

# Population Ageing in New Zealand: The Impact on Living Standards and the Optimal Rate of Saving with a Flexible Real Exchange Rate

Ross Guest, Grant Scobie,  
and John Bryant

---

NEW ZEALAND TREASURY  
WORKING PAPER 03/34

DECEMBER 2003



THE TREASURY  
Kaitohutohu Kaupapa Rawa

**MONTH/YEAR**

December 2003

**AUTHORS**

Ross Guest  
Graduate School of Management  
Griffith University  
PMB 50 Gold Coast Mail Centre  
Queensland 4216  
Australia  
Email r.guest@griffith.edu.au  
Telephone 61 7 5552 8783  
Fax 61 7 5552 8553

Grant Scobie  
New Zealand Treasury  
1 The Terrace  
PO Box 3724  
Wellington  
New Zealand  
Email Grant.Scobie@treasury.govt.nz  
Telephone 64 4 471 5005  
Fax 64 4 499 0992

John Bryant  
New Zealand Treasury  
1 The Terrace  
PO Box 3724  
Wellington  
New Zealand  
Email John.Bryant@treasury.govt.nz  
Telephone 64 4 917 7027  
Fax 64 4 473 1151

**ACKNOWLEDGEMENTS**

This paper has benefited from comments from John Creedy

**NZ TREASURY**

New Zealand Treasury  
PO Box 3724  
Wellington 6008  
NEW ZEALAND  
Email information@treasury.govt.nz  
Telephone 64-4-472 2733  
Website www.treasury.govt.nz

**DISCLAIMER**

The views expressed in this Working Paper are those of the authors and do not necessarily reflect the views of the New Zealand Treasury. The paper is presented not as policy, but with a view to inform and stimulate wider debate.

# Abstract

---

The purpose of this paper is to extend the simulation analysis of population ageing in Guest, Bryant and Scobie (2003). In that paper a single-good Ramsey-Solow model was calibrated for New Zealand and used to simulate the impact of population ageing on optimal national saving and average living standards over the next 100 years. There are several innovations in the present paper. One is to allow for tradable and non-tradable goods and thereby to introduce a real exchange rate. Changes in the real exchange rate due to population ageing produce substitution effects between tradable and non-tradable goods, in both consumption and investment. Other innovations in this paper are an outward-looking model of utility, a proportion of rule-of-thumb consumers, and a vintage capital model. The simulations of population ageing are conducted by first deriving a range of demographic projections from alternative assumptions about fertility, mortality and immigration. The resulting series for population and employment by age group are weighted to account for age-specific labour productivity levels and consumption demands. The model is solved by finding optimal paths of investment and consumption from an initial steady state to a new steady state following a demographic shock. The sanguine assessment of the impact of population ageing on living standards and national saving in Guest, Bryant and Scobie (2003) remains intact following the extensions applied to the model in this paper. That is, the cost of ageing is equivalent in its effect on living standards to an annual loss of labour productivity growth of about a quarter of one percent over the next 50 years. The optimal path for national saving implies a rise of up to 2% of GDP over the next decade, relative to that which would have been optimal in the absence of population ageing. In all the cases considered, the optimal level of savings then trends down, so that by 2051 it would be about 2 percentage points of GDP lower than the level that would have been optimal were the population age structure to have remained unchanged.

## JEL CLASSIFICATION

E21 – Consumption; Saving

E22 – Capital; Investment (including Inventories); Capacity

F20 – International Factor Movements and International Business - General

F21 - International Investment; Long-Term Capital Movements

## KEYWORDS

consumption; saving; inter-temporal paths; Ramsey model; population ageing; foreign exchange; New Zealand

# Table of Contents

---

<b>Abstract</b> .....	<b>i</b>
<b>Table of Contents</b> .....	<b>ii</b>
<b>List of Tables</b> .....	<b>ii</b>
<b>List of Figures</b> .....	<b>ii</b>
<b>1 Introduction</b> .....	<b>1</b>
<b>2 Firms</b> .....	<b>2</b>
<b>3 Consumers</b> .....	<b>5</b>
3.1 The consumer's <i>intra</i> temporal maximization problem.....	5
3.2 The consumer's <i>inter</i> temporal maximization problem.....	6
<b>4 Solving the model</b> .....	<b>8</b>
<b>5 The role of the real exchange rate</b> .....	<b>9</b>
5.1 Substitution in investment between T and N sectors .....	9
5.2 Substitution in consumption between T and N sectors .....	10
<b>6 Demographic projections</b> .....	<b>11</b>
<b>7 Simulation results</b> .....	<b>13</b>
7.1 Living standards.....	13
7.2 National saving .....	16
<b>8 Conclusion</b> .....	<b>17</b>
<b>9 Appendix</b> .....	<b>19</b>
9.1 How important is the flexible real exchange rate?.....	19
9.2 What difference do “rule of thumb” consumers make? .....	21
9.3 Allowing for age-dependent preferences for tradable and non-tradable goods .....	24
<b>10 References</b> .....	<b>26</b>

## List of Tables

---

Table 1 – Symbols for variables and parameters used in the model.....	4
Table 2 – Summary of base and alternative demographic series.....	11
Table 3 – Construction of population projections from fertility, mortality, and migration variants.....	11
Table 4 - The effects of ageing on living standards .....	15
Table 5 – Effects of ageing on national saving: Changes in national saving, relative to case of no ageing, in percentage points of GDP .....	17

## List of Figures

---

Figure 1 – The effect of an ageing shock on the real exchange rate and investment in non-tradables .....	10
Figure 2 – The percentage shares of selected age groups in the total population.....	12
Figure 3 – Projections for the support ratio .....	13
Figure 4 – Loss in living standards due to population ageing.....	14
Figure 5 – Percentage point changes in national saving rate in response to population ageing .....	16

# Population Ageing in New Zealand: The Impact on Living Standards and the Optimal Rate of Saving with a Flexible Real Exchange Rate

## 1 Introduction

---

Population ageing will present new challenges for economic management? What will be the impact on the growth of living standards? Are there implications for the optimal rate of national saving? This paper addresses these questions using an economic model of an open economy with flexible exchange rates.

The paper is an extension of the analysis in Guest, Bryant and Scobie (2003). In that paper the authors made a somewhat sanguine assessment of the impact of population ageing in New Zealand on average living standards and optimal national saving in the coming decades. In particular, two important results emerged. One is that, according to the simulation model, population ageing is expected to reduce living standards by 12% by 2051 given the most likely demographic projection. This can be regarded as small from several perspectives. For example, it is equivalent to an annual reduction of 0.24% in labour productivity growth which is assumed to grow at an annual average rate of 1.5%. From another perspective, living standards will have approximately doubled by 2051 notwithstanding the loss of 12% of living standards due to population ageing.

The second result from the simulation model emerged partly from the first one. That is, population ageing alone would not constitute grounds for a significant boost in the optimal rate of national saving over coming years. The optimal path implied a very small increase in the rate of optimal national saving – probably less than one percent – in the next few years, followed by a fall in the optimal saving rate for several decades. This does not seem to present a strong case for directing fiscal policy toward a significant boost in the rate of national saving on account of population ageing.

These findings depend to varying degrees on the assumptions in the model. Sensitivity analysis was conducted with respect to some key parameters in the utility and production functions and the above conclusions remained intact. However, that model consisted of a single composite good and therefore could not allow for the substitution by firms and consumers between tradable and non-tradable goods in response to a change in their relative prices due to population ageing.

An important aim of the present study is to extend the model to allow for these substitution effects. In addition, three other extensions are adopted. The utility function is modified to allow for habit formation in consumption. The latter is achieved by adopting the outward-looking utility model in Carroll, Overland, and Weil (1997). In addition, it is assumed that a proportion of consumers do not optimise intertemporally but, rather, consume a fixed proportion of their income; they are described as “rule-of-thumb consumers”. The other innovation is to replace the conventional homogeneous capital production function with a vintage production function. These extensions are discussed in further detail below.

Unless stated otherwise, all assumptions and parameter values adopted in the simulation model in Guest, Bryant and Scobie (2003) are maintained in this paper. Readers are referred to that paper, which we will refer to as GBS (2003), for a more comprehensive description of the simulation approach, including literature support, than will be provided here. The description of the model here will focus on the innovations to GBS (2003).

The remainder of the paper proceeds as follows. In the next section we model the behaviour of firms. Two critical features are that we allow for capital to be differentiated by vintage (as distinct from a homogeneous capital good) and firms are assumed to produce both tradable and non-tradable goods. Section 3 models the utility maximising behaviour of consumers, and this is followed in Section 4 by a description of the solution to the model. Particular attention is given to the role of the exchange rate in Section 5 while Section 6 documents the key demographic trends that are employed in the analysis. The key results of the simulations with the model are presented in Section 7, for both living standards and the optimal rate of national saving. The conclusions are presented in Section 8.

## 2 Firms

---

An extension to GBS (2003) with respect to the behaviour of firms is that firms produce output according to a putty-clay, vintage capital model. This differs from the more common approach, adopted in GBS (2003), which is to treat all capital as homogeneous and malleable. In the latter case, all capital goods in use at a particular time have the same productivity and the labour quantum working on any capital good can be varied at any time. In a vintage capital model, however, technical progress is embodied in successive vintages of capital and, in the putty-clay version, once the labour-capital ratio on a particular vintage is chosen it cannot be changed throughout its life.<sup>1</sup>

Although it makes little difference to the optimal values of macroeconomic aggregates whether one adopts a homogeneous or a vintage model of capital in simulating a population ageing shock (Guest and McDonald, 2002), there is a pragmatic reason for doing so here in that it facilitates the solution to the model. This is because there is no need for adjustment costs in the installation of new capital, as there is in homogeneous capital models, in order to avoid unrealistically large jumps in optimal investment in response to a change in the interest rate. In a putty-clay vintage capital model, only the marginal vintage of capital, rather than the whole capital stock, responds to a change in the interest rate and this implies a less volatile series for optimal investment. Therefore no adjustment costs are required.<sup>2</sup>

---

<sup>1</sup> See Greenwood, Hercowitz, and Krussel (1997) for a comparison of embodied versus disembodied models of capital and an argument, on empirical grounds, for modelling technical progress as embodied rather than disembodied.

<sup>2</sup> See Guest and McDonald (2002) for a discussion the different implications of the two models of capital for the volatility of optimal investment.

A more important extension to GBS (2003) in this paper is the distinction between tradable and non-tradable goods which implies the existence of a real exchange rate. A list of variables and notation for the complete model is given in Table 1. Let  $Y_T$  and  $Y_N$  be output of tradable and non-tradable goods, respectively. Let  $V_T$  and  $V_N$  be the number of vintages of capital employed in producing tradable and non-tradable goods, respectively<sup>3</sup>. Assume that output is produced according to a vintage production function with Cobb-Douglas technology:

$$Y_{T,t} = \sum_{k=0}^{V_T-1} \left[ (1-\delta_T)^{k-1} A_{T,t-k} (I_{T,t-k})^\gamma [L_{T,t-k-1} (l_{T,t-k} + \delta_T)]^{1-\gamma} \right] \quad (1)$$

$$Y_{N,t} = \sum_{k=0}^{V_N-1} \left[ (1-\delta_N)^{k-1} A_{N,t-k} (I_{N,t-k})^\gamma [L_{N,t-k-1} (l_{N,t-k} + \delta_N)]^{1-\gamma} \right] \quad (2)$$

which, after suppressing the time subscripts, can be approximated by<sup>4</sup>

$$Y_T = Y_{T,-1} (1-\delta_T) + A_T I_T^\gamma (L_{T,-1} (l_T + \delta_T))^{1-\gamma} \quad (3)$$

$$Y_N = Y_{N,-1} (1-\delta_N) + A_N I_N^\gamma (L_{N,-1} (l_N + \delta_N))^{1-\gamma} \quad (4)$$

which, in turn, can be expressed in intensive form where  $y$  is output in efficiency units per equivalent worker and  $x$  is the growth rate of equivalent workers in efficiency units, that is,  $x=(1+l)(1+g)-1$ :

$$y_T = y_{T,-1} \frac{(1-\delta_T)}{(1+x_T)} + i^\gamma \left[ \frac{l+\delta}{1+l} \right]_T^{1-\gamma} \quad (5)$$

$$y_N = y_{N,-1} \frac{(1-\delta_N)}{(1+x_N)} + i^\gamma \left[ \frac{l+\delta}{1+l} \right]_N^{1-\gamma} \quad (6)$$

Investment in each of the two sectors is determined by the condition that the marginal product of capital is equal to the user cost of capital,  $r+\delta$ . Therefore

$$q_T = \frac{i_T(1+l_T)}{l_T + \delta_T} = \left( \frac{\gamma}{r + \delta_T} \right)^{\frac{1}{1-\gamma}} \quad (7)$$

$$q_N = \frac{i_N(1+l_N)}{l_N + \delta_N} = \left( \frac{e\gamma}{r + \delta_N} \right)^{\frac{1}{1-\gamma}} \quad (8)$$

<sup>3</sup> We follow the assumption in Obstfeld and Rogoff (1996) that only tradable goods can be transformed into capital. They describe this assumption as “inessential but helpful” (p.204). In our version of their model this assumption allows an analytic solution for  $l_N$  and  $l_T$ ; otherwise numerical methods would be required.

<sup>4</sup> These approximations are closer to (1) and (2), the larger number of vintages currently in use.



**Table 1 – Symbols for variables and parameters used in the model**

$N$	Population in natural units
$L$	Labour force in equivalent worker units*
$P$	Population in equivalent person units*
$\alpha$	Support ratio = $L/P$
$n$	Growth rate of $N$
$l$	Growth rate of $L$
$p$	Growth rate of $P$
$A$	Total factor productivity
$g$	Rate of labour productivity growth
$L^E$	Labour force in efficiency units $L_t^E = A_0 e^{gt} L_t$
$P^E$	Population in efficiency units $P_t^E = A_0 e^{gt} P_t$
$Y$	Aggregate output
$D$	Aggregate foreign liabilities, denominated here as debt
$C$	Aggregate consumption
$y$	Output per worker measured in efficiency units ( $Y/L^E$ )
$d$	Debt in efficiency units per worker ( $d = D/L^E$ )
$c$	Consumption in efficiency units per equivalent person ( $c = C/P^E$ )
$U$	Index of welfare maximized by social planner
$\theta$	Rate of time preference
$\delta$	Rate of depreciation
$\gamma$	Capital elasticity of output
$\beta$	The reciprocal of the elasticity of intertemporal consumption
$\lambda$	Change in the interest rate per unit change in foreign liabilities
$x$	$(1+g)(1+l)-1$
$i$	Investment in efficiency units per worker for $T$ and $N$ goods
$w$	The real wage
$e$	The real exchange rate
$b$	The initial, exogenously given, share of tradable goods in GDP
$\mu$	Parameter in consumption index that determines the share of $T$ goods and $N$ goods in the index
$\psi$	Elasticity of <i>intra</i> temporal consumption (between $T$ and $N$ goods)
$\xi$	Proportion of rule-of-thumb consumers
$E$	Expenditure, measured in $T$ goods, on $T$ and $N$ goods
$P^c$	Price, measured in $T$ goods, of a unit of the consumption index
$z$	The reference level of consumption, here set equal to $c_1$
$\omega$	A measure of the importance of the reference stock of consumption
$r^c$	The own rate of interest on the consumption index
$r$	The own rate of interest on tradable goods
$q$	A shorthand variable equal to $i(1+l)/(l+\delta)$ , for $T$ and $N$ goods
$V_T$	Number of vintages of capital employed in the $T$ sector.
$V_N$	Number of vintages of capital employed in the $N$ sector.
$x$	The growth rate of equivalent workers in efficiency units; that is, $x=(1+l)(1+g)-1$ .
$k$	Capital per equivalent worker in efficiency units
$s$	Index of average consumption needs of the all consumers; $s=P/N$

\*See Section 6 for an explanation of “equivalent workers” and “equivalent persons”.



The real wage,  $w$ , is equal to the marginal product of labour in each sector. That is,

$$A_T^{1-\gamma}(1-\gamma)(q_T)^\gamma = w \quad (9)$$

$$e \left[ A_N^{1-\gamma}(1-\gamma)(q_N)^\gamma \right] = w \quad (10)$$

The four equations (7) to (10) can be solved for the four endogenous variables  $q_T$ ,  $q_N$ ,  $w$  and  $e$ . However, in order to determine  $i_T$  and  $i_N$  from  $q_T$  and  $q_N$  it is necessary to allocate the exogenously given growth in aggregate employment between  $i_T$  and  $i_N$ . This is done by assuming that  $N$  goods cannot be capital goods, only consumer goods, implying their output is equal to their consumption (following the assumption in Obstfeld and Rogoff (1996)). To explain how this allows us to separate out  $i_T$  and  $i_N$  we begin by describing the model of consumption.

### 3 Consumers

The model of consumption adopted here follows closely that in Obstfeld and Rogoff (1996, Chapter 4). The intratemporal model of consumption is identical except for different notation. The only difference in the intertemporal model of consumption is in the form of the intertemporal utility function – in particular, we allow for external habit formation.

#### 3.1 The consumer's *intratemporal* maximization problem.

Define an index of total consumption for the representative consumer as<sup>5</sup>:

$$c = \left[ \mu^{1/\psi} c_T^{\psi-1/\psi} + (1-\mu)^{1/\psi} c_N^{\psi-1/\psi} \right]^{\psi/\psi-1} \quad (11)$$

Note the assumption here that  $\mu$ , the parameter determining the share of consumption goods between  $T$  and  $N$  goods (see equation (26) below), is constant. This implies that the preferences for tradable and non-tradable goods are constant across all age groups. In the Appendix we relax this assumption by allowing  $\mu$  to vary with the age distribution. This allows for the fact that health services and, to a lesser extent, education services, are predominantly non-tradable goods. Hence we would expect the relative preference for non-tradable goods to rise as the population ages.

Total expenditure on  $c_T$  and  $c_N$ , measured in units of  $T$  goods is

$$E \equiv c_T + e c_N \quad (12)$$

<sup>5</sup> Persons weighted by their consumption levels: see GBS (2003) for details.

In each period the representative consumer maximizes (11) subject to (12) yielding

$$\frac{\mu c_N}{(1-\mu)c_T} = e^{-\psi} \quad (13)$$

Combining (12) and (13) yields

$$c_T = \frac{\mu E}{\mu + (1-\mu)e^{1-\psi}} \quad \text{and} \quad c_N = \frac{e^{-\psi}(1-\mu)E}{\mu + (1-\mu)e^{1-\psi}} \quad (14)$$

Let  $P^C$  be the minimum  $E$  such that  $c=1$ ; hence it is the price, in  $T$  goods, of a unit of the consumption index. Using this and substituting (14) into (11) yields

$$P^C = [\mu + (1-\mu)e^{1-\psi}]^{1/(1-\psi)} \quad (15)$$

Given (13), (14) and the definition  $E=cP^C$ , gives

$$c_T = \mu \left( \frac{1}{P^C} \right)^{-\psi} c \quad \text{and} \quad c_N = (1-\mu) \left( \frac{e}{P^C} \right)^{-\psi} c \quad (16)$$

This solves the *intra*temporal maximization problem for each equivalent person unit.

## 3.2 The consumer's *inter*temporal maximization problem

In GBS (2003) it was assumed that the economy is centralised and a social planner allocates consumption for all consumers based on fully forward looking behaviour. In this paper, however, we adopt a decentralised economy, in order to allow for outward-looking utility described below, and also to allow for the innovation that not all consumers are intertemporal optimisers. We assume that the economy is populated by infinitely lived dynasties of people who differ only in that their consumption demands are age-specific. It is assumed that a proportion,  $(1-\xi)$ , of consumers are intertemporal optimisers and  $\xi$  are rule-of-thumb consumers who consume a constant proportion of their income. This assumption is adopted in Black et al. (1997) in their model of the New Zealand economy and is common in other applied economy-wide models.<sup>6</sup>

The consumers who optimise intertemporally maximise an outward-looking utility function, as in Carroll et al. (1997), where each consumer compares their consumption against the consumption of others in deriving their utility. Carroll et al. (1997) cite a range of evidence from the literature, both theoretical and empirical, in support of two alternative forms of what they call “comparison utility”.<sup>7</sup> From our point of view, perhaps the most compelling argument is that models of comparison utility generate persistence in the time series of consumption that matches the persistence found in the actual data.

<sup>6</sup> In the context of population ageing see, for example, the application of the MSG3 model in McKibbin and Nguyen (2001) and the OECD's MINILINK model in Turner, Giorno, Serres, Vourch, and Richardson (1998).

<sup>7</sup> One form of comparison utility is the “outward-looking” model that we adopt here. The other form is the “inward-looking” model in which consumers compare their consumption with their own past consumption rather than the consumption of others. Both forms of comparison utility generate a type of habit formation in consumption which implies the sort of persistence in consumption that we observe in the data. This is the main motivation for adopting comparison utility in this paper and from that point of view it does not matter whether we adopt the outward-looking model or the inward-looking model.

The utility that the representative consumer derives from a given level of consumption changes as the consumer's consumption needs change. Consumption needs change with age. Older consumers have greater consumption needs than younger consumers and therefore older consumers derive less utility from a given level of consumption than do younger consumers. To allow for this, the consumption level,  $c_t$ , is divided by an index,  $s_t$ , of consumption needs that vary with age;  $s_t$  is equal to  $(P/N)_t$  and hence  $s$  for the representative consumer varies with the average consumption needs of the population.

This consumer maximizes the following intertemporal utility function:

$$U = \sum_{t=1}^{\infty} \left[ \frac{c/s}{z^{\omega}} \right]_t^{1-\beta} \frac{(1+\theta)^{1-t}}{1-\beta} \quad j=1,\dots,h \quad (17)$$

where  $z$  is a reference stock of consumption.

The first-order condition for intertemporally optimising consumers is

$$\frac{\frac{\partial U_i}{\partial c^{opt}_t}}{\frac{\partial U_i}{\partial c^{opt}_{t+1}}} = (1+r_{t+1}^C) = (1+r_{t+1}) \frac{P_t^C}{P_{t+1}^C} \quad (18)$$

where  $r^C$  is the own rate of interest on the consumption index,  $c$ ; <sup>8</sup> and  $r$  is the rate of interest on tradables. Bonds are indexed to tradables so that  $B$  bonds are a claim on  $rB$  tradables per period (Obstfeld and Rogoff, 1996, p.229). Note that in the steady state,  $P_t^C = P_{t+1}^C$  and therefore  $r_{t+1}^C = r_{t+1}$ .

In equation(18)  $c^{opt}$  denotes the consumption of intertemporally optimising consumers. Consumers are assumed to be outward looking in the sense that their reference stock of consumption,  $z$ , is a function of the average consumption of all consumers across all age groups rather than a function of their own consumption. The parameter  $\omega$  indexes the importance of the reference stock of consumption. Here we set  $\omega = 1$  and define  $z_t = c^{opt}_{t-1}$  which is equal to the average consumption of all optimising consumers in period  $t-1$ .

We follow GBS (2003) in modelling imperfect capital mobility as an upward sloping supply price of foreign capital and adopt the same simple linear function for  $r$  :

$$r = \bar{r} + \lambda d \quad (19)$$

where, in the baseline scenario,  $\lambda=0.02$ .

The solution to (18) given (20), yields:

$$\frac{\dot{c}^{opt}}{c^{opt}} = \frac{1}{\beta} \left( \frac{\partial r}{\partial d} d + r - \dot{P}^C - \theta + (\beta - 1) \left( \frac{\dot{z}}{z} \right) + n - p \right) - g \quad (20)$$

<sup>8</sup> The intuition for the difference between  $r$  and  $r^C$  is as follows.  $P^C$  is a monotonic increasing function of  $p$ ; and  $e=P^N/P^T$ . Hence if  $P$  falls over time (i.e.  $P_t^C/P_{t+1}^C$  is rising), then  $T$  goods are becoming more expensive relative to  $N$  goods. Therefore a dollar of expenditure buys fewer traded goods relative to units of the consumption index than before. Hence the own interest rate on  $T$  goods has to rise to equal a given own interest rate on the consumption index.

The consumers who, as a rule-of-thumb, consume a fixed proportion of their income, have the same consumption as the optimising consumers in the initial steady state but hold that level fixed as a proportion of their income. Hence the ageing shock only affects their consumption only insofar as it alters their level of income, which is the same for all consumers. The consumption of rule-of-thumb consumers is denoted,  $c^{rot}$ ; note that  $c = \xi c^{rot} + (1-\xi)c^{opt}$ .

The national accounting identity gives the equation of motion for debt.

$$\dot{d} = (r(d) - x)d + \frac{c}{\alpha} + i_T + ei_N - y_T - py_N \quad (21)$$

The steady state implies that  $\dot{d} = 0$  and  $\dot{c} = 0$  which yields the following steady state equations:

$$c = \alpha(y_T + py_N - i_T - ei_N - (r(d) - x)d) \quad (22)$$

$$\theta = r - \beta g \quad (23)$$

## 4 Solving the model

As described in GBS (2003), the modelling procedure is to calibrate an initial steady state which implies that both population and employment are initially growing at constant rates. The demographic projections are then introduced as unanticipated “shocks”. The production function is calibrated by setting the rate of depreciation,  $\delta$ , in each of the two sectors such that initial output and capital stock for each sector are in a steady state. Given the first order condition for investment and the condition that output and capital are

in an initial steady state,  $\delta_{T,N} = \gamma \left( \frac{k_0}{y_0} \right)_{T,N}^{-1} - r$  where the value of  $(k_0/y_0)$  is set at 2.0 for the

$N$  sector and 3.0 for the  $T$  sector.<sup>9</sup> This therefore implies a higher rate of depreciation for the  $N$  sector than for the  $T$  sector.

The initial exogenously given employment level is allocated between the  $T$  and  $N$  sectors as follows. We follow the simplifying assumption in Obsfeld and Rogoff (1996) that  $N$  goods cannot be capital goods – they can only be consumption goods. This implies that output in the  $N$  sector is determined by consumption in the  $N$  sector; that is,  $Y_N = C_N$ . In turn,  $C_N$  is given by (13), where the initial steady state value of  $C$  is determined by (21). Hence given the parameter,  $\mu$ , the shares of  $Y_N$  and  $Y_T$  in initial GDP (exogenously given) can be determined endogenously. However, given we know more about the actual shares of  $N$  and  $T$  goods in output from available data than we do about the value of  $\mu$  in the intratemporal utility function, it is more sensible to calibrate  $\mu$ , given exogenous values for the output shares of  $T$  and  $N$  goods. Proceeding in this way, let initial output in the  $T$  sector be an exogenously given proportion,  $b$ , of initial aggregate GDP. This implies that

<sup>9</sup> Capital-labour ratios are typically in the range of two to three, and we assume that tradables are relatively more capital intensive than non-tradables.

$\frac{L_{0,N}}{L_{0,T}} = \frac{y_{0,T}}{y_{0,N}} \frac{(1-b)}{b}$  where  $y_0$  is given for both sectors by assuming an initial steady state in

which  $y_I=y_0$  and  $l=l_N=l_T$ . Given aggregate initial  $L$ , then  $L_{0,N}$  and  $L_{0,T}$  can be determined. The value of  $\mu$  is calibrated by an iterative process such that  $Y_N=C_N$ . Hence it is necessary to set exogenously either of the parameters,  $\mu$  or  $b$  from which the other is determined.

To calculate output in each sector it is necessary to allocate the growth rate of aggregate employment between the  $T$  and  $N$  sectors:  $l_T$  and  $l_N$ . This is done by noting that  $Y_N=C_N$  and using (15) and (19) to determine  $C_N$ . The following analytic solution for  $l_N$  is derived:

$$l_N = \left[ \frac{C_N - Y_{N,-1}(1 - \delta_N)}{A_N^{1-\gamma} L_{N,-1}^\gamma q_N^\gamma} \right] - \delta_N \quad (24)$$

The base case parameter values adopted in the model are the same as those in GBS (2003):  $\beta=1.52$ ,  $\alpha=0.35$ ,  $r$  in a steady state  $=0.05$ ,  $g=0.015$ , debt to output in a steady state  $= 0.8$ , and  $\lambda=0.02$ . In addition, we set  $\xi=0.3$ . These are also the values adopted in Black et al. (1997) for the New Zealand economy (with the exception of the debt to output ratio which they assume is equal to 1.05). The values of parameters that apply only to two good model are:  $\psi=1.0$  (base case),  $b=0.4$ , capital to output ratios in the tradable and non-tradable goods sectors are 3.0 and 2.0, respectively.

## 5 The role of the real exchange rate

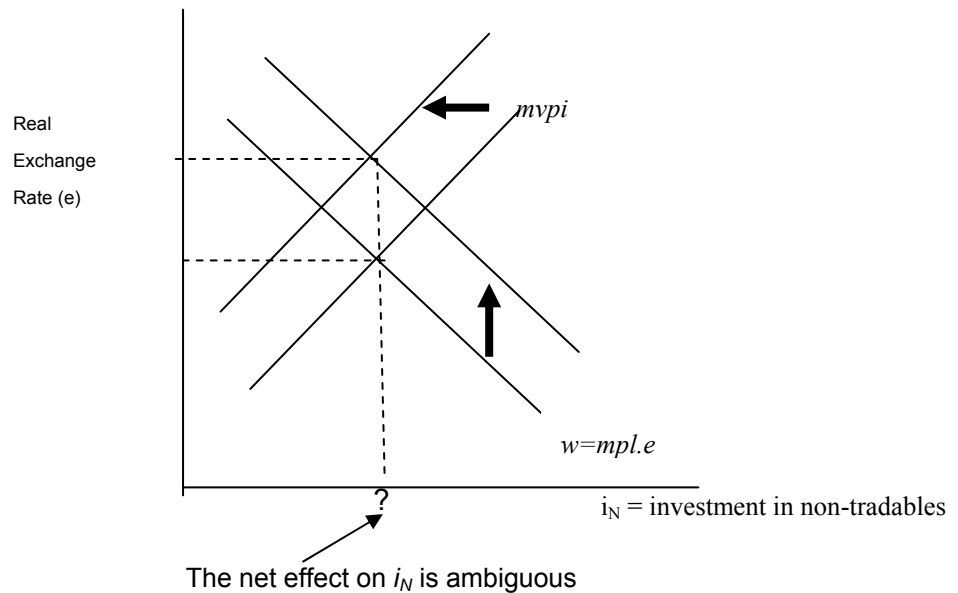
Changes in the real exchange rate generate substitution effects in both investment and consumption emanating from the effect of an ageing shock on the interest rate. With no change in the interest rate there can be no change in the capital-labour ratio and hence no change in the real wage and, in turn, no change in the exchange rate. Therefore in the small open economy model, where  $\lambda=0$ , there are no substitution effects.

### 5.1 Substitution in investment between T and N sectors

An ageing shock is transmitted to the real exchange rate via, initially, a lower interest rate. This is caused by the fall in investment expenditure due to lower capital widening requirements and the initial increase in saving as a consequence of smoothing out the loss of consumption possibilities caused by the ageing shock. The lower interest rate increases the optimal capital-labour ratio and therefore the marginal product of labour which, in the  $T$  goods sector, is equal to the real wage (see equation (9)). (In the  $N$  goods sector the real wage is equal to the marginal product of labour times the real exchange rate, see equation (10)). The higher real wage raises the real exchange rate because  $N$  goods are more labour intensive than  $T$  goods and therefore an increase in the real wage raises the relative price of  $N$  goods, which amounts to an appreciation of the real exchange rate. As Obstfeld and Rogoff point out (1996, p. 209) in the context of a different example, this result is the converse of the Stolper-Samuelson theorem which states that an increase in the relative price of a good benefits the factor used intensively in the production of that good. The converse to this, illustrated here, is that a relative increase in a factor share of income leads to an increase in the relative price of the product that uses that factor intensively.

Figure 1, adapted from the diagram in Obstfeld and Rogoff (1996, p.207), illustrates the relationship between four variables: the real exchange rate,  $e$ , the level of investment in  $N$  goods,  $i_N$ , the marginal value product of investment in  $N$  goods ( $mvpi$ ), and the marginal value product of labour in the  $N$  sector,  $mpl.e$  where  $mpl$  is the marginal product of labour and  $e$  is the real exchange rate. The ageing shock shifts the real wage schedule upwards which implies an upward movement along the  $mpvi$  schedule as the increase in the real exchange rate raises the marginal value product of investment in the  $N$  sector. This offsets the other effect of the ageing shock on investment, which is the effect of lower capital widening requirements. The latter effect shifts the  $mpvi$  schedule to the left. The net effect on investment is ambiguous unless parameter values are specified. The diagram shows the two effects as exactly offsetting which is a special case. Note that the effect of the lower capital requirements on investment is to further increase the real exchange rate because the lower investment lowers the marginal product of labour which requires an increase in  $e$  for a given  $w$ .

**Figure 1 – The effect of an ageing shock on the real exchange rate and investment in non-tradables**



The ratio of investment in the  $T$  sector to that in the  $N$  sector is given by equation (25):

$$\frac{i_T}{i_N} = \left( \frac{r + \delta_N}{e(r + \delta_T)} \right)^{\frac{1}{1-\gamma}} \left( \frac{l_T + \delta_T}{l_N + \delta_N} \right) \left( \frac{1 + l_N}{1 + l_T} \right) \quad (25)$$

The relative investment level depends importantly on the relative capital widening requirements in the two sectors. The latter is determined by the difference in the initial steady state capital to output ratios in the two sectors, which determines  $\delta_N$  and  $\delta_T$ , and the relative employment growth rates in the two sectors,  $l_N$  and  $l_T$ .

## 5.2 Substitution in consumption between T and N sectors

The ratio of consumption in the two sectors is found by dividing the two equations in (16):

$$\frac{c_T}{c_N} = \left( \frac{\mu}{1-\mu} \right) e^{\psi} \quad (26)$$

From (26), the degree of substitution by consumers between  $T$  and  $N$  goods depends on the real exchange rate and their relative preferences for  $T$  and  $N$  goods, the latter being determined by two parameters: the shares of  $T$  and  $N$  goods in their consumption index, determined by  $\mu$ ,<sup>10</sup> and the elasticity of substitution,  $\psi$ , between the two goods in the index.

## 6 Demographic projections

This section briefly describes the derivation of the demographic projections and the ‘support ratio’; further details are available in GBS (2003).

**Table 2 – Summary of base and alternative demographic series**

	Base	Alternative
Fertility	Total fertility rate* falls from 2.0 in 2001 to 1.9 in 2025, and then remains at that level.	Total fertility rate falls from 2.0 in 2001 to 1.2 by 2025, and then remains at that level.
Mortality	Mean life expectancy rises from 78.5 years in 2001 to 86 years in 2053, and then remains at that level. (The Statistics New Zealand median assumption.)	Life expectancy increases by 2.3 years every decade for the entire projection period.
Migration	Net (inward) migration rate of 1.5 per 1000.	Net (inward) migration rate of 0 per 1000.

\*The total fertility rate is the number of births the average woman would expect to have over her lifetime if prevailing age-specific fertility rates were to be maintained indefinitely

We use the base and alternative fertility, mortality, and migration assumptions summarized in Table 2. From these we construct four projection variants, listed in Table 3. Comparison between the variants allows us to investigate the differences in the economic response to lower fertility, mortality, and migration. The projections are carried out using standard cohort component methods (Preston, Heuveline, and Guillot 2001).

**Table 3 – Construction of population projections from fertility, mortality, and migration variants**

Variant	Fertility	Mortality	Migration
Base	Base	Base	Base
Low fertility	Alternative	Base	Base
Low mortality	Base	Alternative	Base
Low migration	Base	Base	Alternative

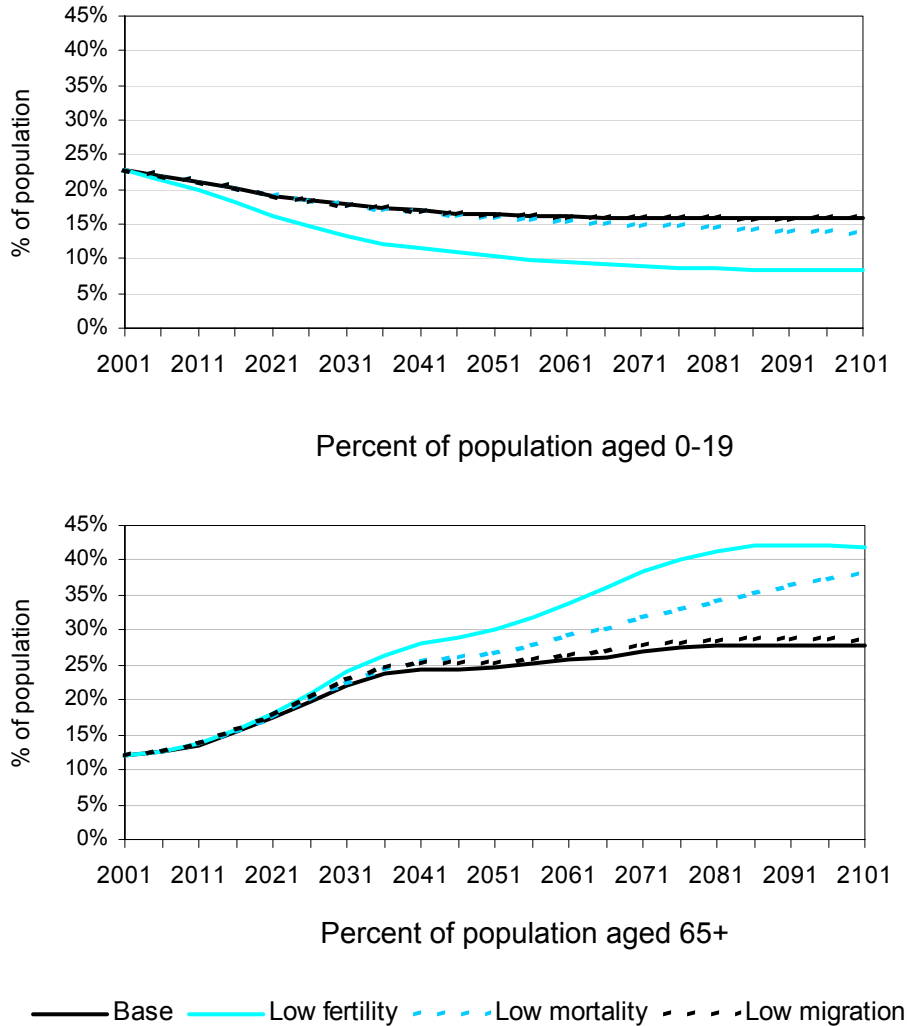
Figure 2 shows projected trends in age shares under the four variants. The upper panel shows trends in the percentage of the population aged 0-19. As is apparent, low fertility sharply reduces the percentage share of young people, relative to the base case. The lower panel shows trends in the percentage aged 65 and over. Low fertility and low mortality both increase the percentage share of old people. Differences in the migration

<sup>10</sup> When the intratemporal utility function is Cobb-Douglas (i.e. when  $\psi=1$ ),  $\mu$  is the share of  $T$  goods in the consumption index.



assumptions have little effect on the shares of either young people or old people; this is a reflection of the fact that the age profile of net migration to New Zealand is not greatly different from the age profile of the population as a whole.

**Figure 2 – The percentage shares of selected age groups in the total population**



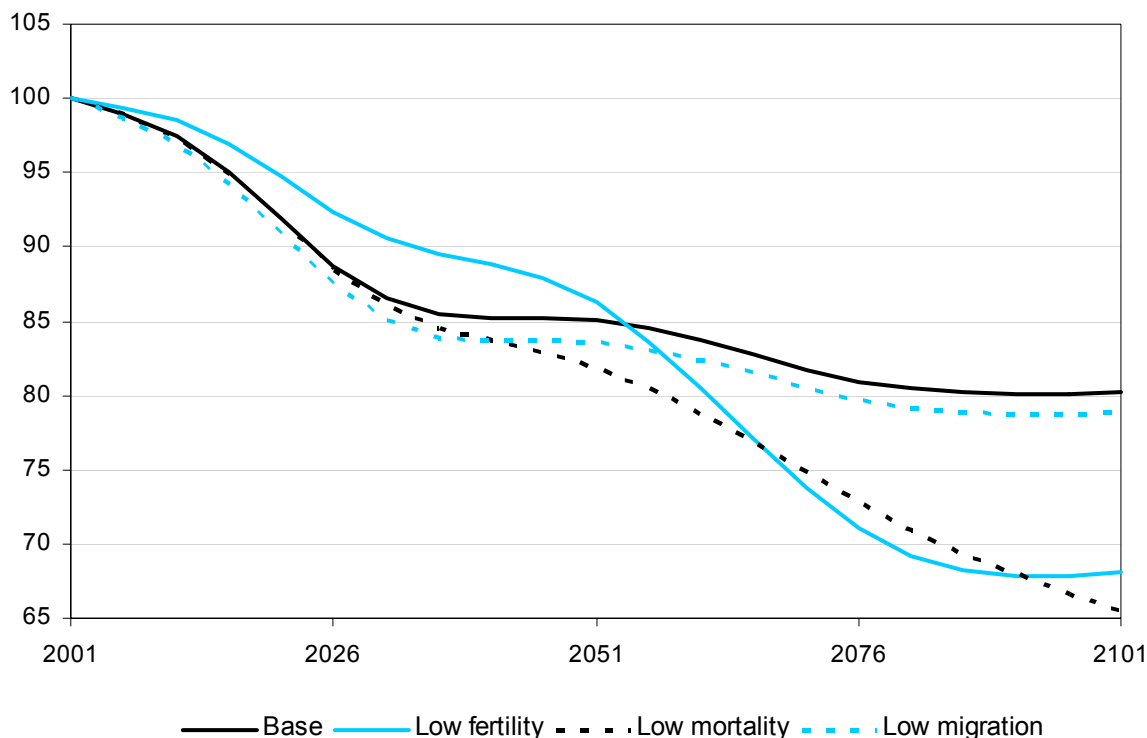
Following Cutler *et al* (1990), we summarize changes in the age structure using the ‘support ratio’. The support ratio weights the age-sex groups by their productivity and consumption. It is defined as follows:

$$\text{Support ratio} \equiv \alpha \equiv \frac{\left[ \begin{array}{c} \text{Labour force in equivalent} \\ \text{worker units} \end{array} \right]}{\left[ \begin{array}{c} \text{Population in equivalent} \\ \text{person units} \end{array} \right]} \equiv \frac{L}{P} \equiv \frac{\sum_i w_i N_i}{\sum_i s_i N_i} \quad (27)$$

where  $N_i$  is the number of people in age-sex group  $i$ ,  $w_i$  is a measure of the productivity of age-sex group  $i$ , and  $s_i$  is a measure of the consumption needs of age-sex group  $i$ . We normalize the support ratio to equal 100 in the base year. Higher support ratios imply higher consumption and production possibilities. The calculation of the weights,  $w_i$  and  $s_i$ , is described in GBS (2003).

Figure 3 shows the projection results for the support ratio. The support ratio undergoes the largest (most adverse) decline in the low-fertility and low-mortality cases.

**Figure 3 – Projections for the support ratio**



## 7 Simulation results

In this section we present the results of the simulations for the path of future living standards and for the optimal path of national saving.

### 7.1 Living standards

We define living standards as consumption per equivalent person, using the age-specific consumption weights described in GBS (2003) to calculate the population in equivalent persons. Figure 4 shows the path of optimal living standards under four alternative demographic scenarios – the base case, low fertility, low migration and low mortality. By 2051 living standards are, under all scenarios, at least 15% below what they otherwise would have been. This represents the cost of ageing. However, this result should be seen in the context of continued productivity growth. As Table 4 indicates, for ageing to result in reductions in the absolute level of living standards, labour productivity growth would have to fall to implausibly low levels – for example, 0.27% per annum in the base case. Given past, and expected future, average labour productivity growth in the order of 1.5% per annum, reductions to 0.27% as a long term average seem highly implausible. The conclusion to be drawn from this analysis is that it is almost impossible to imagine that

population ageing will result in an absolute reduction in living standards from their present level.

The effects on living standards of the alternative demographic scenarios illustrated in Figure 4 and Table 4 are very close, in magnitude and pattern, to those described in GBS (2003) for the one good model with 100% forward-looking behaviour. In both models, a lower (higher) fertility rate results in a higher (lower) standard of living for an initial period compared with the baseline scenario, which may last for decades, before the reverse occurs. The temporary boost to living standards from a lower fertility rate does not seem to be well-understood in popular discussion of fertility trends. The main reason is quite simple – a lower fertility rate lowers youth dependency.<sup>11</sup> In both the low-fertility and low-mortality variants, living standards towards the end of the projection period are significantly lower than in the base variant. However, the low-mortality variant does not have the compensating advantage of an initial period of high consumption. Low migration has much less effect on living standards than low fertility or mortality. This reflects the fact that changes in migration levels generally have less effect on age-structure, and hence support ratios, than changes in fertility and mortality.

**Figure 4 – Loss in living standards due to population ageing**

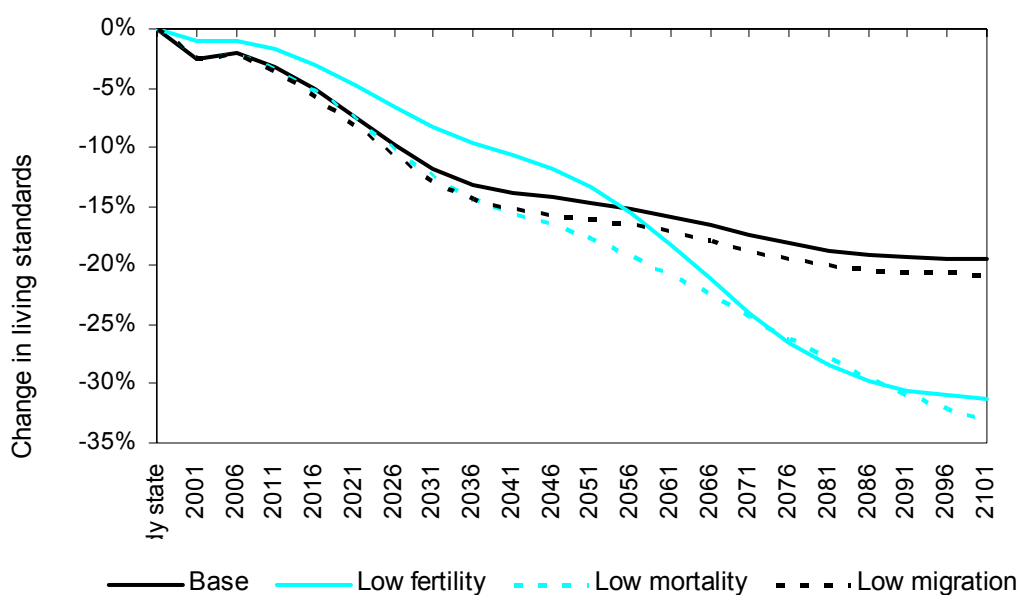


Table 4 presents a sensitivity analysis of the results for living standards to alternative parameters. The reader is referred to the companion paper, GBS (2003), for a detailed discussion of the role that the various alternative parameters play in affecting the response of living standards to an ageing shock. The following results also mirror those in GBS (2003). The impact on living standards is quite robust to alternative demographic assumptions and alternative parameter values. The largest variation in living standards compared with the base case occurs in the perfect capital mobility scenario ( $\lambda=0$ ). This is because the adjustment to a lower rate of consumption occurs immediately resulting in a relatively big initial loss of living standards but a smaller loss of living standards later on.

There is an additional parameter in this model - the intratemporal elasticity of consumption,  $\psi$  - that does not enter the one good model. The difference between the

<sup>11</sup> See GBS (2003) for a discussion of this and two other sources of, albeit much smaller, gains to living standards from lower fertility, that is, a capital-widening effect and a capital intensity effect.

value of this parameter and the value of the intertemporal elasticity of substitution,  $1/\beta$ , determines the rate of adjustment of consumption of  $T$  goods to the ageing shock, and therefore the rate of adjustment of debt to the new steady state (see Obstfeld and Rogoff, 1996, p.234).<sup>12</sup> These parameters have opposite effects on the direction of response of consumption of tradables to a change in the exchange rate. From (26) an increase in  $e$  increases consumption of  $T$  relative to  $N$  goods with elasticity  $\psi$ . But an increase in  $e$ , which also increases  $P^C$  and therefore  $r^C$ , reduces  $c$  (by (18)) and therefore  $c_T$  (by (16)). Table 4 reports sensitivity of the aggregate consumption path to the value of  $(\psi-1/\beta)$ . “High” refers to a high value of  $(\psi-1/\beta)$ , given by  $\psi=10$  and  $1/\beta=0.33$ , and “low” refers to a low value of  $(\psi-1/\beta)$ , where  $\psi=1$  and  $1/\beta=1.0$ . It turns out, however, that the effect on living standards is minimal – no greater than 0.3 of one percentage point at any given time (see Table 4).

**Table 4 - The effects of ageing on living standards**

Year	Demographic assumptions				Sensitivity analysis with base case demographics					
	Base	Low fertility	Low mortality	Low migration	$\lambda=0.0$	$\lambda=0.01$	Exchange rate = 1	“High”*	“Low”*	“High”*; Exchange rate =1
Percentage chance in consumption per effective person relative to the case without population ageing										
2011	-3.2	-1.7	-3.4	-3.6	-7.2	-2.8	-3.2	-3.4	-3.1	-3.4
2021	-7.4	-4.7	-7.7	-8.2	-8.6	-7.7	-7.3	-7.3	-7.4	-7.2
2031	-11.8	-8.3	-12.6	-13.0	-10.1	-12.5	-11.8	-11.7	-11.8	-11.7
2041	-13.9	-10.7	-15.6	-15.2	-10.9	-14.2	-13.9	-14.0	-13.4	-14.0
2051	-14.7	-13.4	-17.8	-16.0	-11.2	-14.6	-14.7	-14.7	-14.6	-14.8
2101	-19.4	-31.2	-33.1	-20.7	-12.2	-19.4	-19.4	-19.4	-19.4	-19.4
Net change in living standards given population ageing and 1.5% per annum productivity growth										
2011	12.8	14.3	12.7	12.5	8.9	13.3	12.9	12.7	12.9	12.7
2021	27.3	29.9	27.0	26.5	26.1	27.0	27.4	27.4	27.3	27.5
2031	44.5	48.0	43.8	43.3	46.2	43.8	44.5	44.6	44.5	44.6
2041	67.5	70.7	65.8	66.2	70.5	67.2	67.5	67.4	67.6	67.4
2051	95.9	97.1	92.7	94.5	99.3	95.9	95.9	95.8	95.9	95.8
2101	323.8	312.0	310.1	322.5	331.0	323.8	323.8	323.8	323.8	323.8
Effect of ageing on living standards measured as the equivalent annual loss of average productivity growth										
2011	-0.32	-0.17	-0.33	-0.35	-0.70	-0.27	-0.31	-0.33	-0.31	-0.33
2021	-0.36	-0.23	-0.37	-0.40	-0.41	-0.37	-0.35	-0.35	-0.36	-0.35
2031	-0.37	-0.27	-0.39	-0.41	-0.32	-0.39	-0.37	-0.37	-0.37	-0.37
2041	-0.33	-0.25	-0.36	-0.35	-0.26	-0.33	-0.33	-0.33	-0.32	-0.33
2051	-0.27	-0.25	-0.33	-0.30	-0.21	-0.27	-0.27	-0.28	-0.27	-0.28
2101	-0.18	-0.27	-0.29	-0.19	-0.11	-0.18	-0.18	-0.18	-0.18	-0.18

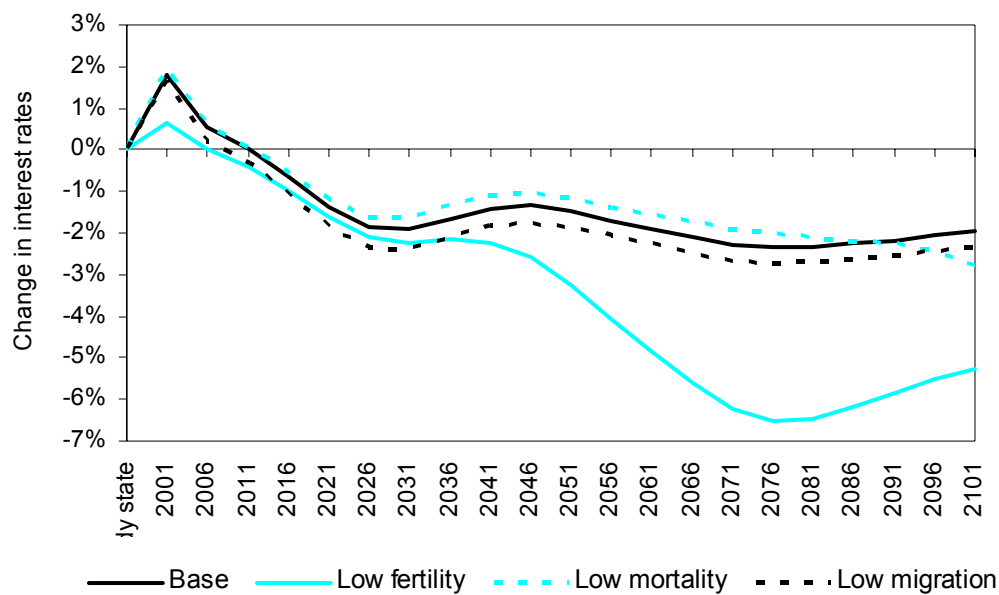
\* “High” and “low” refer to the gap between the *intra*temporal and *inter*temporal elasticities of substitution in consumption.

<sup>12</sup> Only traded goods determine debt because the consumption of non-traded goods is always equal their output by assumption in this model, following Obstfeld and Rogoff, (1996).

## 7.2 National saving

The responses of optimal national saving for the four demographic scenarios are illustrated in Figure 5. There is a small initial increase in optimal saving which varies from 0.9% in the low mortality case to 0.2% in the low fertility case. Again these responses are close (within about 1% point of GDP for each demographic scenario) to those in GBS (2003). A sensitivity analysis with respect to some parameters is reported in Table 5.

**Figure 5 – Percentage point changes in national saving rate in response to population ageing**



In all scenarios optimal saving falls after about ten years following the shock. This is because capital widening requirements fall as the growth rate of employment slows down. The link between lower capital widening requirements and lower saving is indirect rather than direct as it would be in a closed economy model. The link occurs via the interest rate. The lower capital widening requirements put downward pressure on the interest rate which implies a lower return to saving. The lower return to saving in turn slightly offsets the effect of lower capital widening requirements on investment by implying some capital deepening but this effect is not strong enough to prevent investment and saving from falling. By far the biggest effect on saving and investment occurs in the low fertility scenario because this implies the biggest slow down in labour force growth.

**Table 5 – Effects of ageing on national saving: Changes in national saving, relative to case of no ageing, in percentage points of GDP**

Year	Demographic assumptions				Sensitivity analysis with base case demographics					
	Base	Low fertility	Low mortality	Low migration	$\lambda=0.0$	$\lambda=0.01$	Exchange rate = 1	“High”*	“Low”*	“High”*; Exchange rate =1
2004	0.8	0.2	0.9	0.6	4.6	0.3	0.8	0.9	0.7	0.9
2006	0.5	0.0	0.6	0.2	4.2	0.1	0.5	0.7	0.4	0.7
2011	0.0	-0.4	0.1	-0.3	2.9	-0.2	0.0	0.2	0.0	0.1
2021	-1.4	-1.6	-1.2	-1.8	-1.3	-0.9	-1.4	-1.4	-1.4	-1.5
2031	-1.9	-2.2	-1.6	-2.4	-4.4	-1.1	-1.9	-2.0	-1.9	-2.0
2041	-1.4	-2.2	-1.1	-1.8	-4.7	-1.3	-1.4	-1.4	-1.4	-1.4
2051	-1.5	-3.2	-1.2	-1.8	-4.9	-1.6	-1.5	-1.4	-1.5	-1.4
2101	-2.0	-5.3	-2.8	-2.3	-9.1	-2.0	-2.0	-2.0	-2.0	-2.0

\* “High” and “low” refer to the gap between the *intra*temporal and *inter*temporal elasticities of substitution in consumption.

These results have implications for the required change in national saving, in response to population ageing, to achieve growing living standards. The maximum possible growth in living standards can be achieved without significant long-term increases in the rate of national saving in response to ageing. Rather, a modest temporary increase of no more than one percentage point of GDP followed by declines in the rate of saving appears to be the optimal response to ageing. This issue was discussed in more detail in GBS (2003).

## 8 Conclusion

Extending the model in GBS (2003) to a two good model with a real exchange rate and 30% of consumers being rule-of-thumb consumers has reinforced the conclusions in the earlier paper about the effects of population ageing on living standards and national saving. The substitution effects that exist in theory have turned out to be very small in the numerical simulations. The results are robust to a wide range of the key parameters that affect the size of the substitution effects.

The key results can be summarised as follows. In the base case, it is expected that by 2051, living standards measured as real consumption per person could be 15% below the level that they would have been in the absence of population ageing. However, it must be stressed that in any event, even with population ageing, real incomes will be virtually double their level in 2001. Population ageing may well reduce the level of standards but that level will still be much higher for future generations. These results are dependent on the assumption that labour productivity growth, a key driver of real incomes, will grow at an annual average rate of 1.5%. The effect of population ageing is equivalent to reducing the rate of productivity growth from 1.50 to 1.23% annually.

The optimal path for national saving implies a rise of up to 2% of GDP over the next decade, relative to that which would have been optimal in the absence of population ageing. In all the cases considered, the optimal level of savings then trends down, so that by 2051 it would be about 2 percentage points of GDP lower than the level that would have been optimal were the population age structure to have remained unchanged. These changes in the rate of saving are those which would be optimal to achieve the maximum growth in living standards. That the estimates imply modest changes over the

next 50 years, serves to underscore the fact that future living standards are much more dependent on sustaining high rates of economic growth rather than on any significant changes in the level of national savings.



## 9 Appendix

---

The results reported in the text suggested that both the flexible exchange rate and the allowance for rule-of-thumb consumers did not overturn the results in GBS (2003). The Appendix explores this claim in more detail.

### 9.1 How important is the flexible real exchange rate?

No insights into this question can be gained from the small open economy version of the model ( $\lambda=0$ ). In that case the substitution effects in investment and consumption are zero because there is no response of the interest rate to the ageing shock and hence no change in the real wage and the exchange rate. So we restrict our attention to cases where  $\lambda$  is non-zero.

The fact that the exchange rate cannot change unless the interest rate changes suggests the possibility that the parameter,  $\lambda$ , in the interest rate equation is important in determining the size of the substitution effects. Simulations show, however, that this parameter has a minimal effect on the path of the interest rate and therefore the exchange rate. This was illustrated by simulations in the Appendix to GBS (2003). The reason is that changes in the interest rate due to a change in debt end up being largely self-defeating because they set off a response in the saving rate that pushes debt and hence the interest rate back toward their original positions. For example, falling debt levels lead to lower interest rates which reduce the incentive to save and hence increase consumption, debt and hence the interest rate. Similarly, rising debt levels lead to higher interest rates, higher saving, lower consumption and hence lower debt levels. Simulations revealed that the second round effects are powerful in offsetting the first round effects for the entire range of plausible values of the elasticities of intratemporal and intertemporal consumption.

This is an interesting result because it implies that population ageing can only have a small impact on the interest rate, relative factor prices and the exchange rate using the method of determining the exchange rate applied in this paper. The maximum deviation of the interest rate from its steady state value, following the shock, is 35 basis points (0.35 percent). The maximum response of the exchange rate is also 0.35 percent. Such small effects on the exchange rate have small effects on substitution by firms and consumers. These effects are illustrated in Appendix Figures 1 to 3. In the figures, the “Exchange rate=1” case is the simulation holding the exchange rate constant at its initial value which is normalised to one. A comparison of this case with the others shows the effect of allowing the exchange rate to adjust.<sup>13</sup> The  $\lambda=0.10$  case gives an extremely high value for this parameter. The “high” and “low” cases denote high and low values of the gap between the intratemporal and intertemporal elasticities of consumption.

As one would expect, the small substitution effects shown in the figures have small effects on the optimal path of living standards and national saving. Indeed the effect on living standards in any one year is no more than 0.1 percentage point and the difference in

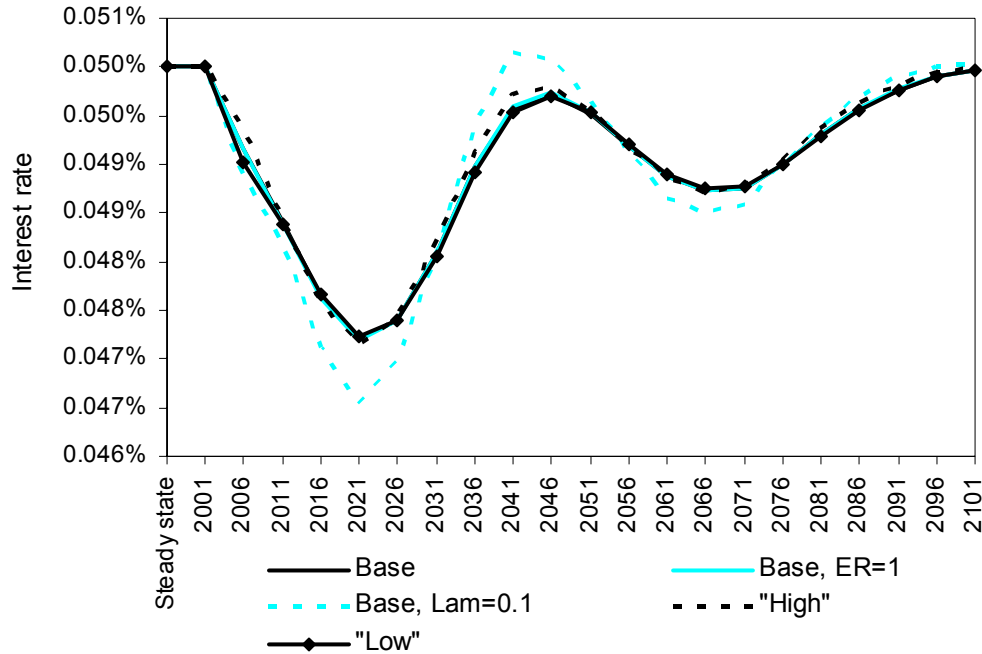
---

<sup>13</sup> Comparing the constant ER case with the base case is as close as one can get to a valid comparison of the two and one good models. It is not possible to map the two good model into the one good model by simply changing a parameter. The assumption in the two good model that output of  $N$  goods is equal to consumption of  $N$  goods, has no counterpart assumption in the one good model.

national saving rates is a little larger but no more than 0.5 of a percentage point in any given year.

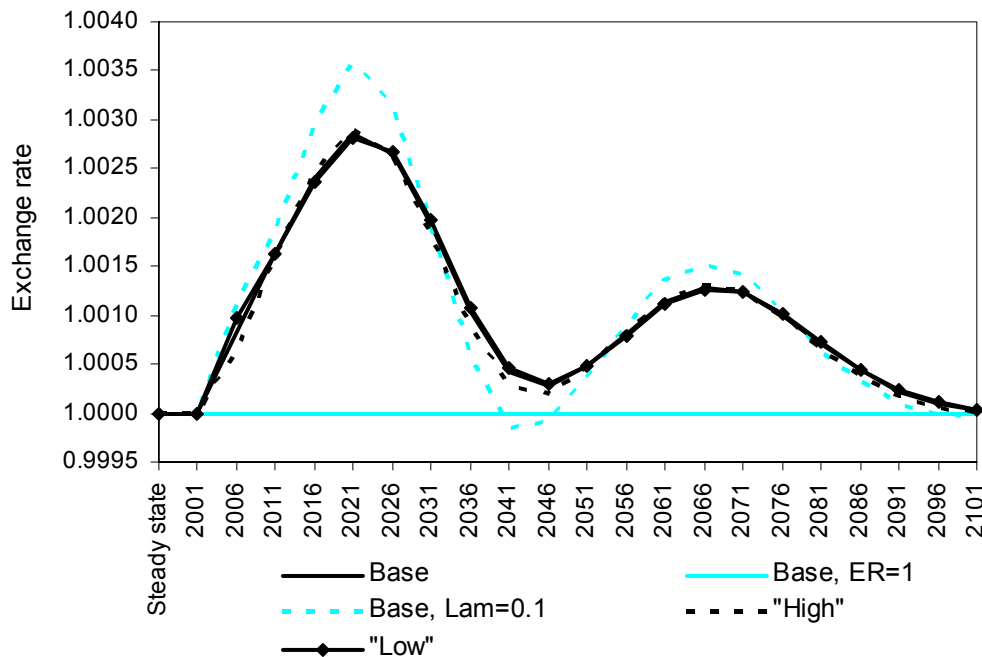
This helps to explain why the results for the one good model in GBS (2003) are so close to those for the two good model in this paper.

**Appendix Figure 1 – Interest rate**



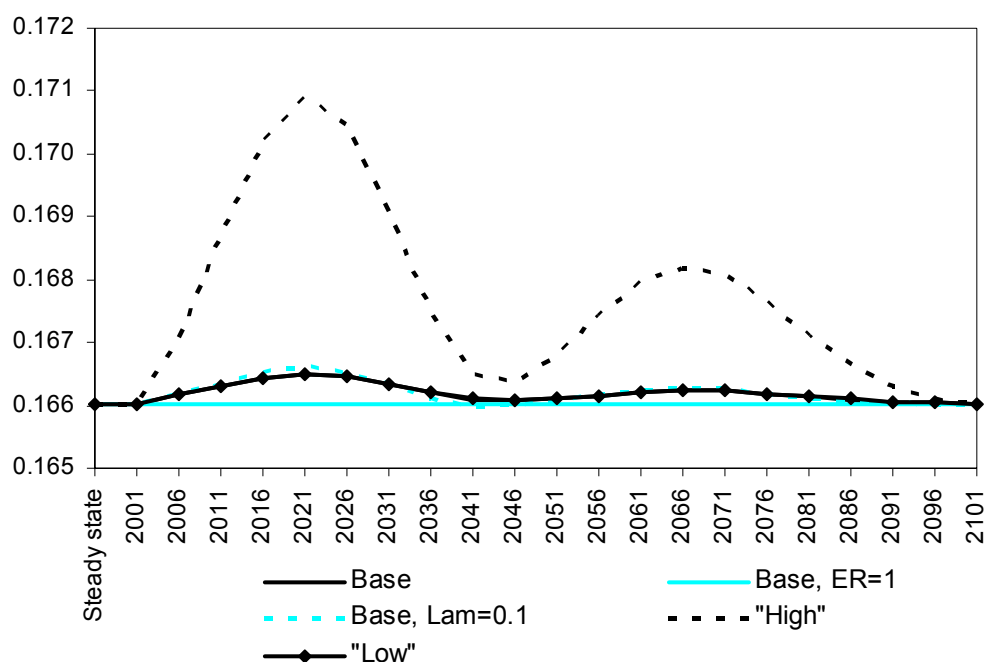
Note – "High and "low" refer to the gap between the *intra*temporal and *inter*temporal elasticities of substitution in consumption.

**Appendix Figure 2 – Real exchange rates**



Note – "High and "low" refer to the gap between the *intra*temporal and *inter*temporal elasticities of substitution in consumption.

**Appendix Figure 3 –  $cT/cN$**



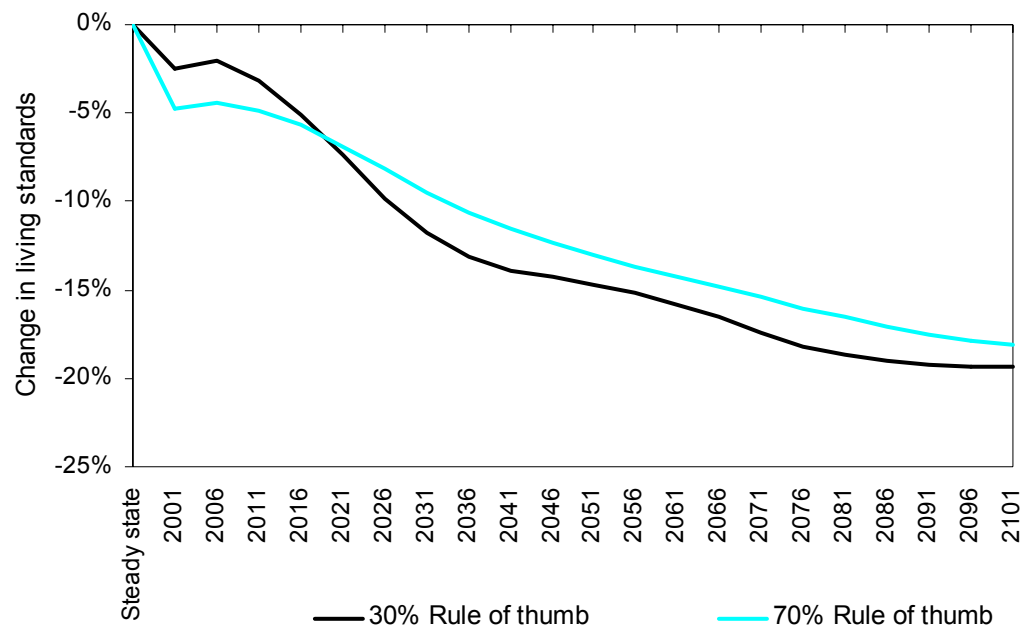
Note – “High and “low” refer to the gap between the *intra*temporal and *inter*temporal elasticities of substitution in consumption.

## 9.2 What difference do “rule of thumb” consumers make?

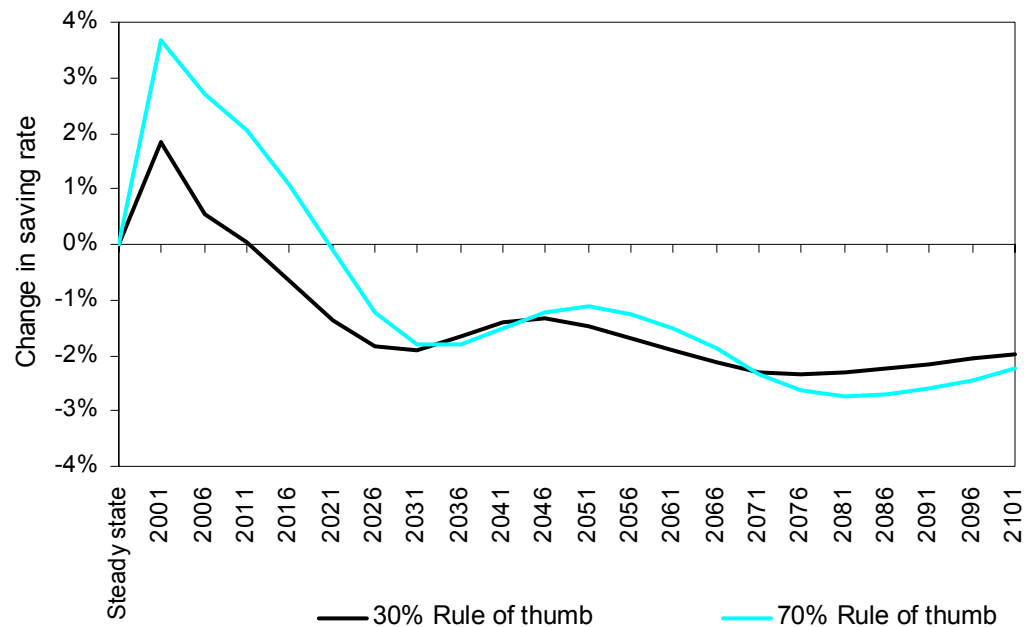
With respect to the proportion,  $\xi$ , of rule-of-thumb consumers, the base case value is 0.3, which is the value applied in the core model of the New Zealand economy (Black et al., 1997). We investigated the sensitivity of the results to this assumption by simulating the case where 70% of consumers are rule-of-thumb consumers; that is,  $\xi=0.70$ .

The results are illustrated in Appendix Figures 4 to 8. The main effect of a higher proportion of rule-of-thumb consumers is that the optimising consumers must save more in order to reach the final steady state. That is, the optimising consumers must compensate for the “undersaving” of the rule-of-thumb consumers, otherwise the debt ratio would not stabilise. Another way of interpreting this is to say that a social planner, constrained by the behaviour of the rule-of-thumb consumers, would restrict the consumption of the remaining consumers to ensure that aggregate consumption remained on the optimal path to the final steady state. The national saving rate in the case of 70% rule-of-thumb consumers is initially two percentage points of GDP higher than in the base case of 30% rule-of-thumb consumers. This extra required saving diminishes over time. The effect of higher national saving is a lower interest rate during the adjustment period (Appendix Figure 6) , a higher real exchange rate (Appendix Figure 7) and a greater degree of substitution by consumers between tradable and non-tradable goods (Appendix Figure 8).

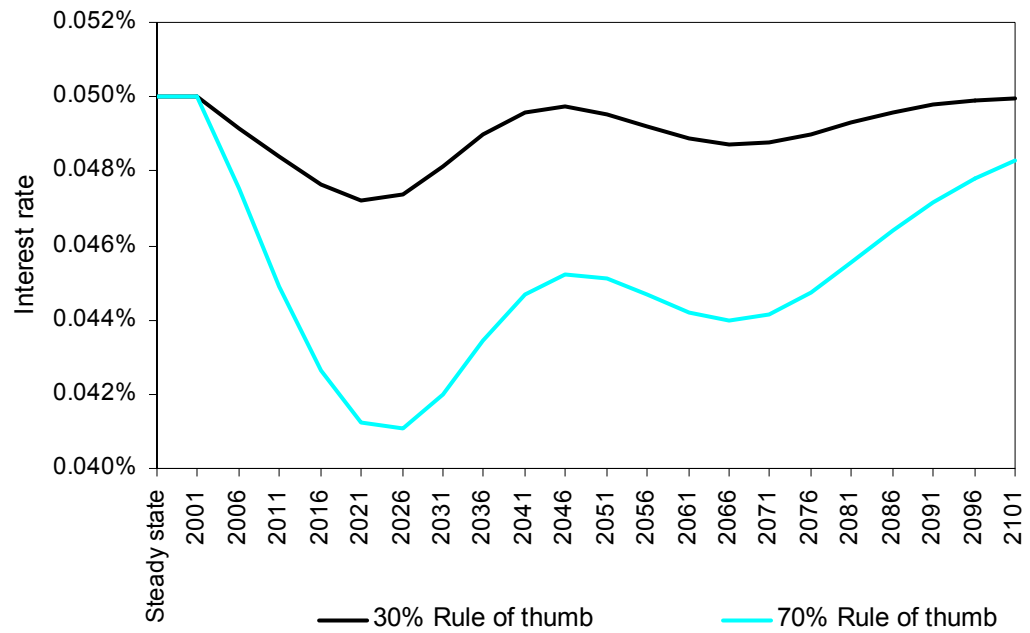
**Appendix Figure 4 – Percent loss in living standards due to population ageing, with 30% and 70% rule of thumb consumers (base demographics)**



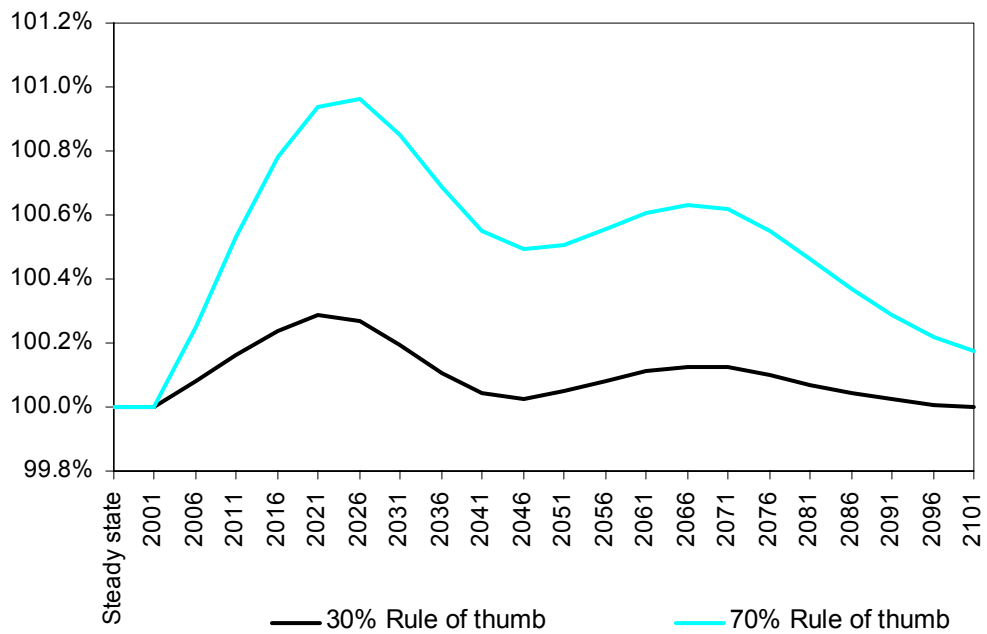
**Appendix Figure 5 – Percentage point change in national saving rate in response to ageing, with 30% and 70% rule of thumb consumers (base demographics)**



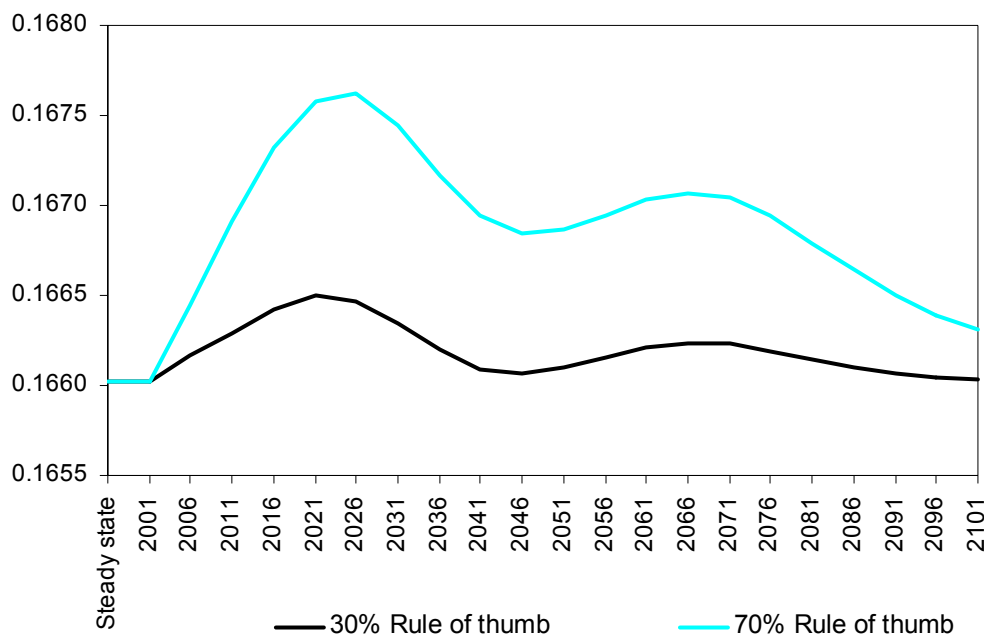
**Appendix Figure 6 – Interest rate, with 30% and 70% rule of thumb consumers (base demographics)**



**Appendix Figure 7 – Real exchange rate, with 30% and 70% rule of thumb consumers (base demographics)**



**Appendix Figure 8 –  $c_T/c_N$ , with 30% and 70% rule of thumb consumers (base demographics)**



### 9.3 Allowing for age-dependent preferences for tradable and non-tradable goods

The assumption made in (10) that  $\mu$  is constant over time for the representative consumer implies that relative preferences for tradable and non-tradable goods do not vary with age. This ignores the observation that preferences for medical and health services, which are essentially non-tradable goods, are age-dependent. In particular, we would expect that dependants, who are predominantly the young and old, have a higher demand for non-tradable relative to tradable goods than do working age people. A simple way to accommodate this is to assume that the preferences for non-tradable goods increase in proportion to the increase in the aggregate dependency ratio, which is equal to the inverse

of the (unweighted) support ratio minus 1.<sup>14</sup> Hence  $\mu_t$  can be defined as,  $\mu_t = \mu_0 \left( \frac{L}{N} \right)_t$ .

Simulations showed that this makes a small difference to the variables of interest. For the base, the dependency ratio declines by about 20 percent over the next 100 years. However, this affects the preference parameter,  $\mu$ , by much less than this because the initial share of tradable goods in total consumption is quite small. In fact, initially

$\left( \frac{C_T}{C_N} \right) = 0.17$ . Allowing for age-based relative preferences for the two goods implies that

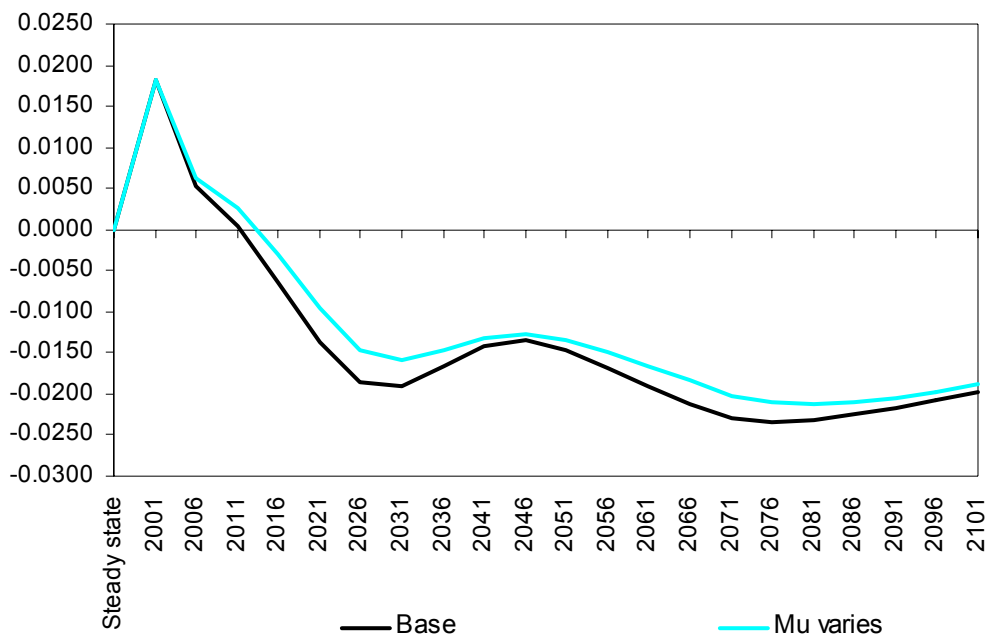
$\left( \frac{C_T}{C_N} \right)$  falls from 0.17 to a low point of 0.14. The effect is to increase the national saving rate slightly compared with the base case – by a maximum of 0.4% in any given year with

rate slightly compared with the base case – by a maximum of 0.4% in any given year with

<sup>14</sup> The dependency ratio  $= (N-L)/L = N/L - 1$ , where  $N$  is the total population and  $L$  is the working age population. The constant term (-1) can be omitted in defining  $\mu$ .

the difference being in the order of 0.2% for most years. The two series are plotted in Figure A9. The saving rate is higher in the case where relative demand for non-tradable goods is higher, because non-tradable goods have lower capital intensity, so the extra production of non-tradable goods means that labour resources are less productive on average – that is, output per worker is lower. This requires a higher saving rate. For other variables the impact is smaller – for example, living standards are 0.1% lower by 2051 than in the base case; and the interest rate differs by less than 0.1 of a percentage point relative to the base case in any year.

**Appendix Figure 9 - Percentage point change in national saving rate for the case of age-dependent preferences for N and T goods (base demographics)**





## 10 References

---

- Black, Richard, Vincenzo Cassino, Aaron Drew, Eric Hansen, Benjamin Hunt, David Rose and Alasdair Scott (1997) "The Forecasting and Policy System: The Core Model." Wellington, The Reserve Bank of New Zealand.
- Carroll, Christopher, Jody Overland and David Weil (1997) "Comparison utility in a growth model." *Journal of Economic Growth* 2(4): 339-367.
- Cutler, David, James Poterba, Louise Sheiner and Lawrence Summers (1990) "An aging society: Opportunity or challenge?" *Brookings Papers on Economic Activity* 1990(1): 1-73.
- Gordon, Roger H. and A. Lans Boverberg (1996) "Why is capital so immobile internationally? Possible explanations and implications for capital income Taxation." *American Economic Review* 86(5): 1057-1075.
- Greenwood, Jeremy, Zvi Hercowitz, and Per Krussell (1997) "Long run implications of investment specific technological change." *American Economic Review* 87(3): 342-362.
- Guest, Ross and Ian McDonald (2002), "Vintage capital versus homogeneous capital in macroeconomic simulations of population ageing: Does it matter?" *Applied Economics Letters* 10(3): 149-54.
- Obstfeld, Maurice and Kenneth Rogoff (1996) *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press.
- Guest, Ross, John Bryant, and Grant Scobie (2003) "Population ageing in New Zealand: Implications for living standards and the optimal rate of saving." Wellington, New Zealand Treasury, Working Paper No 03/10.
- McKibbin, W. and Nguyen, J. (2001), "The impact of demographic change in Japan: some preliminary results from the MSG3 Model." Paper presented at the Fall 2001 International Forum of Collaboration Projects, Economic and Social Research Institute of the Japanese Cabinet Office, Tokyo, September 17-19.
- Preston, Samuel H, Patrick Heuveline and Michel Guillot (2001) *Demography: Measuring and Modelling Population Processes*. Oxford: Blackwell Publishers.
- Turner, D., Giorno, C., Serres, A., Vourch, A, and Richardson, P. (1998) "The Macroeconomic Implications of Ageing in a Global Context", OECD Economics Department Working Paper No. 193.