \right)^{\frac{1}{K_{N,S,t}}} & \epsilon_\Upsilon \geq 0, \quad \epsilon_\Upsilon \neq 1 \end{cases} \tag{266}$$
$$\forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\}$$

Gini inequality measure:

$$\Upsilon_{G,m,t} = \frac{\sum\limits_{h_1} \sum\limits_{h_2 > h_1} K_{N,h_1,t} K_{N,h_2,t} \left| V_{m,h_1,t} - V_{m,h_2,t} \right|}{K_{N,S,t} \sum\limits_{h_3} K_{N,h_3,t} V_{m,h_3,t}} \tag{267}$$
$$\forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\}$$

Theil inequality measure:

$$\Upsilon_{T,m,t} = \sum_h \frac{K_{N,h,t}}{K_{N,S,t}} \frac{V_{m,h,t}}{V_{m,S,t}} \ln \left(\frac{V_{m,h,t}}{V_{m,S,t}} \right) \qquad \forall t, \quad V_{m,h,t} \in \left\{ Y_{h,t}, y_{h,t}, K_{IW,h,t}, \hat{W}_{h,t} \right\} \tag{268}$$

C Derivation of Model Equations

This appendix shows some of the details of the non-trivial derivations of equations in the model.

C.1 General Derivatives

In this section, general derivatives of the Cobb-Douglas and CES functions are given, for use in later sections.

Generic Cobb-Douglas function:

$$\begin{aligned} Q =& A \prod_i {X_i}^{\alpha_i} \\ \frac{\partial Q}{\partial X_j} =& \alpha_j {X_j}^{-1} Q \end{aligned} \tag{269}$$

Generic CES function:

$$\begin{aligned} Q =& A \left(\sum_i \gamma_i {X_i}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \\ \frac{\partial Q}{\partial X_j} =& \left(\frac{{\gamma_j}^{\sigma} Q}{A^{\sigma-1} X_j} \right)^{\frac{1}{\sigma}} \end{aligned} \tag{270}$$

C.2 Household Consumption

Household type h in time period t experiences utility from consumption according to the Cobb-Douglas utility function:

$$U_{C,h,t} = \prod_{k \neq HOT} {C_{h,k,t}}^{\alpha_{C,h,k}} \qquad \forall h, t \tag{271}$$

with derivative:

$$\frac{\partial U_{C,h_1,t}}{\partial C_{h_2,k_2,t}} = \alpha_{C,h_2,k_2} {C_{h_2,k_2,t}}^{-1} U_{C,h_2,t} \qquad \forall h, k \neq HOT, t \tag{272}$$

Note that because $C_{h,k=HOT,t}$ is calculated separately, it must be excluded from this analysis.

Households face a budget constraint given by:

$$B_{h,t} - (1 + \tau_{C,k=HOT})\, P_{C,k=HOT,t} C_{h,k=HOT,t} = \sum_{k \neq HOT} (1 + \tau_{C,k})\, P_{C,k,t} C_{h,k,t} \qquad \forall h,t \tag{273}$$

with derivative:

$$\frac{\partial \left(B_{h_1,t} - (1 + \tau_{C,k_1=HOT})\, P_{C,k_1=HOT,t} C_{h,k_1=HOT,t}\right)}{\partial C_{h_2,k_2,t}} = (1 + \tau_{C,k_2})\, P_{C,k_2,t} \qquad \forall h, k \neq HOT, t \tag{274}$$

Assume we wish to maximise utility due to consumption, subject to the budget constraint. Use the Lagrangian method to find first order conditions. The Lagrangrian is given by:

$$\begin{aligned} \mathcal{L}_{h,t} = & \prod_{k \neq HOT} {C_{h,k,t}}^{\alpha_{C,h,k}} \\ & + \lambda_{h,t} \Bigg(B_{h,t} - (1 + \tau_{C,k=HOT})\, P_{C,k=HOT,t} C_{h,k=HOT,t} \\ & - \sum_{k \neq HOT} (1 + \tau_{C,k})\, P_{C,k,t} C_{h,k,t}\Bigg) \qquad \forall h,t \end{aligned} \tag{275}$$

Differentiating (275) with respect to $C_{h,k,t}$, using (272) and (274), and setting to zero gives the first order condition:

$$\lambda_{h,t} (1 + \tau_{C,k})\, P_{C,k,t} C_{h,k,t} = \alpha_{C,h,k} U_{C,h,t} \qquad \forall h, k \neq HOT, t \tag{276}$$

Summing over $k \neq HOT$:

$$\lambda_{h,t} \sum_{k \neq HOT} (1 + \tau_{C,k})\, P_{C,k,t} C_{h,k,t} = U_{C,h,t} \sum_{k \neq HOT} \alpha_{C,h,k} \qquad \forall h,t \tag{277}$$

Use $\sum_{k \neq HOT} \alpha_{C,h,k} = 1$:

$$\lambda_{h,t} \sum_{k \neq HOT} (1 + \tau_{C,k})\, P_{C,k,t} C_{h,k,t} = U_{C,h,t} \qquad \forall h,t \tag{278}$$

Substitute in (273)

$$\lambda_{h,t} = \frac{U_{C,h,t}}{B_{h,t} - (1 + \tau_{C,k=HOT})\, P_{C,k=HOT,t} C_{h,k=HOT,t}} \qquad \forall h,t \tag{279}$$

Substitute (279) back into (276):

$$C_{h,k,t} = \frac{\alpha_{C,h,k} \left(B_{h,t} - (1 + \tau_{C,k=HOT})\, P_{C,k=HOT,t} C_{h,k_1=HOT,t}\right)}{(1 + \tau_{C,k})\, P_{C,k,t}} \qquad \forall h, k \neq HOT, t \tag{280}$$

This gives the level of consumption of good $k \neq HOT$ by household type h in time period t as required.

C.3 Government Consumption from Investment

The government in time period experiences utility from consumption according to the Cobb-Douglas utility function:

$$U_{C,G,t} = \prod_k C_{G,I,k,t}{}^{\alpha_{C,G,k}} \qquad \forall t \tag{281}$$

with derivative:

$$\frac{\partial U_{C,G,t}}{\partial C_{G,I,k,t}} = \alpha_{C,G,k} C_{G,I,k,t}{}^{-1} U_{C,G,t} \qquad \forall k,t \tag{282}$$

The government faces a budget constraint given by:

$$I_{G,t} = \sum_k P_{C,k,t} C_{G,I,k,t} \qquad \forall t \tag{283}$$

with derivative:

$$\frac{\partial I_{G,t}}{\partial C_{G,I,k,t}} = P_{C,k,t} \qquad \forall k,t \tag{284}$$

Assume we wish to maximise utility due to consumption, subject to the budget constraint. Use the Lagrangian method to find first order conditions. The Lagrangrian is given by:

$$\mathcal{L}_{G,t} = \prod_k C_{G,I,k,t}{}^{\alpha_{C,G,k}} + \lambda_{G,t} \left(I_{G,t} - \sum_k P_{C,k,t} C_{G,I,k,t} \right) \qquad \forall t \tag{285}$$

Differentiating (285) with respect to $C_{G,k,t}$, using (282) and (284), and setting to zero gives the first order condition:

$$\lambda_{G,t} P_{C,k,t} C_{G,I,k,t} = \alpha_{C,G,k} U_{C,G,t} \qquad \forall k,t \tag{286}$$

Summing over k:

$$\lambda_{G,t} \sum_k P_{C,k,t} C_{G,I,k,t} = U_{C,G,t} \sum_k \alpha_{C,G,k} \qquad \forall t \tag{287}$$

Use $\sum_k \alpha_{C,G,k} = 1$:

$$\lambda_{G,t} \sum_k P_{C,k,t} C_{G,I,k,t} = U_{C,G,t} \qquad \forall t \tag{288}$$

Substitute in (283)

$$\lambda_{G,t} = \frac{U_{C,G,t}}{I_{G,t}} \qquad \forall t \tag{289}$$

Substitute (289) back into (286):

$$C_{G,I,k,t} = \frac{\alpha_{C,G,k} I_{G,t}}{P_{C,k,t}} \qquad \forall k, t \tag{290}$$

This gives the level of consumption of good k for government investment in time period t as required.

C.4 Production

C.4.1 Domestic Production

The production process e for extracted good j in time period t obeys the Cobb-Douglas production function:

$$\begin{gathered} Q_{e,j,t} = A_{Q,e,j,t} \prod_g V_{g,t}{}^{\eta_{g,e,j}} \prod_f V_{f,e,j,t}{}^{\alpha_{f,e,j}} \\ \forall e, j, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad V_f \in \{L_{Q,h}, K_Q\} \end{gathered} \tag{291}$$

with derivative

$$\frac{\partial Q_{e_1,j_1,t}}{\partial V_{f_2,e_2,j_2,t}} = \alpha_{f_2,e_2,j_2} V_{f_2,e_2,j_2,t}{}^{-1} Q_{e_2,j_2,t} \qquad \forall e, j, t, \quad V_f \in \{L_{Q,h}, K_Q\} \tag{292}$$

The production process e for final good k in time period t also obeys the Cobb-Douglas production function:

$$\begin{gathered} Q_{e,k,t} = A_{Q,e,k,t} \prod_g V_{g,t}{}^{\eta_{g,e,k}} \prod_f V_{f,e,k,t}{}^{\alpha_{f,e,k}} \prod_j V_{j,e,k,t}{}^{\alpha_{j,e,k}} \\ \forall e, k, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{293}$$

with derivatives

$$\frac{\partial Q_{e_1,k_1,t}}{\partial V_{f_2,e_2,k_2,t}} = \alpha_{f_2,e_2,k_2} V_{f_2,e_2,k_2,t}{}^{-1} Q_{e_2,k_2,t} \qquad \forall e, k, t, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \tag{294}$$

$$\frac{\partial Q_{e_1,k_1,t}}{\partial V_{j_2,e_2,k_2,t}} = \alpha_{j_2,e_2,k_2} V_{j_2,e_2,k_2,t}{}^{-1} Q_{e_2,k_2,t} \qquad \forall e, j, k, t \tag{295}$$

These processes produce pollution:

$$\Psi_{e,q,t} = \xi_{\Psi,e,q} Q_{e,q,t} \qquad \forall e, q, t \tag{296}$$

In addition to the labour used for production, producers also pay for labour time for developing skills:

$$\tilde{L}_{Q,h,e,q,t} = V_{f=\tilde{L},h,e,q,t} + \tilde{L}_{ESS,h,e,q,t} \qquad \forall h, e, q, t \tag{297}$$

where

$$V_{f=\tilde{L},h,e,q,t} = (1 - \xi_{Q,ESS,h,e,q})\,\tilde{L}_{Q,h,e,q,t} \qquad \forall h,e,q,t \tag{298}$$

If we define

$$P_{\tilde{L},Q,h,e,q,t} = \frac{P_{\tilde{L},S,h,t}}{1 - \xi_{Q,ESS,h,e,q}} \qquad \forall h,e,q,t \tag{299}$$

then the total amount spent by firm q on labour from household h for use in process e in time period t will be:

$$P_{\tilde{L},S,h,t}\tilde{L}_{Q,h,e,q,t} = P_{\tilde{L},Q,h,e,q,t}V_{f=\tilde{L},h,e,q,t} \qquad \forall h,e,q,t \tag{300}$$

The budget/zero profit constraint for this process for firm j is then:

$$\begin{gathered} P_{Q,e,j,t}Q_{e,j,t} = P_{\Psi}\Psi_{e,j,t} + \sum_f P_{f,e,j,t}V_{f,e,j,t} \\ \forall e,j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{301}$$

with derivative:

$$\begin{aligned} \frac{\partial\left(P_{Q,e_1,j_1,t}Q_{e_1,j_1,t}\right)}{\partial V_{f_2,e_2,j_2,t}} =& P_{f_2,e_2,j_2,t} + \alpha_{f_2,e_2,j_2}{V_{f_2,e_2,j_2,t}}^{-1}P_{\Psi}\Psi_{e_2,j_2,t} \\ & \forall e,j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{302}$$

The budget/zero profit constraint for this process for firm k is then:

$$\begin{gathered} P_{Q,e,k,t}Q_{e,k,t} = P_{\Psi}\Psi_{e,k,t} + \sum_f P_{f,e,k,t}V_{f,e,k,t} + \sum_j P_{Q,j,t}V_{j,e,k,t} \\ \forall e,k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{gathered} \tag{303}$$

with derivatives:

$$\begin{aligned} \frac{\partial\left(P_{Q,e_1,k_1,t}Q_{e_1,k_1,t}\right)}{\partial V_{f_2,e_2,k_2,t}} =& P_{f_2,e_2,k_2,t} + \alpha_{f_2,e_2,k_2}{V_{f_2,e_2,k_2,t}}^{-1}P_{\Psi}\Psi_{e_2,k_2,t} \\ & \forall e,k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{304}$$

$$\frac{\partial\left(P_{Q,e_1,k_1,t}Q_{e_1,k_1,t}\right)}{\partial V_{j_2,e_2,k_2,t}} = P_{Q,j_2,t} + \alpha_{j_2,e_2,k_2}{V_{j_2,e_2,k_2,t}}^{-1}P_{\Psi}\Psi_{e_2,k_2,t} \qquad \forall e,j,k,t \tag{305}$$

The total production of extraction good j in time period t is given by the CES function:

$$Q_{q,t} = A_{Q,q}\left(\sum_e \gamma_{e,q}{Q_{e,q,t}}^{\frac{\sigma_q - 1}{\sigma_q}}\right)^{\frac{\sigma_q}{\sigma_q - 1}} \qquad \forall q,t \tag{306}$$

with derivative

$$\frac{\partial Q_{q_1,t}}{\partial Q_{e_2,q_2,t}} = \left(\frac{{\gamma_{e_2,q_2}}^{\sigma_{q2}}Q_{q_2,t}}{{A_{Q,q_2}}^{\sigma_{q2}-1}Q_{e_2,q_2,t}}\right)^{\frac{1}{\sigma_{q2}}} \qquad \forall e,q,t \tag{307}$$

The budget/zero profit constraint for this good is:

$$P_{Q,q,t}Q_{q,t} = \sum_{e} P_{Q,e,q,t}Q_{e,q,t} \qquad \forall q,t \tag{308}$$

Eliminate $P_{Q,e,j,t}$ from (308) using the process budget constraint (301) to give

$$\begin{aligned} P_{Q,j,t}Q_{j,t} =& P_{\Psi}\sum_{e}\Psi_{e,j,t} + \sum_{e}\sum_{f} P_{f,e,j,t}V_{f,e,j,t} \\ & \forall j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{309}$$

with derivative:

$$\begin{aligned} \frac{\partial\left(P_{Q,j_1,t}Q_{j_1,t}\right)}{\partial V_{f_2,e_2,j_2,t}} =& P_{f_2,e_2,j_2,t} + \alpha_{f_2,e_2,j_2}V_{f_2,e_2,j_2,t}{}^{-1}P_{\Psi}\Psi_{e_2,j_2,t} \\ & \forall e,j,t, \quad P_f \in \left\{P_{L,Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{L_{Q,h}, K_Q\right\} \end{aligned} \tag{310}$$

and eliminate $P_{Q,e,k,t}$ from (308) using the process budget constraint (303) to give

$$\begin{aligned} P_{Q,k,t}Q_{k,t} =& P_{\Psi}\sum_{e}\Psi_{e,k,t} + \sum_{e}\sum_{f} P_{f,e,k,t}V_{f,e,k,t} + \sum_{e}\sum_{j} P_{Q,j,t}V_{j,e,k,t} \\ & \forall k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{311}$$

with derivatives:

$$\begin{aligned} \frac{\partial\left(P_{Q,k_1,t}Q_{k_1,t}\right)}{\partial V_{f_2,e_2,k_2,t}} =& P_{f_2,e_2,k_2,t} + \alpha_{f_2,e_2,k_2}V_{f_2,e_2,k_2,t}{}^{-1}P_{\Psi}\Psi_{e_2,k_2,t} \\ & \forall e,k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{312}$$

$$\frac{\partial\left(P_{Q,k_1,t}Q_{k_1,t}\right)}{\partial V_{j_2,e_2,k_2,t}} = P_{Q,j_2,t} + \alpha_{j_2,e_2,k_2}V_{j_2,e_2,k_2,t}{}^{-1}P_{\Psi}\Psi_{e_2,k_2,t} \qquad \forall e,j,k,t \tag{313}$$

Assume we wish to maximise production of the final good, subject to the budget/zero profit constraint. Use the Lagrangian method to find first order conditions. The Lagrangrians are given by:

$$\begin{aligned} \mathcal{L}_{j,t} =& A_{Q,j}\left(\sum_{e}\gamma_{e,j}Q_{e,j,t}{}^{\frac{\sigma_j-1}{\sigma_j}}\right)^{\frac{\sigma_j}{\sigma_j-1}} \\ & + \lambda_{j,t}\left(P_{Q,j,t}Q_{j,t} - P_{\Psi}\sum_{e}\Psi_{e,j,t} - \sum_{e}\sum_{f}P_{f,e,j,t}V_{f,e,j,t}\right) \\ & \forall j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\} \end{aligned} \tag{314}$$

$$
\begin{aligned}
\mathcal{L}_{k,t} =& A_{Q,k}\left(\sum_e \gamma_{e,k} {Q_{e,k,t}}^{\frac{\sigma_k-1}{\sigma_k}}\right)^{\frac{\sigma_k}{\sigma_k-1}} \\
&+ \lambda_{k,t}\Bigg(P_{Q,k,t}Q_{k,t} - P_\Psi \sum_e \Psi_{e,k,t} \\
&\qquad - \sum_e \sum_f P_{f,e,k,t}V_{f,e,k,t} - \sum_e \sum_j P_{Q,j,t}V_{j,e,k,t}\Bigg) \\
&\forall k,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_K\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\}
\end{aligned} \tag{315}
$$

Differentiating (314) and (315) with respect to $V_{f,e,q,t}$, using (302), (307), (310), and (312) then setting to zero gives the first order condition:

$$
\begin{aligned}
0 =& \alpha_{f,e,q}\gamma_{e,q}{A_{Q,q}}^{\frac{1-\sigma_q}{\sigma_q}} Q_{e,q,t}\left(\frac{Q_{q,t}}{Q_{e,q,t}}\right)^{\frac{1}{\sigma_q}} - \lambda_{q,t}\left(P_{f,e,q,t}V_{f,e,q,t} + \alpha_{f,e,q}P_\Psi \Psi_{e,q,t}\right) \\
&\forall q,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\}
\end{aligned} \tag{316}
$$

Summing over f:

$$
\begin{aligned}
0 =& \gamma_{e,q}{A_{Q,q}}^{\frac{1-\sigma_q}{\sigma_q}} Q_{e,q,t}\left(\frac{Q_{q,t}}{Q_{e,q,t}}\right)^{\frac{1}{\sigma_q}} \sum_f \alpha_{f,e,q} \\
&- \lambda_{q,t}\left(\sum_f P_{f,e,q,t}V_{f,e,q,t} + P_\Psi \Psi_{e,q,t}\sum_f \alpha_{f,e,q}\right) \\
&\forall e,q,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{\tilde{L}_{Q,h}, K_Q\right\}
\end{aligned} \tag{317}
$$

Use $\sum_f \alpha_{f,e,j} = 1$:

$$
\begin{aligned}
0 = \gamma_{e,j}{A_{Q,j}}^{\frac{1-\sigma_j}{\sigma_j}} Q_{e,j,t}\left(\frac{Q_{j,t}}{Q_{e,j,t}}\right)^{\frac{1}{\sigma_j}} - \lambda_{j,t}\left(\sum_f P_{f,e,j,t}V_{f,e,j,t} + P_\Psi \Psi_{e,j,t}\right) \\
\forall e,j,t, \quad P_f \in \left\{P_{L,Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{L_{Q,h}, K_Q\right\}
\end{aligned} \tag{318}
$$

Sum over e:

$$
\begin{aligned}
0 =& {A_{Q,j}}^{\frac{1-\sigma_j}{\sigma_j}} {Q_{j,t}}^{\frac{1}{\sigma_j}} \sum_e \gamma_{e,j}{Q_{e,j,t}}^{\frac{\sigma_j-1}{\sigma_j}} - \lambda_{j,t}\left(\sum_e \sum_f P_{f,e,j,t}V_{f,e,j,t} + P_\Psi \sum_e \Psi_{e,j,t}\right) \\
&\forall j,t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_f \in \left\{L_{Q,h}, K_Q\right\}
\end{aligned} \tag{319}
$$

Differentiating (315) with respect to $V_{j,e,k,t}$, using (295), (305), (313), and setting to zero:

$$\begin{aligned} 0 =& \alpha_{j_2,e_2,k_2}\gamma_{e_2,k_2}A_{Q,k_2}{}^{\frac{1-\sigma_{k_2}}{\sigma_{k_2}}}Q_{e_2,k_2,t}\left(\frac{Q_{k_2,t}}{Q_{e_2,k_2,t}}\right)^{\frac{1}{\sigma_{k_2}}} \\ &-\lambda_{k_2,t}\left(P_{Q,j_2,t}V_{j_2,e_2,k_2,t}+\alpha_{j_2,e_2,k_2}P_\Psi\Psi_{e_2,k_2,t}\right) \qquad \forall e,j,k,t \end{aligned} \tag{320}$$

Summing over j:

$$\begin{aligned} 0 =& \gamma_{e,k}A_{Q,k}{}^{\frac{1-\sigma_k}{\sigma_k}}Q_{e,k,t}\left(\frac{Q_{k,t}}{Q_{e,k,t}}\right)^{\frac{1}{\sigma_k}}\sum_j\alpha_{j,e,k} \\ &-\lambda_{k,t}\left(\sum_j P_{Q,j,t}V_{j,e,k,t}+P_\Psi\Psi_{e,k,t}\sum_j\alpha_{j,e,k}\right) \qquad \forall e,j,k,t \end{aligned} \tag{321}$$

Adding together (317) and (321):

$$\begin{aligned} 0 =& \gamma_{e,k}A_{Q,k}{}^{\frac{1-\sigma_k}{\sigma_k}}Q_{e,k,t}\left(\frac{Q_{k,t}}{Q_{e,k,t}}\right)^{\frac{1}{\sigma_k}}\left(\sum_f\alpha_{f,e,k}+\sum_j\alpha_{j,e,k}\right) \\ &-\lambda_{k,t}\Bigg(\sum_f P_{f,e,k,t}V_{f,e,k,t}+\sum_j P_{Q,j,t}V_{j,e,k,t} \\ &\qquad +P_\Psi\Psi_{e,k,t}\left(\sum_f\alpha_{f,e,k}+\sum_j\alpha_{j,e,k}\right)\Bigg) \\ &\forall e,k,t, \quad P_f\in\left\{P_{\tilde{L},Q,h},P_{C,K}\right\}, \quad V_f\in\left\{\tilde{L}_{Q,h},K_Q\right\} \end{aligned} \tag{322}$$

Use $\sum\limits_f\alpha_{f,e,k}+\sum\limits_j\alpha_{j,e,k}=1$:

$$\begin{aligned} 0 =& \gamma_{e,k}A_{Q,k}{}^{\frac{1-\sigma_k}{\sigma_k}}Q_{e,k,t}\left(\frac{Q_{k,t}}{Q_{e,k,t}}\right)^{\frac{1}{\sigma_k}} \\ &-\lambda_{k,t}\left(\sum_f P_{f,e,k,t}V_{f,e,k,t}+\sum_j P_{Q,j,t}V_{j,e,k,t}+P_\Psi\Psi_{e,k,t}\right) \\ &\forall e,k,t, \quad P_f\in\left\{P_{\tilde{L},Q,h},P_{C,K}\right\}, \quad V_f\in\left\{\tilde{L}_{Q,h},K_Q\right\} \end{aligned} \tag{323}$$

Sum over e:

$$\begin{aligned} 0 =& A_{Q,k}{}^{\frac{1-\sigma_k}{\sigma_k}}Q_{k,t}{}^{\frac{1}{\sigma_k}}\sum_e\gamma_{e,k}Q_{e,k,t}{}^{\frac{\sigma_k-1}{\sigma_k}} \\ &-\lambda_{k,t}\left(\sum_e\sum_f P_{f,e,k,t}V_{f,e,k,t}+\sum_e\sum_j P_{Q,j,t}V_{j,e,k,t}+P_\Psi\sum_e\Psi_{e,k,t}\right) \\ &\forall k,t, \quad P_f\in\left\{P_{\tilde{L},Q,h},P_K\right\}, \quad V_f\in\left\{\tilde{L}_{Q,h},K_Q\right\} \end{aligned} \tag{324}$$

Substitute (306), (309), and (311) into (319) and (324):

$$\lambda_{q,t} = P_{Q,q,t}{}^{-1} A_{Q,q}{}^{-2\frac{\sigma_q - 1}{\sigma_q}} \qquad \forall q, t \tag{325}$$

Substitute (325) back into (316):

$$V_{f,e,q,t} = \alpha_{f,e,q} P_{f,e,q,t}{}^{-1} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \tag{326}$$
$$\forall e, q, t, \quad P_f \in \{ P_{\tilde{L},Q,h}, P_{C,K} \}, \quad V_f \in \left\{ \tilde{L}_{Q,h}, K_Q \right\}$$

This gives the demand for factor of production f for producing good q using production process e in time period t.

Substitute (325) back into (323)

$$V_{j,e,k,t} = \alpha_{j,e,k} P_{Q,j,t}{}^{-1} \left(\gamma_{e,k} P_{Q,k,t} Q_{k,t} A_{Q,k}{}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}} \right)^{\frac{\sigma_k - 1}{\sigma_k}} - P_\Psi \Psi_{e,k,t} \right) \tag{327}$$
$$\forall e, j, k, t$$

This gives the demand for intermediate input j for producing final good k using production process e in time period t.

Substitute (326) into the production equation (291):

$$\gamma_{e,j} P_{Q,j,t} A_{Q,j}{}^{\frac{\sigma_j - 1}{\sigma_j}} \left(\frac{Q_{e,j,t}}{Q_{j,t}} \right)^{\frac{-1}{\sigma_j}} = A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}} + P_\Psi \xi_{\Psi,e,j} \tag{328}$$
$$\forall e, j, t, \quad V_g \in \{ K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j \}, \quad P_f \in \{ P_{\tilde{L},Q,h}, P_{C,K} \}$$

and substitute (326) and (327) into the production equation (293)

$$\gamma_{e,k} P_{Q,k,t} A_{Q,k}{}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}} \right)^{\frac{-1}{\sigma_k}} = A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} + P_\Psi \xi_{\Psi,e,k} \tag{329}$$
$$\forall e, k, t, \quad V_g \in \{ K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S} \}, \quad P_f \in \{ P_{\tilde{L},Q,h}, P_{C,K} \}$$

For notational convenience, define the price factor for extraction goods as

$$X_{e,j,t} = P_\Psi \xi_{\Psi,e,j} + A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}}$$
$$\forall e,j,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\} \tag{330}$$

and the price factor for final goods as

$$X_{e,k,t} = P_\Psi \xi_{\Psi,e,k} + A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}}$$
$$\times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \tag{331}$$
$$\forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_K\right\}$$

Substitute (330) into (328) and (331) into (329) to get

$$Q_{e,q,t} = Q_{q,t} \gamma_{e,q}{}^{\sigma_q} P_{Q,q,t}{}^{\sigma_q} A_{Q,q,t}{}^{\sigma_q - 1} X_{e,q,t}{}^{-\sigma_q} \qquad \forall e,q,t \tag{332}$$

which is the production of good q using production process e in time period t.

Substitute (332) into the good CES production function (306) to get

$$P_{Q,q,t} = A_{Q,q}{}^{-1} \left(\sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{1-\sigma_q} \right)^{\frac{-1}{\sigma_q - 1}} \qquad \forall q,t \tag{333}$$

which is the price of good q in time period t.

C.4.2 Price of Factors of Production

Assume markets for factors of production clear:

$$V_{S,f,t} = \sum_q \sum_e V_{f,e,q,t} \qquad \forall t, \quad V_{S,f} \in \{K_{N,S} K_{IW,S}\} \tag{334}$$

Substitute in (326)

$$V_{S,f,t} = \sum_q \sum_e \alpha_{f,e,q} P_{f,e,q,t}{}^{-1} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right)$$
$$\forall t, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}, \quad V_{S,f} \in \left\{K_{N,S} \tilde{L}_{Q,h}, K_{N,S} K_{IW,S}\right\} \tag{335}$$

For labour, where different industries have different effective labour costs based on the amount of skills training they provide, (334) becomes

$$K_{N,S} \tilde{L}_{S,h,t} = \sum_q \sum_e \tilde{L}_{Q,h,e,q,t} \qquad \forall h,t \tag{336}$$

Using (300) and (326), this becomes

$$P_{\tilde{L},S,h,t} = \sum_q \sum_e \alpha_{f,e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \times K_{N,S,t}{}^{-1} \tilde{L}_{S,h,t}{}^{-1} \quad \forall h,t \tag{337}$$

For all other factors of production, which have the same cost to any industry, (335) becomes

$$P_{f,t} = V_{S,f,t}{}^{-1} \sum_q \sum_e \alpha_{f,e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \quad \forall t, \quad P_f \in \{P_{C,K}\}, \quad V_{S,f} \in \{K_{N,S} K_{IW,S}\} \tag{338}$$

C.4.3 Wages

Wages received by the household can be derived by assuming that the labour income paid by producers equals the total income from labour received by households:

$$K_{N,h,t} P_{L,h,t} T_{IW,h,t} = K_{S,t} P_{\tilde{L},S,h,t} \tilde{L}_{S,h,t} \qquad \forall h,t \tag{339}$$

Using

$$\tilde{L}_{S,h,t} = A_{\tilde{L},h} K_{ESF,h,t}{}^{\eta_{L,ESF,h}} K_{ESS,h,t}{}^{\eta_{L,ESS,h}} K_{HS,h,t}{}^{\eta_{L,HS,h}} T_{IW,h,t} \frac{K_{N,h,t}}{K_{N,S,t}} \qquad \forall h,t \tag{340}$$

equation (339) becomes:

$$P_{L,h,t} = A_{\tilde{L},h} K_{ESF,h,t}{}^{\eta_{L,ESF,h}} K_{ESS,h,t}{}^{\eta_{L,ESS,h}} K_{HS,h,t}{}^{\eta_{L,HS,h}} P_{\tilde{L},S,h,t} \qquad \forall h,t \tag{341}$$

which is the wages received by household type h in time period t.

C.5 Rest of the World

C.5.1 Exports of Goods

Exports of goods of type q are determined using the CET function:

$$Q_{q,t} = A_{X,q} \left(\gamma_{X,q} Q_{X,q,t}{}^{\frac{\sigma_{X,q} - 1}{\sigma_{X,q}}} + (1 - \gamma_{X,q}) Q_{D,q,t}{}^{\frac{\sigma_{X,q} - 1}{\sigma_{X,q}}} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q} - 1}} \qquad \forall q,t \tag{342}$$

with derivatives:

$$\frac{\partial Q_{q_1,t}}{\partial Q_{X,q_2,t}} = \left(\frac{\gamma_{X,q_2}{}^{\sigma_{X,q_2}} Q_{q_2,t}}{A_{X,q_2}{}^{\sigma_{X,q_2}-1} Q_{X,q_2,t}} \right)^{\frac{1}{\sigma_{X,q_2}}} \qquad \forall q,t \tag{343}$$

$$\frac{\partial Q_{q_1,t}}{\partial Q_{D,q_2,t}} = \left(\frac{(1-\gamma_{X,q_2})^{\sigma_{X,q_2}} Q_{q_2,t}}{A_{X,q_2}}^{\sigma_{X,q_2}-1} Q_{D,q_2,t} \right)^{\frac{1}{\sigma_{X,q_2}}} \qquad \forall q,t \tag{344}$$

The revenue generated by this good is:

$$P_{Q,q,t} Q_{q,t} = P_{X,q,t} Q_{X,q,t} + P_{D,q,t} Q_{D,q,t} \qquad \forall q,t \tag{345}$$

with derivatives:

$$\frac{\partial \left(P_{Q,q_1,t} Q_{q_1,t} \right)}{\partial Q_{X,q_2,t}} = P_{X,q_2,t} \qquad \forall q,t \tag{346}$$

$$\frac{\partial \left(P_{Q,q_1,t} Q_{q_1,t} \right)}{\partial Q_{D,q_2,t}} = P_{D,q_2,t} \qquad \forall q,t \tag{347}$$

Assume we wish to maximise revenue from the good, subject to the CET function. Use the Lagrangian method to find first order conditions. The Lagrangrian is given by:

$$\begin{aligned} \mathcal{L}_{q,t} =& P_{X,q,t} Q_{X,q,t} + P_{D,q,t} Q_{D,q,t} \\ &+ \lambda_{q,t} \left(Q_{q,t} - A_{X,q} \left(\gamma_{X,q} Q_{X,q,t}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} + (1-\gamma_{X,q}) Q_{D,q,t}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \right) \\ &\forall q,t \end{aligned} \tag{348}$$

Differentiating (348) with respect to $Q_{X,q,t}$, using (343), (346), and setting to zero gives the first order condition:

$$A_{X,q}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} P_{X,q,t} Q_{X,q,t}{}^{\frac{1}{\sigma_{X,q}}} = \lambda_{q,t} \gamma_{X,q} Q_{q,t}{}^{\frac{1}{\sigma_{X,q}}} \qquad \forall q,t \tag{349}$$

Differentiating (348) with respect to $Q_{D,q,t}$, using (343), (347), and setting to zero gives the first order condition:

$$A_{X,q}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} P_{D,q,t} Q_{D,q,t}{}^{\frac{1}{\sigma_{X,q}}} = \lambda_{q,t} (1-\gamma_{X,q}) Q_{q,t}{}^{\frac{1}{\sigma_{X,q}}} \qquad \forall q,t \tag{350}$$

Adding together (349) and (350):

$$\lambda_{q,t} = Q_{q,t}{}^{\sigma_{X,q}} A_{X,q}{}^{\frac{\sigma_{X,q}-1}{\sigma_{X,q}}} \left(P_{X,q,t} Q_{X,q,t}{}^{\frac{1}{\sigma_{X,q}}} + P_{D,q,t} Q_{D,q,t}{}^{\frac{1}{\sigma_{X,q}}} \right) \qquad \forall q,t \tag{351}$$

Substitute (351) back into (349):

$$Q_{X,q,t} = \left(\frac{\gamma_{X,q}}{(1-\gamma_{X,q})} \frac{P_{D,q,t}}{P_{X,q,t}} \right)^{\sigma_{X,q}} Q_{D,q,t} \qquad \forall q, t \tag{352}$$

Substituting this back in to (342) and solving for $Q_{D,q,t}$:

$$\begin{aligned} Q_{D,q,t} = &\left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-(\sigma_{X,q}-1)} + (1-\gamma_{X,q})^{\sigma_{X,q}} {P_{D,q,t}}^{-(\sigma_{X,q}-1)} \right)^{-\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\ &\times (1-\gamma_{X,q})^{\sigma_{X,q}} {P_{D,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} Q_{q,t} \qquad \forall q, t \end{aligned} \tag{353}$$

This gives the locally-produced goods q in time period t that are not exported.

Substituting (353) into (352):

$$\begin{aligned} Q_{X,q,t} = &\left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-(\sigma_{X,q}-1)} + (1-\gamma_{X,q})^{\sigma_{X,q}} {P_{D,q,t}}^{-(\sigma_{X,q}-1)} \right)^{-\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\ &\times {\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} Q_{q,t} \qquad \forall q, t \end{aligned} \tag{354}$$

This gives the exported goods q in time period t.

C.5.2 Exports of Capital

Exports of financial capital are determined using the CET function:

$$K_{IW,S,t} K_{N,S,t} = A_{XK} \left(\gamma_{XK} {K_{IW,X,t}}^{\frac{\sigma_{XK}-1}{\sigma_{XK}}} + (1-\gamma_{XK}) {K_{IW,D,t}}^{\frac{\sigma_{XK}-1}{\sigma_{XK}}} \right)^{\frac{\sigma_{XK}}{\sigma_{XK}-1}} \qquad \forall t \tag{355}$$

The revenue generated by this capital is:

$$P_{Q,K,t} K_{IW,S,t} K_{N,S,t} = P_{X,K,t} K_{IW,X,t} + P_{D,K,t} K_{IW,D,t} \qquad \forall t \tag{356}$$

This problem has an almost identical functional form to that for goods of type q above, with $Q_{q,t}$ replaced by $K_{IW,S,t} K_{N,S,t}$. Using an identical derivative method, the locally-supplied capital in time period t that is not exported is given by:

$$\begin{aligned} K_{IW,D,t} = &\left({\gamma_{XK}}^{\sigma_{XK}} {P_{X,K,t}}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \\ &\times (1-\gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \end{aligned} \tag{357}$$

and the exported capital in time period t is given by:

$$\begin{aligned} K_{IW,X,t} = &\left({\gamma_{XK}}^{\sigma_{XK}} {P_{X,K,t}}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \\ &\times {\gamma_{XK}}^{\sigma_{XK}} {P_{X,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \qquad \forall t \end{aligned} \tag{358}$$

C.5.3 Imports of Goods

Imports of goods of type q are determined using the Armington function:

$$Q_{C,q,t} = A_{M,q}\left(\gamma_{M,q} Q_{M,q,t}^{\frac{\sigma_{M,q}-1}{\sigma_{M,q}}} + (1-\gamma_{M,q})\, Q_{D,q,t}^{\frac{\sigma_{M,q}-1}{\sigma_{M,q}}}\right)^{\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \qquad \forall q,t \tag{359}$$

The total cost of this good is:

$$P_{C,q,t}Q_{C,q,t} = P_{M,q,t}Q_{M,q,t} + P_{D,q,t}Q_{D,q,t} \qquad \forall q,t \tag{360}$$

This problem has an identical functional form to that for export of goods of type q above. Using an identical derivative method, the locally-produced goods q in time period t that are not exported are given by:

$$Q_{D,q,t} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-(\sigma_{M,q}-1)} + (1-\gamma_{M,q})^{\sigma_{M,q}}\, P_{D,q,t}{}^{-(\sigma_{M,q}-1)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \times (1-\gamma_{M,q})^{\sigma_{M,q}}\, P_{D,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \qquad \forall q,t \tag{361}$$

and the imported goods q in time period t are given by:

$$Q_{M,q,t} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-(\sigma_{M,q}-1)} + (1-\gamma_{M,q})^{\sigma_{M,q}}\, P_{D,q,t}{}^{-(\sigma_{M,q}-1)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \times \gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \qquad \forall q,t \tag{362}$$

C.5.4 Imports of Capital

Imports of financial capital are determined using the Armington function:

$$K_{IW,C,t} = A_{MK}\left(\gamma_{MK} K_{IW,M,t}^{\frac{\sigma_{MK}-1}{\sigma_{MK}}} + (1-\gamma_{MK})\, K_{IW,D,t}^{\frac{\sigma_{MK}-1}{\sigma_{MK}}}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \qquad \forall t \tag{363}$$

The total cost of this capital is:

$$P_{C,K,t}K_{IW,C,t} = P_{M,K,t}K_{IW,M,t} + P_{D,K,t}K_{IW,D,t} \qquad \forall t \tag{364}$$

This problem has an identical functional form to that for goods of type q above. Using an identical derivative method, the locally-supplied capital in time period t that is not exported are given by:

$$K_{IW,D,t} = \left(\gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)} + (1-\gamma_{MK})^{\sigma_{MK}}\, P_{D,K,t}{}^{-(\sigma_{MK}-1)}\right)^{-\frac{\sigma_{MK}}{\sigma_{MK}-1}} \times (1-\gamma_{MK})^{\sigma_{MK}}\, P_{D,K,t}{}^{-\sigma_{MK}} A_{MK}{}^{-1} K_{IW,C,t} \qquad \forall t \tag{365}$$

and the imported capital in time period t is given by:

$$K_{IW,M,t} = \left(\gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)} + (1-\gamma_{MK})^{\sigma_{MK}} P_{D,K,t}{}^{-(\sigma_{MK}-1)} \right)^{-\frac{\sigma_{MK}}{\sigma_{MK}-1}} \times \gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-\sigma_{MK}} A_{MK}{}^{-1} K_{IW,C,t} \qquad \forall t \tag{366}$$

C.5.5 Import and Export Equations for Goods

The first-order equations for importing and exporting goods q are given by the budget constraints (345) and (360), the export equations (353) and (354), and the import equations (361) and (362). From these, we need to derive explicit equations for $Q_{q,t}$, $Q_{X,q,t}$, $Q_{M,q,t}$, and $P_{C,q,t}$. $P_{Q,q,t}$, $P_{X,q,t}$, $P_{M,q,t}$, and $Q_{C,q,t}$ (dependent on $P_{C,q,t}$) are given exogenous to this subsystem. First, substitute (353) and (354) into (345) to get $P_{D,q,t}$ in terms of $P_{X,q,t}$ and $P_{Q,q,t}$:

$$P_{Q,q,t} = A_{X,q}{}^{-1} \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-(\sigma_{X,q}-1)} + (1-\gamma_{X,q})^{\sigma_{X,q}} P_{D,q,t}{}^{-(\sigma_{X,q}-1)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \qquad \forall q,t \tag{367}$$

$$P_{D,q,t} = (1-\gamma_{X,q})^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left((A_{X,q} P_{Q,q,t})^{-(\sigma_{X,q}-1)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-(\sigma_{X,q}-1)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \qquad \forall q,t \tag{368}$$

Similarly, substitute (361) and (362) into (360) to get $P_{C,q,t}$ in terms of $P_{M,q,t}$ and $P_{D,q,t}$:

$$P_{C,q,t} = A_{M,q}{}^{-1} \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-(\sigma_{M,q}-1)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-(\sigma_{M,q}-1)} \right)^{\frac{-1}{\sigma_{M,q}-1}} \qquad \forall j,t \tag{369}$$

$Q_{X,q,t}$ can then be calculated from (354), $Q_{M,q,t}$ from (362), and $Q_{D,j,t}$ from (361). $Q_{j,t}$ can be calculated by rearranging (353):

$$Q_{q,t} = \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-(\sigma_{X,q}-1)} + (1-\gamma_{X,q})^{\sigma_{X,q}} P_{D,q,t}{}^{-(\sigma_{X,q}-1)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \times (1-\gamma_{X,q})^{-\sigma_{X,q}} P_{D,q,t}{}^{\sigma_{X,q}} A_{X,q} Q_{D,q,t} \qquad \forall q,t \tag{370}$$

C.5.6 Import and Export Equations for Capital

The first-order equations for importing and exporting capital are given by the budget constraints (356) and (364), the export equations (357) and (358), and the import equations (365) and (366). From these, we need to derive explicit equations for

$P_{Q,K,t}$, $K_{IW,X,t}$, $K_{IW,M,t}$, and $P_{C,K,t}$. $K_{IW,S,t}$, $P_{X,K,t}$, $P_{M,K,t}$, and $K_{IW,C,t}$ (dependent on $P_{C,K,t}$) are given exogenous to this subsystem. This will need to be solved. First, assume we know $P_{C,K,t}$, and substitute (365) and (366) into (364) to get $P_{D,K,t}$ in terms of $P_{M,K,t}$ and $P_{C,K,t}$. The derivation is identical to that for $P_{D,q,t}$:

$$P_{D,K,t} = (1-\gamma_{MK})^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \left((A_{MK} P_{C,K,t})^{-(\sigma_{MK}-1)} - \gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)} \right)^{\frac{-1}{\sigma_{MK}-1}} \quad \forall t \tag{371}$$

Similarly, substitute (357) and (358) into (356) to get $P_{Q,K,t}$ in terms of $P_{X,K,t}$ and $P_{D,K,t}$. The derivation is identical to that for $P_{C,q,t}$:

$$P_{Q,K,t} = A_{XK}{}^{-1} \left(\gamma_{XK}{}^{\sigma_{XK}} P_{X,K,t}{}^{-(\sigma_{XK}-1)} + (1-\gamma_{XK})^{\sigma_{XK}} P_{D,K,t}{}^{-(\sigma_{XK}-1)} \right)^{\frac{-1}{\sigma_{XK}-1}} \quad \forall t \tag{372}$$

$K_{IW,X,t}$ can then be calculated from (358), $K_{IW,M,t}$ from (366), and $K_{IW,D,t}$ from (357). $P_{C,K,t}$ can then be re-calculated by rearranging (371) and substituting into (365):

$$(A_{MK} P_{C,K,t})^{\sigma_{MK}} = \left[(1-\gamma_{MK})^{\sigma_{MK}} P_{D,K,t}{}^{-(\sigma_{MK}-1)} + \gamma_{MK}{}^{\sigma_{MK}} P_{M,K,t}{}^{-(\sigma_{MK}-1)} \right]^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}} \quad \forall t \tag{373}$$

$$P_{C,K,t} = (1-\gamma_{MK})^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} P_{D,K,t} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}} \right)^{\frac{1}{\sigma_{MK}}} \quad \forall t \tag{374}$$

D Derivatives for the Solver

D.1 Derivation of the Derivatives

In this section is the derivation of the derivatives needed for a Newton-Rhaphson solver.

D.1.1 Price factor

Intermediate goods:

$$X_{e,j,t} = P_\Psi \xi_{\Psi,e,j} + A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}} \tag{375}$$

$$X_{e,j,t} - P_\Psi \xi_{\Psi,e,j} = A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}}$$

$$\forall e, j, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}$$

$$\begin{aligned} \frac{\partial X_{e,j,t}}{\partial P_{f,t}} &= \alpha_{f,e,j} P_{f,e,j,t}{}^{-1} \frac{\partial P_{f,e,j,t}}{\partial P_{f,t}} A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}} \\ &= \alpha_{f,e,j} P_{f,e,j,t}{}^{-1} \left(X_{e,j,t} - P_\Psi \xi_{\Psi,e,j}\right) \frac{\partial P_{f,e,j,t}}{\partial P_{f,t}} \\ &\forall e, j, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\} \end{aligned} \tag{376}$$

Final goods:

$$\begin{aligned} X_{e,k,t} &= P_\Psi \xi_{\Psi,e,k} + A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \\ &\times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \end{aligned} \tag{377}$$

$$\begin{aligned} X_{e,k,t} - P_\Psi \xi_{\Psi,e,k} &= A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \\ &\times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \\ &\forall e, k, t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \left\{P_{\tilde{L},Q,h,e}, P_{C,K,e}\right\} \end{aligned}$$

$$
\begin{aligned}
\frac{\partial X_{e,k,t}}{\partial P_{f,t}} =&\alpha_{f,e,k} P_{f,e,k,t}{}^{-1} \frac{\partial P_{f,e,k,t}}{\partial P_{f,t}} A_{Q,e,k,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \\
&\times \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \\
&+ \prod_g V_{g,t}{}^{-\eta_{g,e,k}} \prod_f \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_f P_{f,e,k,t}{}^{\alpha_{f,e,k}} \prod_j \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_j P_{Q,j,t}{}^{\alpha_{j,e,k}} \\
&\times \sum_j \alpha_{j,e,k} P_{Q,j,t}{}^{-1} \frac{\partial P_{Q,j,t}}{\partial P_{f,t}} \\
=&\alpha_{f,e,k} P_{f,e,k,t}{}^{-1} \frac{\partial P_{f,e,k,t}}{\partial P_{f,t}} \left(X_{e,k,t} - P_\Psi \xi_{\Psi,e,k} \right) \\
&+ \left(X_{e,k,t} - P_\Psi \xi_{\Psi,e,k} \right) \sum_j \alpha_{j,e,k} P_{Q,j,t}{}^{-1} \frac{\partial P_{Q,j,t}}{\partial P_{f,t}} \\
=& \left(X_{e,k,t} - P_\Psi \xi_{\Psi,e,k} \right) \left(\alpha_{f,e,k} P_{f,e,k,t}{}^{-1} \frac{\partial P_{f,e,k,t}}{\partial P_{f,t}} + \sum_j \alpha_{j,e,k} P_{Q,j,t}{}^{-1} \frac{\partial P_{Q,j,t}}{\partial P_{f,t}} \right) \\
&\forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \left\{ P_{\tilde{L},Q,h,e}, P_{C,K,e} \right\}
\end{aligned} \tag{378}
$$

D.1.2 Price of extracted goods

$$
P_{Q,q,t} = A_{Q,q}{}^{-1} \left(\sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{1-\sigma_q} \right)^{\frac{-1}{\sigma_q - 1}} \tag{379}
$$

$$
[A_{Q,q} P_{Q,q,t}]^{-(\sigma_q - 1)} = \left(\sum_e \gamma_{e,q}{}^{\sigma_j} X_{e,q,t}{}^{1-\sigma_q} \right) \qquad \forall q,t
$$

$$
\begin{aligned}
\frac{\partial P_{Q,q,t}}{\partial P_{f,t}} =& \frac{-1}{\sigma_q - 1} A_{Q,q}{}^{-1} \left(\sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{1-\sigma_q} \right)^{\frac{-1}{\sigma_q - 1} - 1} \left(\sum_e (1-\sigma_q) \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{-\sigma_q} \frac{\partial X_{e,q,t}}{\partial P_{f,t}} \right) \\
=& A_{Q,q}{}^{-1} \left(\sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{1-\sigma_q} \right)^{\frac{-\sigma_q}{\sigma_q - 1}} \left(\sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{-\sigma_q} \frac{\partial X_{e,q,t}}{\partial P_{f,t}} \right) \\
=& A_{Q,q}{}^{-1} \left([A_{Q,q} P_{Q,q,t}]^{-(\sigma_q - 1)} \right)^{\frac{-\sigma_q}{\sigma_q - 1}} \sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{-\sigma_q} \frac{\partial X_{e,q,t}}{\partial P_{f,t}} \\
=& A_{Q,q}{}^{-1} \left(A_{Q,q} P_{Q,q,t} \right)^{\sigma_q} \sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{-\sigma_q} \frac{\partial X_{e,q,t}}{\partial P_{f,t}} \\
=& A_{Q,q}{}^{\sigma_q - 1} P_{Q,q,t}{}^{\sigma_q} \sum_e \gamma_{e,q}{}^{\sigma_q} X_{e,q,t}{}^{-\sigma_q} \frac{\partial X_{e,q,t}}{\partial P_{f,t}} \qquad \forall q,t
\end{aligned} \tag{380}
$$

D.1.3 Price of domestically produced and sold goods

$$P_{D,q,t} = \left(\left(A_{X,q} P_{Q,q,t} \right)^{-\left(\sigma_{X,q}-1\right)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-1}{\sigma_{X,q}-1}} \times \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \tag{381}$$

$$\left[\left(1-\gamma_{X,q}\right)^{\frac{-\sigma_{X,q}}{\sigma_{X,q}-1}} P_{D,q,t} \right]^{-\left(\sigma_{X,q}-1\right)} = \left(\left(A_{X,q} P_{Q,q,t} \right)^{-\left(\sigma_{X,q}-1\right)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \right) \quad \forall q,t$$

$$\begin{aligned}
\frac{\partial P_{D,q,t}}{\partial P_{f,t}} =& \frac{-1}{\sigma_{X,q}-1} \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left(\left(A_{X,q} P_{Q,q,t} \right)^{-\left(\sigma_{X,q}-1\right)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-1}{\sigma_{X,q}-1}-1} \\
& \times \left(-\left(\sigma_{X,q}-1\right) A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \right) \\
=& \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left(\left(A_{X,q} P_{Q,q,t} \right)^{-\left(\sigma_{X,q}-1\right)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-\sigma_{X,q}}{\sigma_{X,q}-1}} \\
& \times A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \\
=& \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left(\left[\left(1-\gamma_{X,q}\right)^{\frac{-\sigma_{X,q}}{\sigma_{X,q}-1}} P_{D,q,t} \right]^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{-\sigma_{X,q}}{\sigma_{X,q}-1}} \\
& \times A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \\
=& \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \left(\left(1-\gamma_{X,q}\right)^{\frac{-\sigma_{X,q}}{\sigma_{X,q}-1}} P_{D,q,t} \right)^{\sigma_{X,q}} \\
& \times A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \\
=& \left(1-\gamma_{X,q}\right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}+\frac{-\sigma_{X,q}{}^{2}}{\sigma_{X,q}-1}} P_{D,q,t}{}^{\sigma_{X,q}} A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \\
=& \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} P_{D,q,t}{}^{\sigma_{X,q}} A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} P_{Q,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \\
=& \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} A_{X,q}{}^{-\left(\sigma_{X,q}-1\right)} \left(\frac{P_{D,q,t}}{P_{Q,q,t}} \right)^{\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \qquad \forall q,t
\end{aligned} \tag{382}$$

D.1.4 Price of consumption

$$P_{C,q,t} = A_{M,q}{}^{-1}\left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{\frac{-1}{\sigma_{M,q}-1}} \tag{383}$$

$$\left[A_{M,q}P_{C,q,t}\right]^{-\left(\sigma_{M,q}-1\right)} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right) \quad \forall q,t$$

$$\begin{aligned}
\frac{\partial P_{C,q,t}}{\partial P_{f,t}} =& \frac{-1}{\sigma_{M,q}-1} A_{M,q}{}^{-1}\left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{\frac{-1}{\sigma_{M,q}-1}-1} \\
& \times \left(-\left(\sigma_{M,q}-1\right)\left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}}\right) \\
=& A_{M,q}{}^{-1}\left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{\frac{-\sigma_{M,q}}{\sigma_{M,q}-1}} \\
& \times \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
=& A_{M,q}{}^{-1}\left(\left[A_{M,q}P_{C,q,t}\right]^{-\left(\sigma_{M,q}-1\right)}\right)^{\frac{-\sigma_{M,q}}{\sigma_{M,q}-1}} \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
=& A_{M,q}{}^{\sigma_{M,q}-1} P_{C,q,t}{}^{\sigma_{M,q}} \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
=& A_{M,q}{}^{\sigma_{M,q}-1} \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} \left(\frac{P_{C,q,t}}{P_{D,q,t}}\right)^{\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \qquad \forall q,t
\end{aligned} \tag{384}$$

D.1.5 Household consumption

Transport:

$$C_{h,k=HOT,t} = A_{HOT,h} T_{HO,h,t} - \sum_{h_1} Q_{h_1,h,k=HOT,t} \qquad \forall h,t \tag{385}$$

$$\frac{\partial C_{h,k=HOT,t}}{\partial P_{f,t}} = 0 \qquad \forall h,t \tag{386}$$

Other consumption:

$$C_{h,k,t} = \frac{\alpha_{C,h,k}\left(B_{h,t} - \left(1+\tau_{C,k_2=HOT}\right)P_{C,k_2=HOT,t}C_{h,k_2=HOT,t}\right)}{\left(1+\tau_{C,k}\right)P_{C,k,t}} \tag{387}$$

$$C_{h,k,t} - \frac{\alpha_{C,h,k}B_{h,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}} = -\frac{\alpha_{C,h,k}\left(1+\tau_{C,k_2=HOT}\right)P_{C,k_2=HOT,t}C_{h,k_2=HOT,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}}$$

$$\forall h, k \neq HOT, t$$

$$\begin{aligned}
\frac{\partial C_{h,k,t}}{\partial P_{f,t}} =& \frac{-\alpha_{C,h,k}\left(B_{h,t} - \left(1+\tau_{C,k_2=HOT}\right)P_{C,k_2=HOT,t}C_{h,k_2=HOT,t}\right)}{\left(1+\tau_{C,k}\right){P_{C,k,t}}^2}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} \\
&+ \frac{-\alpha_{C,h,k}\left(1+\tau_{C,k_2=HOT}\right)P_{C,k_2=HOT,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}}\frac{\partial C_{h,k_2=HOT,t}}{\partial P_{f,t}} \\
&+ \frac{-\alpha_{C,h,k}\left(1+\tau_{C,k_2=HOT}\right)C_{h,k_2=HOT,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}}\frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}} \\
=& -\frac{C_{h,k,t}}{P_{C,k,t}}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} - \frac{\alpha_{C,h,k}\left(1+\tau_{C,k_2=HOT}\right)C_{h,k_2=HOT,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}}\frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}} \\
=& -\frac{C_{h,k,t}}{P_{C,k,t}}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} - {P_{C,k_2=HOT,t}}^{-1}\left(C_{h,k,t} - \frac{\alpha_{C,h,k}B_{h,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}}\right)\frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}} \\
=& \frac{\alpha_{C,h,k}B_{h,t}}{\left(1+\tau_{C,k}\right)P_{C,k,t}P_{C,k_2=HOT,t}}\frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}} \\
&- \frac{C_{h,k,t}}{P_{C,k_2=HOT,t}}\frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}} - \frac{C_{h,k,t}}{P_{C,k,t}}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} \qquad \forall h, k \neq HOT, t
\end{aligned} \tag{388}$$

D.1.6 Government consumption

$$C_{G,k,t} = \frac{\tilde{S}_{G,k,t}}{P_{C,k,t}} + \frac{\alpha_{C,G,k}I_{G,t}}{P_{C,k,t}} \qquad \forall k, t \tag{389}$$

$$\begin{aligned}
\frac{\partial C_{G,k,t}}{\partial P_{f,t}} &= -\frac{\tilde{S}_{G,k,t}}{{P_{C,k,t}}^2}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} - \frac{\alpha_{C,G,k}I_{G,t}}{{P_{C,k,t}}^2}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} \\
&= -{P_{C,k,t}}^{-1}\frac{\partial P_{C,k,t}}{\partial P_{f,t}}\left(\frac{\tilde{S}_{G,k,t}}{P_{C,k,t}} + \frac{\alpha_{C,G,k}I_{G,t}}{P_{C,k,t}}\right) \\
&= -\frac{C_{G,k,t}}{P_{C,k,t}}\frac{\partial P_{C,k,t}}{\partial P_{f,t}} \qquad \forall k, t
\end{aligned} \tag{390}$$

D.1.7 Quantity of domestically produced and sold goods

$$Q_{D,q,t} = (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \tag{391}$$

$$\left[(1-\gamma_{M,q})^{-\sigma_{M,q}} P_{D,q,t}{}^{\sigma_{M,q}} A_{M,q} Q_{C,q,t}{}^{-1} Q_{D,q,t}\right]^{-\frac{\sigma_{M,q}-1}{\sigma_{M,q}}} = \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right) \quad \forall q,t$$

$$\begin{aligned}
\frac{\partial Q_{D,q,t}}{\partial P_{f,t}} =& -\sigma_{M,q} (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}-1} A_{M,q}{}^{-1} Q_{C,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \\
&+ (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} \frac{\partial Q_{C,q,t}}{\partial P_{f,t}} \\
&\times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}} \\
&- \frac{\sigma_{M,q}}{\sigma_{M,q}-1} (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \\
&\times \left(-\left(\sigma_{M,q}-1\right)(1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}}\right) \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-\frac{\sigma_{M,q}}{\sigma_{M,q}-1}-1} \\
=& -\sigma_{M,q} \frac{Q_{D,q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} + \frac{Q_{D,q,t}}{Q_{C,q,t}} \frac{\partial Q_{C,q,t}}{\partial P_{f,t}} \\
&+ \sigma_{M,q} (1-\gamma_{M,q})^{2\sigma_{M,q}} P_{D,q,t}{}^{-2\sigma_{M,q}} A_{M,q}{}^{-1} Q_{C,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\times (1-\gamma_{M,q})^{-\sigma_{M,q}} P_{D,q,t}{}^{\sigma_{M,q}} A_{M,q} Q_{C,q,t}{}^{-1} Q_{D,q,t} \\
&\times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-1} \\
=& -\sigma_{M,q} \frac{Q_{D,q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} + \frac{Q_{D,q,t}}{Q_{C,q,t}} \frac{\partial Q_{C,q,t}}{\partial P_{f,t}} \\
&+ \sigma_{M,q} (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\sigma_{M,q}} Q_{D,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\times \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + (1-\gamma_{M,q})^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)}\right)^{-1}
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial Q_{D,q,t}}{\partial P_{f,t}} =& \frac{Q_{D,q,t}}{Q_{C,q,t}} \frac{\partial Q_{C,q,t}}{\partial P_{f,t}} \\
&- \sigma_{M,q} \frac{Q_{D,q,t}}{P_{D,q,t}} \left({\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} {P_{D,q,t}}^{-\left(\sigma_{M,q}-1\right)} \right. \\
&\qquad \left. - \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} {P_{D,q,t}}^{-\left(\sigma_{M,q}-1\right)} \right) \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\quad \times \left({\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} {P_{D,q,t}}^{-\left(\sigma_{M,q}-1\right)} \right)^{-1} \\
=& \frac{Q_{D,q,t}}{Q_{C,q,t}} \frac{\partial Q_{C,q,t}}{\partial P_{f,t}} - \sigma_{M,q} {\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-\left(\sigma_{M,q}-1\right)} \frac{Q_{D,q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\quad \times \left({\gamma_{M,q}}^{\sigma_{M,q}} {P_{M,q,t}}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} {P_{D,q,t}}^{-\left(\sigma_{M,q}-1\right)} \right)^{-1}
\end{aligned} \tag{392}
$$

D.1.8 Quantity of domestically-produced goods

$$
\begin{aligned}
Q_{q,t} =& \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\
&\times \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} {P_{D,q,t}}^{\sigma_{X,q}} A_{X,q} Q_{D,q,t} \\
&Q_{q,t} \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\sigma_{X,q}} {A_{X,q}}^{-1} {Q_{D,q,t}}^{-1} \\
&= \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\
&\qquad \forall q, t
\end{aligned} \tag{393}
$$

$$
\begin{aligned}
\frac{\partial Q_{q,t}}{\partial P_{f,t}} =& \frac{\sigma_{X,q}}{\sigma_{X,q}-1} \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}-1} \\
&\times \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\sigma_{X,q}} \times - \left(\sigma_{X,q}-1\right) \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\times \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} {P_{D,q,t}}^{\sigma_{X,q}} A_{X,q} Q_{D,q,t} \\
&+ \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\
&\times \sigma_{X,q} \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} {P_{D,q,t}}^{\sigma_{X,q}-1} A_{X,q} Q_{D,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&+ \left({\gamma_{X,q}}^{\sigma_{X,q}} {P_{X,q,t}}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} {P_{D,q,t}}^{-\left(\sigma_{X,q}-1\right)} \right)^{\frac{\sigma_{X,q}}{\sigma_{X,q}-1}} \\
&\times \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} {P_{D,q,t}}^{\sigma_{X,q}} A_{X,q} \frac{\partial Q_{D,q,t}}{\partial P_{f,t}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial Q_{q,t}}{\partial P_{f,t}} =& -\sigma_{X,q} Q_{q,t} \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\sigma_{X,q}} A_{X,q}{}^{-1} Q_{D,q,t}{}^{-1} \\
&\quad \times \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)}\right)^{-1} \\
&\quad \times A_{X,q} Q_{D,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&+ Q_{q,t} \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\sigma_{X,q}} A_{X,q}{}^{-1} Q_{D,q,t}{}^{-1} \\
&\quad \times \sigma_{X,q} \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} P_{D,q,t}{}^{\sigma_{X,q}-1} A_{X,q} Q_{D,q,t} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&+ Q_{q,t} \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\sigma_{X,q}} A_{X,q}{}^{-1} Q_{D,q,t}{}^{-1} \\
&\quad \times \left(1-\gamma_{X,q}\right)^{-\sigma_{X,q}} P_{D,q,t}{}^{\sigma_{X,q}} A_{X,q} \frac{\partial Q_{D,q,t}}{\partial P_{f,t}} \\
=& -\sigma_{X,q} Q_{q,t} \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\sigma_{X,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\quad \times \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)}\right)^{-1} \\
&+ \sigma_{X,q} \frac{Q_{q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} + \frac{Q_{q,t}}{Q_{D,q,t}} \frac{\partial Q_{D,q,t}}{\partial P_{f,t}} \\
=& \frac{Q_{q,t}}{Q_{D,q,t}} \frac{\partial Q_{D,q,t}}{\partial P_{f,t}} - \sigma_{X,q} \frac{Q_{q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\quad \times \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)}\right)^{-1} \\
&\quad \times \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \right. \\
&\qquad \left. - \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)}\right) \\
=& \frac{Q_{q,t}}{Q_{D,q,t}} \frac{\partial Q_{D,q,t}}{\partial P_{f,t}} + \sigma_{X,q} \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} \frac{Q_{q,t}}{P_{D,q,t}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \\
&\quad \times \left(\gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q}-1\right)} + \left(1-\gamma_{X,q}\right)^{\sigma_{X,q}} P_{D,q,t}{}^{-\left(\sigma_{X,q}-1\right)}\right)^{-1} \\
&\quad \forall q, t
\end{aligned}
\tag{394}
$$

D.1.9 Production process

$$
Q_{e,q,t} = Q_{q,t} \gamma_{e,q}{}^{\sigma_q} P_{Q,q,t}{}^{\sigma_q} A_{Q,q}{}^{\sigma_q - 1} X_{e,q,t}{}^{-\sigma_q} \qquad \forall e, q, t \tag{395}
$$

$$
\begin{aligned}
\frac{\partial Q_{e,q,t}}{\partial P_{f,t}} =& \frac{\partial Q_{q,t}}{\partial P_{f,t}} \gamma_{e,q}{}^{\sigma_q} P_{Q,q,t}{}^{\sigma_q} A_{Q,q}{}^{\sigma_q - 1} X_{e,q,t}{}^{-\sigma_q} \\
&+ \sigma_q Q_{q,t} \gamma_{e,q}{}^{\sigma_q} P_{Q,q,t}{}^{\sigma_q - 1} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} A_{Q,q}{}^{\sigma_q - 1} X_{e,q,t}{}^{-\sigma_q} \\
&- \sigma_q Q_{q,t} \gamma_{e,q}{}^{\sigma_q} P_{Q,q,t}{}^{\sigma_q} A_{Q,q}{}^{\sigma_q - 1} X_{e,q,t}{}^{-\sigma_q - 1} \frac{\partial X_{e,q,t}}{\partial P_{f,t}}
\end{aligned}
$$

$$\frac{\partial Q_{e,q,t}}{\partial P_{f,t}} = \frac{Q_{e,q,t}}{Q_{q,t}}\frac{\partial Q_{q,t}}{\partial P_{f,t}} + \sigma_k \frac{Q_{e,q,t}}{P_{Q,q,t}}\frac{\partial P_{Q,q,t}}{\partial P_{f,t}} - \sigma_k \frac{Q_{e,q,t}}{X_{e,q,t}}\frac{\partial X_{e,q,t}}{\partial P_{f,t}} \quad \forall e,q,t \tag{396}$$

D.1.10 Extraction products demanded for final production

$$V_{j,e,k,t} = \alpha_{j,e,k} {P_{C,j,t}}^{-1} \left(\gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} - P_\Psi \Psi_{e,k,t} \right)$$

$$ {\alpha_{j,e,k}}^{-1} P_{C,j,t} V_{j,e,k,t} + P_\Psi \Psi_{e,k,t} = \gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} \quad \forall e,j,k,t$$

$$\begin{aligned}
\frac{\partial V_{j,e,k,t}}{\partial P_{f,t}} =& - \alpha_{j,e,k} {P_{C,j,t}}^{-2} \frac{\partial P_{C,j,t}}{\partial P_{f,t}} \left(\gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} - P_\Psi \Psi_{e,k,t} \right) \\
&+ \alpha_{j,e,k} {P_{C,j,t}}^{-1} \left(\gamma_{e,k} \frac{\partial P_{Q,k,t}}{\partial P_{f,t}} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} \right. \\
&\qquad + \gamma_{e,k} P_{Q,k,t} \frac{\partial Q_{k,t}}{\partial P_{f,t}} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} \\
&\qquad + \frac{\sigma_k - 1}{\sigma_k} \gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} \\
&\qquad \times \left({Q_{e,k,t}}^{-1} \frac{\partial Q_{e,k,t}}{\partial P_{f,t}} - {Q_{k,t}}^{-1} \frac{\partial Q_{k,t}}{\partial P_{f,t}} \right) \\
&\qquad \left. - P_\Psi \frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}} \right) \\
=& - \frac{V_{j,e,k,t}}{P_{C,j,t}} \frac{\partial P_{C,j,t}}{\partial P_{f,t}} - \alpha_{j,e,k} {P_{C,j,t}}^{-1} P_\Psi \frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}} \\
&+ \alpha_{j,e,k} {P_{C,j,t}}^{-1} \gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k - 1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}}\right)^{\frac{\sigma_k-1}{\sigma_k}} \\
&\times \left({P_{Q,k,t}}^{-1} \frac{\partial P_{Q,k,t}}{\partial P_{f,t}} + {Q_{k,t}}^{-1} \frac{\partial Q_{k,t}}{\partial P_{f,t}} \right. \\
&\qquad \left. + \frac{\sigma_k - 1}{\sigma_k} \left({Q_{e,k,t}}^{-1} \frac{\partial Q_{e,k,t}}{\partial P_{f,t}} - {Q_{k,t}}^{-1} \frac{\partial Q_{k,t}}{\partial P_{f,t}} \right) \right)
\end{aligned}$$

$$\begin{aligned}\frac{\partial V_{j,e,k,t}}{\partial P_{f,t}} =& -\frac{V_{j,e,k,t}}{P_{C,j,t}}\frac{\partial P_{C,j,t}}{\partial P_{f,t}} - \alpha_{j,e,k}{P_{C,j,t}}^{-1}P_{\Psi}\frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}}\\ &+\alpha_{j,e,k}{P_{C,j,t}}^{-1}\left({\alpha_{j,e,k}}^{-1}P_{C,j,t}V_{j,e,k,t}+P_{\Psi}\Psi_{e,k,t}\right)\\ &\times\left({P_{Q,k,t}}^{-1}\frac{\partial P_{Q,k,t}}{\partial P_{f,t}}+\left(1-\frac{\sigma_k-1}{\sigma_k}\right){Q_{k,t}}^{-1}\frac{\partial Q_{k,t}}{\partial P_{f,t}}\right.\\ &\left.+\frac{\sigma_k-1}{\sigma_k}{Q_{e,k,t}}^{-1}\frac{\partial Q_{e,k,t}}{\partial P_{f,t}}\right)\\ =& -\frac{V_{j,e,k,t}}{P_{C,j,t}}\frac{\partial P_{C,j,t}}{\partial P_{f,t}} - \alpha_{j,e,k}{P_{C,j,t}}^{-1}P_{\Psi}\frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}}\\ &+\left(V_{j,e,k,t}+\alpha_{j,e,k}{P_{C,j,t}}^{-1}P_{\Psi}\Psi_{e,k,t}\right)\\ &\times\left({P_{Q,k,t}}^{-1}\frac{\partial P_{Q,k,t}}{\partial P_{f,t}}+{\sigma_k}^{-1}{Q_{k,t}}^{-1}\frac{\partial Q_{k,t}}{\partial P_{f,t}}+\frac{\sigma_k-1}{\sigma_k}{Q_{e,k,t}}^{-1}\frac{\partial Q_{e,k,t}}{\partial P_{f,t}}\right)\\ &\forall e,j,k,t \qquad (397)\end{aligned}$$

D.1.11 Price of domestically produced and sold capital

$$\begin{aligned}P_{D,K,t} =& \left(\left(A_{MK}P_{C,K,t}\right)^{-(\sigma_{MK}-1)}-{\gamma_{MK}}^{\sigma_{MK}}{P_{MK,t}}^{-(\sigma_{MK}-1)}\right)^{\frac{-1}{\sigma_{MK}-1}}\\ &\times\left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \qquad (398)\\ \left[\left(1-\gamma_{MK}\right)^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}}P_{D,K,t}\right]^{-(\sigma_{MK}-1)} =& \left(\left(A_{MK}P_{C,K,t}\right)^{-(\sigma_{MK}-1)}-{\gamma_{MK}}^{\sigma_{MK}}{P_{MK,t}}^{-(\sigma_{MK}-1)}\right)\\ &\forall t\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{D,K,t}}{\partial P_{f,t}} =& \frac{-1}{\sigma_{MK}-1}\left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}}\left(\left(A_{MK}P_{C,K,t}\right)^{-(\sigma_{MK}-1)}-{\gamma_{MK}}^{\sigma_{MK}}{P_{MK,t}}^{-(\sigma_{MK}-1)}\right)^{\frac{-1}{\sigma_{MK}-1}-1}\\ &\times\left(-\left(\sigma_{MK}-1\right){A_{MK}}^{-(\sigma_{MK}-1)}{P_{C,K,t}}^{-\sigma_{MK}}\frac{\partial P_{C,K,t}}{\partial P_{f,t}}\right)\\ =& \left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}}\left(\left(A_{MK}P_{C,K,t}\right)^{-(\sigma_{MK}-1)}-{\gamma_{MK}}^{\sigma_{MK}}{P_{MK,t}}^{-(\sigma_{MK}-1)}\right)^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}}\\ &\times {A_{MK}}^{-(\sigma_{MK}-1)}{P_{C,K,t}}^{-\sigma_{MK}}\frac{\partial P_{C,K,t}}{\partial P_{f,t}}\\ =& \left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}}\left(\left[\left(1-\gamma_{MK}\right)^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}}P_{C,K,t}\right]^{-(\sigma_{MK}-1)}\right)^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}}\\ &\times {A_{MK}}^{-(\sigma_{MK}-1)}{P_{C,K,t}}^{-\sigma_{MK}}\frac{\partial P_{C,K,t}}{\partial P_{f,t}}\\ =& \left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}}\left(\left(1-\gamma_{MK}\right)^{\frac{-\sigma_{MK}}{\sigma_{MK}-1}}P_{C,K,t}\right)^{\sigma_{MK}}\\ &\times {A_{MK}}^{-(\sigma_{MK}-1)}{P_{C,K,t}}^{-\sigma_{MK}}\frac{\partial P_{C,K,t}}{\partial P_{f,t}}\\ =& \left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}+\frac{-{\sigma_{MK}}^2}{\sigma_{MK}-1}}{P_{D,K,t}}^{\sigma_{MK}}{A_{MK}}^{-(\sigma_{MK}-1)}{P_{C,K,t}}^{-\sigma_{MK}}\frac{\partial P_{C,K,t}}{\partial P_{f,t}}\end{aligned}$$

$$
\begin{aligned}
\frac{\partial P_{D,K,t}}{\partial P_{f,t}} &= (1-\gamma_{MK})^{-\sigma_{MK}} P_{D,K,t}{}^{\sigma_{MK}} A_{MK}{}^{-(\sigma_{MK}-1)} P_{C,K,t}{}^{-\sigma_{MK}} \frac{\partial P_{C,K,t}}{\partial P_{f,t}} \\
&= (1-\gamma_{MK})^{-\sigma_{MK}} A_{MK}{}^{-(\sigma_{MK}-1)} \left(\frac{P_{D,K,t}}{P_{C,K,t}}\right)^{\sigma_{MK}} \frac{\partial P_{C,K,t}}{\partial P_{f,t}} \qquad \forall t
\end{aligned} \tag{399}
$$

D.1.12 Factors demanded for production

$$
\begin{aligned}
V_{f,e,q,t} =& \alpha_{f,e,q} P_{f,e,q,t}{}^{-1} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \\
\alpha_{f,e,q}{}^{-1} P_{f,e,q,t} V_{f,e,q,t} + P_\Psi \Psi_{e,q,t} =& \gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} \\
& \forall e, f, q, t
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_{f_1,e,q,t}}{\partial P_{f_2,t}} =& -\alpha_{f_1,e,q} P_{f_1,e,q,t}{}^{-2} \frac{\partial P_{f_1,e,q,t}}{\partial P_{f_2,t}} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \\
& + \alpha_{f_1,e,q} P_{f_1,e,q,t}{}^{-1} \left(\gamma_{e,q} \frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} \right. \\
& \qquad + \gamma_{e,q} P_{Q,q,t} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} A_{Q,q}{}^{\frac{\sigma_k-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} \\
& \qquad + \frac{\sigma_q-1}{\sigma_q} \gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} \\
& \qquad \times \left(Q_{e,q,t}{}^{-1} \frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} - Q_{q,t}{}^{-1} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} \right) \\
& \qquad \left. - P_\Psi \frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \right) \\
=& -\frac{V_{f_1,e,q,t}}{P_{f_1,e,q,t}} \frac{\partial P_{f_1,e,q,t}}{\partial P_{f_2,t}} - \alpha_{f_1,e,q} P_{f_1,e,q,t}{}^{-1} P_\Psi \frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \\
& + \alpha_{f_1,e,q} P_{f_1,e,q,t}{}^{-1} \gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} \\
& \times \left(P_{Q,q,t}{}^{-1} \frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + Q_{q,t}{}^{-1} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} \right. \\
& \qquad \left. + \frac{\sigma_q-1}{\sigma_q} \left(Q_{e,q,t}{}^{-1} \frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} - Q_{q,t}{}^{-1} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} \right) \right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V_{f_1,e,q,t}}{\partial P_{f_2,t}} &= -\frac{V_{f_1,e,q,t}}{P_{f_1,e,q,t}}\frac{\partial P_{f_1,e,q,t}}{\partial P_{f_2,t}} - \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1} P_{\Psi}\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \\
&\quad + \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1}\left({\alpha_{f_1,e,q}}^{-1} P_{f_1,e,q,t} V_{f_1,e,q,t} + P_{\Psi}\Psi_{e,q,t}\right) \\
&\quad \times \left({P_{Q,q,t}}^{-1}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + \left(1 - \frac{\sigma_q - 1}{\sigma_q}\right) {Q_{q,t}}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}} \right. \\
&\qquad \left. + \frac{\sigma_q - 1}{\sigma_q} {Q_{e,q,t}}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} \right) \\
&= -\frac{V_{f_1,e,q,t}}{P_{f_1,e,q,t}}\frac{\partial P_{f_1,e,q,t}}{\partial P_{f_2,t}} - \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1} P_{\Psi}\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \\
&\quad + \left(V_{f_1,e,q,t} + \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1} P_{\Psi}\Psi_{e,q,t}\right) \\
&\quad \times \left({P_{Q,q,t}}^{-1}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + {\sigma_q}^{-1} {Q_{q,t}}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}} + \frac{\sigma_q - 1}{\sigma_q} {Q_{e,q,t}}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} \right) \\
&\quad \forall e, f, q, t
\end{aligned} \tag{400}
$$

D.1.13 Quantity of domestically-demanded capital

$$
K_{IW,C,t} = \sum_e \sum_q V_{f=K,e,q,t} + D_{G,t} \qquad \forall t \tag{401}
$$

$$
\frac{\partial K_{IW,C,t}}{\partial P_{f_2,t}} = \sum_e \sum_q \frac{\partial V_{f=K,e,q,t}}{\partial P_{f_2,t}} \qquad \forall t \tag{402}
$$

D.1.14 Quantity of domestically produced and sold capital

$$
\begin{aligned}
K_{IW,D,t} &= \left(1 - \gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \\
&\quad \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1 - \gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}}
\end{aligned} \tag{403}
$$

$$
\begin{aligned}
&\left[\left(1 - \gamma_{XK}\right)^{-\sigma_{XK}} {P_{D,K,t}}^{\sigma_{XK}} A_{XK} \left(K_{IW,S,t} K_{N,S,t}\right)^{-1} K_{IW,D,t}\right]^{-\frac{\sigma_{XK}-1}{\sigma_{XK}}} \\
&\quad = \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1 - \gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right) \\
&\quad \forall t
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial K_{IW,D,t}}{\partial P_{f,t}} =& -\sigma_{XK}\left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}-1} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \\
& - \frac{\sigma_{XK}}{\sigma_{XK}-1}\left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \\
& \times \left(-\left(\sigma_{XK}-1\right)\left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}}\right) \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}-1} \\
=& -\sigma_{XK} \frac{K_{IW,D,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& + \sigma_{XK}\left(1-\gamma_{XK}\right)^{2\sigma_{XK}} {P_{D,K,t}}^{-2\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left(1-\gamma_{XK}\right)^{-\sigma_{XK}} {P_{D,K,t}}^{\sigma_{XK}} A_{XK} \left(K_{IW,S,t} K_{N,S,t}\right)^{-1} K_{IW,D,t} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-1} \\
=& -\sigma_{XK} \frac{K_{IW,D,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& + \sigma_{XK}\left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}} K_{IW,D,t} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-1} \\
=& -\sigma_{XK} \frac{K_{IW,D,t}}{P_{D,K,t}} \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right.\\
& \left. - \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right) \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-1} \\
=& -\sigma_{XK} {\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} \frac{K_{IW,D,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\
& \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + \left(1-\gamma_{XK}\right)^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)}\right)^{-1}
\end{aligned}
\tag{404}
$$

D.1.15 Price of factors of production (labour)

$$
\begin{aligned}
P_{f=\tilde{L},S,h,t} =& \sum_q \sum_e \alpha_{f=\tilde{L},e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} {A_{Q,q}}^{\frac{\sigma_q-1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}} - P_\Psi \Psi_{e,q,t}\right) \\
& \times \left(K_{N,S,t} \tilde{L}_{S,h,t}\right)^{-1} \qquad \forall t
\end{aligned}
\tag{405}
$$

$$
\begin{aligned}
\frac{\partial P_{f=\tilde{L},S,h,t}}{\partial P_{f_2,t}} = & \left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\left(\gamma_{e,q}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\right.\\
& + \gamma_{e,q}P_{Q,q,t}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}}A_{Q,q}{}^{\frac{\sigma_k-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\\
& + \frac{\sigma_q-1}{\sigma_q}\gamma_{e,q}P_{Q,q,t}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\\
& \times\left(Q_{e,q,t}{}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} - Q_{q,t}{}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}}\right)\\
& \left. -P_\Psi\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}}\right)\\
= & -\left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}\sum_q\sum_e \alpha_{f=\tilde{L},e,q}P_\Psi\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}}\\
& + \left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\gamma_{e,q}P_{Q,q,t}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\\
& \times\left(P_{Q,q,t}{}^{-1}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + Q_{q,t}{}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}}\right.\\
& \left. + \frac{\sigma_q-1}{\sigma_q}\left(Q_{e,q,t}{}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} - Q_{q,t}{}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}}\right)\right)\\
= & -\left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}P_\Psi\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}}\\
& + \left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\gamma_{e,q}P_{Q,q,t}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\\
& \times\left(P_{Q,q,t}{}^{-1}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + \left(1-\frac{\sigma_q-1}{\sigma_q}\right)Q_{q,t}{}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}}\right.\\
& \left. + \frac{\sigma_q-1}{\sigma_q}Q_{e,q,t}{}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}}\right)\\
= & -\left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}P_\Psi\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}}\\
& + \left(K_{N,S,t}\tilde{L}_{S,h,t}\right)^{-1}\sum_q\sum_e \alpha_{f=\tilde{L},e,q}\gamma_{e,q}P_{Q,q,t}Q_{q,t}A_{Q,q}{}^{\frac{\sigma_q-1}{\sigma_q}}\left(\frac{Q_{e,q,t}}{Q_{q,t}}\right)^{\frac{\sigma_q-1}{\sigma_q}}\\
& \times\left(P_{Q,q,t}{}^{-1}\frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + \sigma_q{}^{-1}Q_{q,t}{}^{-1}\frac{\partial Q_{q,t}}{\partial P_{f_2,t}} + \frac{\sigma_q-1}{\sigma_q}Q_{e,q,t}{}^{-1}\frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}}\right)\\
& \forall e,f,q,t
\end{aligned} \tag{406}
$$

D.1.16 Price of consumption of capital

$$
P_{C,K,t} = \left(1-\gamma_{MK}\right)^{-1}A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}}P_{D,K,t}\left(\frac{K_{IW,D,t}}{K_{IW,C,t}}\right)^{\frac{1}{\sigma_{MK}}} \qquad \forall t \tag{407}
$$

$$
\begin{aligned}
\frac{\partial P_{C,K,t}}{\partial P_{f,t}} =& \left(1-\gamma_{MK}\right)^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}}\right)^{\frac{1}{\sigma_{MK}}} \\
&+ \sigma_{MK}{}^{-1} \left(1-\gamma_{MK}\right)^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} \frac{P_{D,K,t}}{K_{IW,D,t}} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}}\right)^{\frac{1}{\sigma_{MK}}} \frac{\partial K_{IW,D,t}}{\partial P_{f,t}} \\
&- \sigma_{MK}{}^{-1} \left(1-\gamma_{MK}\right)^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} \frac{P_{D,K,t}}{K_{IW,C,t}} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}}\right)^{\frac{1}{\sigma_{MK}}} \frac{\partial K_{IW,C,t}}{\partial P_{f,t}} \\
=& \frac{P_{C,K,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} + \sigma_{MK}{}^{-1} \left(\frac{P_{C,K,t}}{K_{IW,D,t}} \frac{\partial K_{IW,D,t}}{\partial P_{f,t}} - \frac{P_{C,K,t}}{K_{IW,C,t}} \frac{\partial K_{IW,C,t}}{\partial P_{f,t}} \right) \\
& \forall t
\end{aligned} \quad (408)
$$

D.2 The Final Results

In this section, each equation in the solved section of the model is included along with it's derivative.

Effective price of labour:

$$P_{\tilde{L},Q,h,e,q,t} = \frac{P_{\tilde{L},S,h,t}}{1-\xi_{Q,ESS,h,e,q}} \qquad \forall h,e,q,t \quad (409)$$

$$\frac{\partial P_{\tilde{L},Q,h,e,q,t}}{\partial P_{f,t}} = \frac{1}{1-\xi_{Q,ESS,h,e,q}} \frac{\partial P_{\tilde{L},S,h,t}}{\partial P_{f,t}} \qquad \forall h,e,q,t \quad (410)$$

Effective price of factors of production:

$$P_{C,K,e,q,t} = P_{C,K,t} \qquad \forall e,q,t \quad (411)$$

$$\frac{\partial P_{C,K,e,q,t}}{\partial P_{f,t}} = \frac{\partial P_{C,K,t}}{\partial P_{f,t}} \qquad \forall e,q,t \quad (412)$$

Price factor (extraction goods):

$$
\begin{aligned}
X_{e,j,t} =& P_{\Psi}\xi_{\Psi,e,j} + A_{Q,e,j,t}{}^{-1} \prod_g V_{g,t}{}^{-\eta_{g,e,j}} \prod_f \alpha_{f,e,j}{}^{-\alpha_{f,e,j}} \prod_f P_{f,e,j,t}{}^{\alpha_{f,e,j}} \\
& \forall e,j,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}
\end{aligned} \quad (413)
$$

$$
\begin{aligned}
\frac{\partial X_{e,j,t}}{\partial P_{f,t}} =& \alpha_{f,e,j} P_{f,e,j,t}{}^{-1} \left(X_{e,j,t} - P_{\Psi}\xi_{\Psi,e,j}\right) \frac{\partial P_{f,e,j,t}}{\partial P_{f,t}} \\
& \forall e,j,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}, K_j\}, \quad P_f \in \left\{P_{\tilde{L},Q,h}, P_{C,K}\right\}
\end{aligned} \quad (414)
$$

Price of extracted (intermediate) goods:

$$P_{Q,j,t} = A_{Q,j}{}^{-1} \left(\sum_{e} \gamma_{e,j}{}^{\sigma_j} X_{e,j,t}{}^{1-\sigma_j} \right)^{\frac{-1}{\sigma_j - 1}} \qquad \forall j,t \tag{415}$$

$$\frac{\partial P_{Q,j,t}}{\partial P_{f,t}} = A_{Q,j}{}^{\sigma_j - 1} P_{Q,j,t}{}^{\sigma_j} \sum_{e} \gamma_{e,j}{}^{\sigma_j} X_{e,j,t}{}^{-\sigma_j} \frac{\partial X_{e,j,t}}{\partial P_{f,t}} \qquad \forall j,t \tag{416}$$

Price factor (final goods):

$$\begin{aligned} X_{e,k,t} = & P_{\Psi} \xi_{\Psi,e,k} + A_{Q,e,k,t}{}^{-1} \prod_{g} V_{g,t}{}^{-\eta_{g,e,k}} \prod_{f} \alpha_{f,e,k}{}^{-\alpha_{f,e,k}} \prod_{f} P_{f,e,k,t}{}^{\alpha_{f,e,k}} \\ & \times \prod_{j} \alpha_{j,e,k}{}^{-\alpha_{j,e,k}} \prod_{j} P_{Q,j,t}{}^{\alpha_{j,e,k}} \\ & \forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \left\{P_{\tilde{L},Q,h,e}, P_{C,K,e}\right\} \end{aligned} \tag{417}$$

$$\begin{aligned} \frac{\partial X_{e,k,t}}{\partial P_{f,t}} = & (X_{e,k,t} - P_{\Psi} \xi_{\Psi,e,k}) \left(\alpha_{f,e,k} P_{f,e,k,t}{}^{-1} \frac{\partial P_{f,e,k,t}}{\partial P_{f,t}} + \sum_{j} \alpha_{j,e,k} P_{Q,j,t}{}^{-1} \frac{\partial P_{Q,j,t}}{\partial P_{f,t}} \right) \\ & \forall e,k,t, \quad V_g \in \{K_G, K_{EQ,S}, K_{PS,S}, K_{SC,S}\}, \quad P_{f,e} \in \left\{P_{\tilde{L},Q,h,e}, P_{C,K,e}\right\} \end{aligned} \tag{418}$$

Price of produced goods and services:

$$P_{Q,k,t} = A_{Q,k}{}^{-1} \left(\sum_{e} \gamma_{e,k}{}^{\sigma_k} X_{e,k,t}{}^{1-\sigma_k} \right)^{\frac{-1}{\sigma_k - 1}} \tag{419}$$

$$\frac{\partial P_{Q,k,t}}{\partial P_{f,t}} = A_{Q,k}{}^{\sigma_k - 1} P_{Q,k,t}{}^{\sigma_k} \sum_{e} \gamma_{e,k}{}^{\sigma_k} X_{e,k,t}{}^{-\sigma_k} \frac{\partial X_{e,k,t}}{\partial P_{f,t}} \qquad \forall k,t \tag{420}$$

Price of domestically produced and sold goods:

$$\begin{aligned} P_{D,q,t} = & \left((A_{X,q} P_{Q,q,t})^{-\left(\sigma_{X,q} - 1\right)} - \gamma_{X,q}{}^{\sigma_{X,q}} P_{X,q,t}{}^{-\left(\sigma_{X,q} - 1\right)} \right)^{\frac{-1}{\sigma_{X,q} - 1}} \\ & \times (1 - \gamma_{X,q})^{\frac{\sigma_{X,q}}{\sigma_{X,q} - 1}} \; \forall q,t \end{aligned} \tag{421}$$

$$\frac{\partial P_{D,q,t}}{\partial P_{f,t}} = (1 - \gamma_{X,q})^{-\sigma_{X,q}} A_{X,q}{}^{-\left(\sigma_{X,q} - 1\right)} \left(\frac{P_{D,q,t}}{P_{Q,q,t}} \right)^{\sigma_{X,q}} \frac{\partial P_{Q,q,t}}{\partial P_{f,t}} \qquad \forall q,t \tag{422}$$

Price of consumption:

$$P_{C,q,t} = A_{M,q}{}^{-1} \left(\gamma_{M,q}{}^{\sigma_{M,q}} P_{M,q,t}{}^{-\left(\sigma_{M,q}-1\right)} + \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} P_{D,q,t}{}^{-\left(\sigma_{M,q}-1\right)} \right)^{\frac{-1}{\sigma_{M,q}-1}} \tag{423}$$

$$\forall q, t$$

$$\frac{\partial P_{C,q,t}}{\partial P_{f,t}} = A_{M,q}{}^{\sigma_{M,q}-1} \left(1-\gamma_{M,q}\right)^{\sigma_{M,q}} \left(\frac{P_{C,q,t}}{P_{D,q,t}} \right)^{\sigma_{M,q}} \frac{\partial P_{D,q,t}}{\partial P_{f,t}} \qquad \forall q, t \tag{424}$$

Household transport consumption:

$$C_{h,k=HOT,t} = A_{HOT,h} T_{HO,h,t} - \sum_{h_1} Q_{h_1,h,k=HOT,t} \qquad \forall h, t \tag{425}$$

$$\frac{\partial C_{h,k=HOT,t}}{\partial P_{f,t}} = 0 \qquad \forall h, t \tag{426}$$

Household consumption:

$$C_{h,k,t} = \frac{\alpha_{C,h,k} \left(B_{h,t} - \left(1+\tau_{C,k_2=HOT}\right) P_{C,k_2=HOT,t} C_{h,k_2=HOT,t} \right)}{\left(1+\tau_{C,k}\right) P_{C,k,t}} \qquad \forall h, k \neq HOT, t \tag{427}$$

$$\frac{\partial C_{h,k,t}}{\partial P_{f,t}} = - \frac{C_{h,k,t}}{P_{C,k,t}} \frac{\partial P_{C,k,t}}{\partial P_{f,t}} - \frac{\alpha_{C,h,k} \left(1+\tau_{C,k_2=HOT}\right) C_{h,k_2=HOT,t}}{\left(1+\tau_{C,k}\right) P_{C,k,t}} \frac{\partial P_{C,k_2=HOT,t}}{\partial P_{f,t}}$$
$$\forall h, k \neq HOT, t \tag{428}$$

Government consumption:

$$C_{G,k,t} = \frac{\tilde{S}_{G,k,t}}{P_{C,k,t}} + \frac{\alpha_{C,G,k} I_{G,t}}{P_{C,k,t}} \qquad \forall k, t \tag{429}$$

$$\frac{\partial C_{G,k,t}}{\partial P_{f,t}} = - \frac{C_{G,k,t}}{P_{C,k,t}} \frac{\partial P_{C,k,t}}{\partial P_{f,t}} \qquad \forall k, t \tag{430}$$

Market-clearing demand for final production:

$$Q_{C,k,t} = \sum_{h} K_{N,h,t} C_{h,k,t} + C_{G,k,t} \qquad \forall k, t \tag{431}$$

$$\frac{\partial Q_{C,k,t}}{\partial P_{f,t}} = \sum_{h} K_{N,h,t} \frac{\partial C_{h,k,t}}{\partial P_{f,t}} + \frac{\partial C_{G,k,t}}{\partial P_{f,t}} \qquad \forall k,t \tag{432}$$

Quantity of domestically produced and sold final goods:

$$\begin{aligned} Q_{D,k,t} = & (1-\gamma_{M,k})^{\sigma_{M,k}} P_{D,k,t}{}^{-\sigma_{M,k}} A_{M,k}{}^{-1} Q_{C,k,t} \\ & \times \left(\gamma_{M,k}{}^{\sigma_{M,k}} P_{M,k,t}{}^{-\left(\sigma_{M,k}-1\right)} + (1-\gamma_{M,k})^{\sigma_{M,k}} P_{D,k,t}{}^{-\left(\sigma_{M,k}-1\right)}\right)^{-\frac{\sigma_{M,k}}{\sigma_{M,k}-1}} \forall k,t \end{aligned} \tag{433}$$

$$\begin{aligned} \frac{\partial Q_{D,k,t}}{\partial P_{f,t}} = & \frac{Q_{D,k,t}}{Q_{C,k,t}} \frac{\partial Q_{C,k,t}}{\partial P_{f,t}} - \sigma_{M,k} \gamma_{M,k}{}^{\sigma_{M,k}} P_{M,k,t}{}^{-\left(\sigma_{M,k}-1\right)} \frac{Q_{D,k,t}}{P_{D,k,t}} \frac{\partial P_{D,k,t}}{\partial P_{f,t}} \\ & \times \left(\gamma_{M,k}{}^{\sigma_{M,k}} P_{M,k,t}{}^{-\left(\sigma_{M,k}-1\right)} + (1-\gamma_{M,k})^{\sigma_{M,k}} P_{D,k,t}{}^{-\left(\sigma_{M,k}-1\right)}\right)^{-1} \end{aligned} \tag{434}$$

Quantity of domestically-produced final goods:

$$\begin{aligned} Q_{k,t} = & \left(\gamma_{X,k}{}^{\sigma_{X,k}} P_{X,k,t}{}^{-\left(\sigma_{X,k}-1\right)} + (1-\gamma_{X,k})^{\sigma_{X,k}} P_{D,k,t}{}^{-\left(\sigma_{X,k}-1\right)}\right)^{\frac{\sigma_{X,k}}{\sigma_{X,k}-1}} \\ & \times (1-\gamma_{X,k})^{-\sigma_{X,k}} P_{D,k,t}{}^{\sigma_{X,k}} A_{X,k} Q_{D,k,t} \\ & \forall k,t \end{aligned} \tag{435}$$

$$\begin{aligned} \frac{\partial Q_{k,t}}{\partial P_{f,t}} = & \frac{Q_{k,t}}{Q_{D,k,t}} \frac{\partial Q_{D,k,t}}{\partial P_{f,t}} + \sigma_{X,k} \gamma_{X,k}{}^{\sigma_{X,k}} P_{X,k,t}{}^{-\left(\sigma_{X,k}-1\right)} \frac{Q_{k,t}}{P_{D,k,t}} \frac{\partial P_{D,k,t}}{\partial P_{f,t}} \\ & \times \left(\gamma_{X,k}{}^{\sigma_{X,k}} P_{X,k,t}{}^{-\left(\sigma_{X,k}-1\right)} + (1-\gamma_{X,k})^{\sigma_{X,k}} P_{D,k,t}{}^{-\left(\sigma_{X,k}-1\right)}\right)^{-1} \\ & \forall k,t \end{aligned} \tag{436}$$

Production process (final products):

$$Q_{e,k,t} = Q_{k,t} \gamma_{e,k}{}^{\sigma_k} P_{Q,k,t}{}^{\sigma_k} A_{Q,k}{}^{\sigma_k - 1} X_{e,k,t}{}^{-\sigma_k} \qquad \forall e,k,t \tag{437}$$

$$\begin{aligned} \frac{\partial Q_{e,k,t}}{\partial P_{f,t}} = & \frac{Q_{e,k,t}}{Q_{k,t}} \frac{\partial Q_{k,t}}{\partial P_{f,t}} + \sigma_k \frac{Q_{e,k,t}}{P_{Q,k,t}} \frac{\partial P_{Q,k,t}}{\partial P_{f,t}} - \sigma_k \frac{Q_{e,k,t}}{X_{e,k,t}} \frac{\partial X_{e,k,t}}{\partial P_{f,t}} \\ & \forall e,k,t \end{aligned} \tag{438}$$

Pollution (final products):

$$\Psi_{e,k,t} = \xi_{\Psi,e,k} Q_{e,k,t} \qquad \forall e,k,t \tag{439}$$

$$\frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}} = \xi_{\Psi,e,k} \frac{\partial Q_{e,k,t}}{\partial P_{f,t}} \qquad \forall e,k,t \tag{440}$$

Extraction products demanded for final production:

$$\begin{aligned} V_{j,e,k,t} =& \alpha_{j,e,k} {P_{C,j,t}}^{-1} \left(\gamma_{e,k} P_{Q,k,t} Q_{k,t} {A_{Q,k}}^{\frac{\sigma_k-1}{\sigma_k}} \left(\frac{Q_{e,k,t}}{Q_{k,t}} \right)^{\frac{\sigma_k-1}{\sigma_k}} - P_\Psi \Psi_{e,k,t} \right) \\ & \forall e,j,k,t \end{aligned} \tag{441}$$

$$\begin{aligned} \frac{\partial V_{j,e,k,t}}{\partial P_{f,t}} =& - \frac{V_{j,e,k,t}}{P_{C,j,t}} \frac{\partial P_{C,j,t}}{\partial P_{f,t}} - \alpha_{j,e,k} {P_{C,j,t}}^{-1} P_\Psi \frac{\partial \Psi_{e,k,t}}{\partial P_{f,t}} \\ & + \left(V_{j,e,k,t} + \alpha_{j,e,k} {P_{C,j,t}}^{-1} P_\Psi \Psi_{e,k,t} \right) \\ & \times \left({P_{Q,k,t}}^{-1} \frac{\partial P_{Q,k,t}}{\partial P_{f,t}} + {\sigma_k}^{-1} {Q_{k,t}}^{-1} \frac{\partial Q_{k,t}}{\partial P_{f,t}} + \frac{\sigma_k - 1}{\sigma_k} {Q_{e,k,t}}^{-1} \frac{\partial Q_{e,k,t}}{\partial P_{f,t}} \right) \\ & \forall e,j,k,t \end{aligned} \tag{442}$$

Market-clearing demand for extraction goods:

$$Q_{C,j,t} = \sum_k \sum_e V_{j,e,k,t} \qquad \forall j,t \tag{443}$$

$$\frac{\partial Q_{C,j,t}}{\partial P_{f,t}} = \sum_k \sum_e \frac{\partial V_{j,e,k,t}}{\partial P_{f,t}} \qquad \forall j,t \tag{444}$$

Quantity of domestically produced and sold extraction goods:

$$\begin{aligned} Q_{D,j,t} =& (1-\gamma_{M,j})^{\sigma_{M,j}} {P_{D,j,t}}^{-\sigma_{M,j}} {A_{M,j}}^{-1} Q_{C,j,t} \\ & \times \left({\gamma_{M,j}}^{\sigma_{M,j}} {P_{M,j,t}}^{-(\sigma_{M,j}-1)} + (1-\gamma_{M,j})^{\sigma_{M,j}} {P_{D,j,t}}^{-(\sigma_{M,j}-1)} \right)^{-\frac{\sigma_{M,j}}{\sigma_{M,j}-1}} \end{aligned} \tag{445}$$

$$\forall j,t$$

$$\begin{aligned} \frac{\partial Q_{D,j,t}}{\partial P_{f,t}} =& \frac{Q_{D,j,t}}{Q_{C,j,t}} \frac{\partial Q_{C,j,t}}{\partial P_{f,t}} - \sigma_{M,j} {\gamma_{M,j}}^{\sigma_{M,j}} {P_{M,j,t}}^{-(\sigma_{M,j}-1)} \frac{Q_{D,j,t}}{P_{D,j,t}} \frac{\partial P_{D,j,t}}{\partial P_{f,t}} \\ & \times \left({\gamma_{M,j}}^{\sigma_{M,j}} {P_{M,j,t}}^{-(\sigma_{M,j}-1)} + (1-\gamma_{M,j})^{\sigma_{M,j}} {P_{D,j,t}}^{-(\sigma_{M,j}-1)} \right)^{-1} \end{aligned} \tag{446}$$

Quantity of domestically-produced extraction goods:

$$
\begin{aligned}
Q_{j,t} = &\left(\gamma_{X,j}{}^{\sigma_{X,j}} P_{X,j,t}{}^{-\left(\sigma_{X,j}-1\right)} + \left(1-\gamma_{X,j}\right)^{\sigma_{X,j}} P_{D,j,t}{}^{-\left(\sigma_{X,j}-1\right)}\right)^{\frac{\sigma_{X,j}}{\sigma_{X,j}-1}} \\
&\times \left(1-\gamma_{X,j}\right)^{-\sigma_{X,j}} P_{D,j,t}{}^{\sigma_{X,j}} A_{X,j} Q_{D,j,t} \\
&\quad \forall j,t
\end{aligned}
\tag{447}
$$

$$
\begin{aligned}
\frac{\partial Q_{j,t}}{\partial P_{f,t}} = &\frac{Q_{j,t}}{Q_{D,j,t}}\frac{\partial Q_{D,j,t}}{\partial P_{f,t}} + \sigma_{X,j}\gamma_{X,j}{}^{\sigma_{X,j}} P_{X,j,t}{}^{-\left(\sigma_{X,j}-1\right)} \frac{Q_{j,t}}{P_{D,j,t}}\frac{\partial P_{D,j,t}}{\partial P_{f,t}} \\
&\times \left(\gamma_{X,j}{}^{\sigma_{X,j}} P_{X,j,t}{}^{-\left(\sigma_{X,j}-1\right)} + \left(1-\gamma_{X,j}\right)^{\sigma_{X,j}} P_{D,j,t}{}^{-\left(\sigma_{X,j}-1\right)}\right)^{-1} \\
&\forall j,t
\end{aligned}
\tag{448}
$$

Production of extraction goods:

$$
Q_{e,j,t} = Q_{j,t}\gamma_{e,j}{}^{\sigma_j} P_{Q,j,t}{}^{\sigma_j} A_{Q,j}{}^{\sigma_j-1} X_{e,j,t}{}^{-\sigma_j} \qquad \forall e,j,t \tag{449}
$$

$$
\frac{\partial Q_{e,j,t}}{\partial P_{f,t}} = \frac{Q_{e,j,t}}{Q_{j,t}}\frac{\partial Q_{j,t}}{\partial P_{f,t}} + \sigma_j \frac{Q_{e,j,t}}{P_{Q,j,t}}\frac{\partial P_{Q,j,t}}{\partial P_{f,t}} - \sigma_j \frac{Q_{e,j,t}}{X_{e,j,t}}\frac{\partial X_{e,j,t}}{\partial P_{f,t}} \forall e,j,t \tag{450}
$$

Pollution (extraction goods):

$$
\Psi_{e,j,t} = \xi_{\Psi,e,j} Q_{e,j,t} \qquad \forall e,j,t \tag{451}
$$

$$
\frac{\partial \Psi_{e,j,t}}{\partial P_{f,t}} = \xi_{\Psi,e,j}\frac{\partial Q_{e,j,t}}{\partial P_{f,t}} \qquad \forall e,j,t \tag{452}
$$

Price of domestically produced and sold capital:

$$
\begin{aligned}
P_{D,K,t} = &\left(\left(A_{MK} P_{C,K,t}\right)^{-\left(\sigma_{MK}-1\right)} - \gamma_{MK}{}^{\sigma_{MK}} P_{MK,t}{}^{-\left(\sigma_{MK}-1\right)}\right)^{\frac{-1}{\sigma_{MK}-1}} \\
&\times \left(1-\gamma_{MK}\right)^{\frac{\sigma_{MK}}{\sigma_{MK}-1}} \qquad \forall t
\end{aligned}
\tag{453}
$$

$$
\frac{\partial P_{D,K,t}}{\partial P_{f,t}} = \left(1-\gamma_{MK}\right)^{-\sigma_{MK}} A_{MK}{}^{-\left(\sigma_{MK}-1\right)} \left(\frac{P_{D,K,t}}{P_{C,K,t}}\right)^{\sigma_{MK}} \frac{\partial P_{C,K,t}}{\partial P_{f,t}} \qquad \forall t \tag{454}
$$

Factors demanded for production:

$$V_{f,e,q,t} = \alpha_{f,e,q} {P_{f,e,q,t}}^{-1} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} {A_{Q,q}}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q - 1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \quad (455)$$

$$\forall e, f, q, t$$

$$\begin{aligned} \frac{\partial V_{f_1,e,q,t}}{\partial P_{f_2,t}} = & - \frac{V_{f_1,e,q,t}}{P_{f_1,e,q,t}} \frac{\partial P_{f_1,e,q,t}}{\partial P_{f_2,t}} - \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1} P_\Psi \frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \\ & + \left(V_{f_1,e,q,t} + \alpha_{f_1,e,q} {P_{f_1,e,q,t}}^{-1} P_\Psi \Psi_{e,q,t} \right) \\ & \times \left({P_{Q,q,t}}^{-1} \frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + {\sigma_q}^{-1} {Q_{q,t}}^{-1} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} + \frac{\sigma_q - 1}{\sigma_q} {Q_{e,q,t}}^{-1} \frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} \right) \\ & \forall e, f, q, t \end{aligned} \quad (456)$$

Quantity of domestically-demanded capital:

$$K_{IW,C,t} = \sum_e \sum_q V_{f=K,e,q,t} + D_{G,t} \qquad \forall t \quad (457)$$

$$\frac{\partial K_{IW,C,t}}{\partial P_{f_2,t}} = \sum_e \sum_q \frac{\partial V_{f=K,e,q,t}}{\partial P_{f_2,t}} \qquad \forall t \quad (458)$$

Quantity of domestically produced and sold capital

$$\begin{aligned} K_{IW,D,t} = & (1 - \gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-\sigma_{XK}} {A_{XK}}^{-1} K_{IW,S,t} K_{N,S,t} \\ & \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + (1 - \gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-\frac{\sigma_{XK}}{\sigma_{XK}-1}} \end{aligned} \quad (459)$$

$$\forall t$$

$$\begin{aligned} \frac{\partial K_{IW,D,t}}{\partial P_{f,t}} = & - \sigma_{XK} {\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} \frac{K_{IW,D,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} \\ & \times \left({\gamma_{XK}}^{\sigma_{XK}} {P_{XK,t}}^{-(\sigma_{XK}-1)} + (1 - \gamma_{XK})^{\sigma_{XK}} {P_{D,K,t}}^{-(\sigma_{XK}-1)} \right)^{-1} \end{aligned} \quad (460)$$

Price of factors of production (labour):

$$P_{f=\tilde{L},S,h,t} = \sum_q \sum_e \alpha_{f=\tilde{L},e,q} \left(\gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q-1}{\sigma_q}} - P_\Psi \Psi_{e,q,t} \right) \times \left(K_{N,S,t} \tilde{L}_{S,h,t} \right)^{-1} \qquad \forall t \tag{461}$$

$$\begin{aligned} \frac{\partial P_{f=\tilde{L},S,h,t}}{\partial P_{f_2,t}} =& - \left(K_{N,S,t} \tilde{L}_{S,h,t} \right)^{-1} P_\Psi \sum_q \sum_e \alpha_{f=\tilde{L},e,q} \frac{\partial \Psi_{e,q,t}}{\partial P_{f_2,t}} \\ & + \left(K_{N,S,t} \tilde{L}_{S,h,t} \right)^{-1} \sum_q \sum_e \alpha_{f=\tilde{L},e,q} \gamma_{e,q} P_{Q,q,t} Q_{q,t} A_{Q,q}{}^{\frac{\sigma_q - 1}{\sigma_q}} \left(\frac{Q_{e,q,t}}{Q_{q,t}} \right)^{\frac{\sigma_q-1}{\sigma_q}} \\ & \times \left(P_{Q,q,t}{}^{-1} \frac{\partial P_{Q,q,t}}{\partial P_{f_2,t}} + \sigma_q{}^{-1} Q_{q,t}{}^{-1} \frac{\partial Q_{q,t}}{\partial P_{f_2,t}} + \frac{\sigma_q - 1}{\sigma_q} Q_{e,q,t}{}^{-1} \frac{\partial Q_{e,q,t}}{\partial P_{f_2,t}} \right) \\ & \forall e, f, q, t \end{aligned} \tag{462}$$

Price of consumption of capital:

$$P_{C,K,t} = \left(1 - \gamma_{MK}\right)^{-1} A_{MK}{}^{-\frac{\sigma_{MK}-1}{\sigma_{MK}}} P_{D,K,t} \left(\frac{K_{IW,D,t}}{K_{IW,C,t}} \right)^{\frac{1}{\sigma_{MK}}} \qquad \forall t \tag{463}$$

$$\begin{aligned} \frac{\partial P_{C,K,t}}{\partial P_{f,t}} =& \frac{P_{C,K,t}}{P_{D,K,t}} \frac{\partial P_{D,K,t}}{\partial P_{f,t}} + \sigma_{MK}{}^{-1} \left(\frac{P_{C,K,t}}{K_{IW,D,t}} \frac{\partial K_{IW,D,t}}{\partial P_{f,t}} - \frac{P_{C,K,t}}{K_{IW,C,t}} \frac{\partial K_{IW,C,t}}{\partial P_{f,t}} \right) \\ & \forall t \end{aligned} \tag{464}$$