Estimating the Cost of Capital for Crown Entities and State-Owned Enterprises

October 1997

A Handbook Prepared for the Treasury

Preface

This Handbook is a guide for the staff of Treasury when estimating the cost of the capital in relation to Crown entities or SOEs, or reviewing estimates undertaken by others. Other readers may also find the Handbook useful when undertaking these estimates.

The primary impetus for the Handbook arises from the need, in the context of value based reporting, to report a cost of capital. In addition to this application, the Handbook covers similar issues related to capital budgeting, setting output prices in non-contestable markets, and valuation.

The cost of capital for an entity can be estimated in a number of different ways. The methods set out in this Handbook have been selected on the basis that they are relatively straightforward to use, are commonly used in practice, and are consistent with the relevant economic and finance literature.

The Handbook draws on a March 1996 paper by Dr Martin Lally of Victoria University entitled *The Cost of Capital for Crown Companies*. This paper was presented to Treasury staff and circulated to a number of reviewers. The paper was revised by Dr Lally to reflect reviewers' comments, and is included as a set of Appendices to the Handbook. These Appendices provide a comprehensive discussion of the issues underlying the recommended methods of estimation.

The formulae for estimating the cost of capital as set out in the Handbook differ in some minor ways from that used for calculating the capital charge applying to government departments. The Handbook assumes tax neutrality between equity and debt capital, reflecting the dividend imputation regime now in place in New Zealand; in contrast the departmental capital charge calculation assumes that returns to equity are taxed more heavily than returns to debt. The two methods are compared in Appendix 6. The implications (if any) of these differences for the departmental capital charge will be addressed in a subsequent paper.

Familiarity with the Capital Asset Pricing Model (CAPM) is desirable, but not essential, for anyone using the Handbook. Two texts that include an introduction to the CAPM are:

- Copeland TE, Weston JF *Financial Theory and Corporate Policy (Third Edition)* Addison-Wesley 1988
- Brealey R, Myers S *Principles of Corporate Finance (Fifth Edition)* McGraw-Hill 1996.

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Introduction

This Handbook is to help staff of the Treasury to:

- review other people's estimates of the cost of capital
- prepare their own estimates of the cost of capital for Crown entities or SOEs.

The Handbook explains how to calculate the cost of capital for value based reporting, and then reviews how this calculation could change if the cost of capital was required for:

- capital budgeting
- setting output prices in non-contestable markets
- valuation.

The Handbook is not a detailed treatise on alternative methods of calculating cost of capital; such discussion is in the appendices. Where individual judgement is necessary, the Handbook seeks a practical approach; again detailed discussion of such judgements is in the appendices.

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Estimating the Cost of Capital for Value Based Reporting

This section sets out how to estimate the cost of capital for Crown entities and SOEs for value based reporting purposes.

Value based reporting presents the financial performance of a company net of a charge (capital charge) for the capital employed by the reporting entity. This charge is calculated by multiplying the total capital employed by the reporting entity (invested capital) by a cost of capital rate expressed as a percentage[1].

This section outlines how to estimate the cost of capital rate, and how to use that rate to calculate the capital charge.

The recommended method for estimating the cost of capital rate is set out below in two steps:

- the basic formula and its modifications
- determining each variable of the formula.

2.1 The basic formula and its modifications

The basic formula calculates a nominal capital charge rate (**k**), and is based on assumptions that:

- the reporting entity is subject to income tax
- **k** is independent of the proportions of debt and equity in the entity's capital structure; this assumption means **k** is an unlevered (or zero debt) cost of capital. The Capital Asset Pricing Model (CAPM) is used to specify this unlevered cost of capital.

[1] The value based reporting protocol defines invested capital, cost of capital and capital charge as:

"*Invested Capital*" is the total capital (equity, debt and their equivalents) employed by the entity.

"*Cost of Capital*" is the expected percentage return to the providers of capital.

"*Capital Charge*" is the dollar cost of capital for a period.

The formula is:

$$\mathbf{k = R_f(1 - T) + \varnothing\beta_a} \qquad \textbf{(1)}$$

where

$\mathbf{R_f}$ = the riskless rate, proxied by the current five year government stock rate, at the beginning of the reporting period

$\mathbf{T}$ = the marginal personal tax rate on interest income, set at 33% to reflect current tax law

$\varnothing$ = the tax adjusted market risk premium, estimated at 9%

β_a = the relevant asset beta for the reporting entity

This formula calculates the nominal capital charge rate for an entity that pays tax. A modified formula is needed if:

- a real capital charge rate is required; and / or
- the entity does not pay income tax.

Appendix 5 discusses the choice between a real capital charge rate and a nominal capital charge rate.

The modification of the formula to reflect an entity's tax free status is essential when comparisons with tax paying entities are likely.

Calculating a real rate

To convert a nominal rate (**k**) to a real rate ($\mathbf{k_r}$), the impact of expected price inflation needs to be removed from the nominal rate. The formula is:

$$\mathbf{k_r = \frac{1+k}{1+i} - 1 = \frac{1+R_f(1-T)+\varnothing\beta_a}{1+i} - 1} \qquad \textbf{(2)}$$

where

$\mathbf{i}$ = the expected inflation rate, proxied by the expected change in the consumer price index

Calculating a rate for non-taxed entities

Where the reporting entity is not subject to income tax, the formulae for **k** and for $\mathbf{k_r}$ are modified by dividing **k** in both formulae by $(\mathbf{1 - T_c})$.

Therefore, the nominal rate applicable to reporting entities that do not pay income tax, $(\mathbf{k_t})$ is calculated from **k** as follows:

$$\mathbf{k_t} = \frac{\mathbf{k}}{\mathbf{1 - T_c}} = \frac{\mathbf{R_f(1 - T) + \varnothing\beta_a}}{\mathbf{1 - T_c}} \qquad \mathbf{(3)}$$

where

$\mathbf{T_c}$ = the marginal company tax rate, set at 33% to reflect current tax law

Also, the real rate applicable to reporting entities that do not pay income tax, $(\mathbf{k_{rt}})$ is calculated from $\mathbf{k_t}$ as follows:

$$\mathbf{k_{rt}} = \frac{\mathbf{1 + k_t}}{\mathbf{1 + i}} - \mathbf{1} = \frac{\mathbf{1 + \{[R_f(1 - T) + \varnothing\beta_a] \,/\, (1 - T_c)\}}}{\mathbf{1 + i}} - \mathbf{1} \qquad \mathbf{(4)}$$

2.2 Determining each variable of the formula

This section works through how to determine each of the variables in formulae (1) to (4) above.

2.2.1 R_f - the riskless rate

Domestic government debt offers the best proxy for a riskless rate of return. However, as the rate often varies with the term of the debt, the choice of term is likely to be significant. For value based reporting, it is desirable to have a rate that reflects the value weighted average duration of the entity's future cash flows. Unless there is a clear reason to use a different term, the five year rate at the beginning of the reporting period is recommended. Appendix 3 discusses the choice of rates in more detail.

2.2.2 *T - marginal personal tax rate on interest income*

T is set equal to the (current) top marginal personal tax rate of 33%. This rate is consistent with the corporate tax rate of 33%. This consistency arises from the assumption of tax neutrality between debt and equity finance, and this, in turn arises from the assumption that **k** is independent of the proportions of debt and equity financing. Appendix 2 discusses the assumption of tax neutrality.

2.2.3 *∅ - tax adjusted market risk premium*

The tax adjusted market risk premium is the tax adjusted excess over the riskless rate ($\mathbf{R_f}$) of the expected rate of return to investors in a fully diversified portfolio of equities.

This parameter has been estimated as 9% from tax adjusted New Zealand data. Most other empirical work in this area has focussed on the non-tax adjusted market risk premium, and has provided a range of values broadly consistent with the 9%, after allowing for the tax difference (the 9% value is widely accepted in practice and corresponds to 6.4% without the tax adjustment). A tax adjusted market risk premium of 9% is therefore recommended. Appendix 3 outlines some of the issues in measuring the tax adjusted market risk premium.

2.2.4 β_a *- asset beta*

The CAPM uses the term beta (**β**) to denote systematic risk, that is the degree of association of returns from a particular investment with the returns from the whole market. Beta can be expressed as an equity beta, which includes the effects of debt on equity returns, or as an asset beta, which has any debt effects removed.

The equity beta of the whole market is necessarily 1.0. Investments with less systematic risk than the market have equity betas less than 1.0. Conversely investments with greater systematic risk than the market have betas greater than 1.0. An asset beta will be lower than the equity beta for any given investment; how much lower will depend on the level of debt in the capital structure of the firm.

The CAPM implies that investors will expect a return that reflects the investment's systematic risk. For Crown entities and SOEs, this approach estimates the opportunity cost of the capital invested in these entities.

For value based reporting, where the reporting entity has different lines of business, the systematic risk to be assessed is the risk of the separate lines of business. The beta for the entity as a whole needs to reflect a weighted average of the betas of the individual businesses that make up the entity.

Beta is likely to be the most difficult variable to determine when estimating the cost of capital rate. The process involves the following two steps:

- as the equity of Crown entities and SOEs is not traded, the returns to the equity holder (and therefore the betas of these returns) are not observable. This problem can be addressed by identifying comparator firms that do have traded equity, and using their returns to estimate an equity beta. A comparator firm needs to have economic characteristics similar to the Crown entity or SOE being assessed. These characteristics include the type of output supplied, nature of customers and suppliers, types of trading contracts, and so on. Any such comparison involves a degree of judgement
- the recommended formula for estimating the cost of capital rate requires an asset beta. An asset beta is a measure of systematic risk in the absence of debt. Most comparator firms have debt, and their equity betas must be converted to asset betas. An average of the asset betas of the comparator firms is then calculated to arrive at the asset beta for the particular investment or line of business being assessed.

Identify comparator firms

The selection of comparator firms calls for the exercise of professional judgement to identify firms that reflect the same systematic risk as the Crown entity or SOE. To gain a sufficient number of comparators, overseas data will normally need to be used. Where the entity has more than one line of business, comparators for each line of business should be identified.

Choosing appropriate comparators

To identify an appropriate comparator requires some knowledge of what underlies betas. Since beta measures systematic risk, betas arise from the firm's exposure to macro-economic events ("factors") affecting the returns of most firms in the economy. These factors are real GNP growth, inflation, changes in market risk aversion and changes in the long term real interest rate. Amongst equities the responsiveness of returns to inflation, market risk aversion and the long term real interest rate are expected to be similar. However responsiveness to real GNP growth will vary significantly over equities. Thus differences in equity betas are expected to arise largely from differences in responsiveness to real GNP.

Sensitivities of equity returns to real GNP may be expected to be affected materially by:

(1) nature of the output - the returns of firms producing necessities should have lower sensitivity to real GNP than for firms producing luxuries, because the demand for their product is less sensitive to real GNP

(2) nature of the customer - the returns of firms delivering an output to government should have lower sensitivity to real GNP than for firms whose output is delivered to the private sector, because the demand by their customer (government) should be less sensitive to real GNP

(3) nature of contracts with suppliers and customers - the returns of firms with, for example, long term fixed price contracts covering both inputs and outputs should have lower sensitivity to real GNP than for firms without these, because prices are not adjusted in response to real GNP shocks

(4) degree of regulation - the return of a firm subject to, for example, rate of return regulation should exhibit less sensitivity to real GNP, because demand changes arising from real GNP changes will be countered by price changes in the opposite direction

(5) degree of monopoly - assuming monopolists react to demand shocks by varying the “cushion” provided by suboptimal price setting and cost control more than do non-monopolists, then their returns will exhibit less sensitivity to demand, and hence real GNP, fluctuations

(6) the nature of the firm’s real options - to adopt a new project, close-down an existing one, proceed to the next stage of a multi-stage project, and to expand / contract existing operations. In general, the existence of options involving expansions of the firm will tend to increase beta, whilst those involving contractions will tend to decrease beta

(7) operating leverage - the returns of firms with higher operating leverage (i.e., higher fixed operating costs to total operating costs) should have higher sensitivity to real GNP, because cash flows are more sensitive to demand, and hence real GNP, fluctuations

(8) market weight - firms with a non-trivial weighting in the market index against which betas are calculated will have their beta drawn towards the market average of 1

(9) capital structure - the principal issue here is the proportion of capital represented by “straight” debt, i.e., the level of financial leverage. The (equity) returns

of firms with higher financial leverage should have higher sensitivity to real GNP, because cash flows (to equity) are more sensitive to demand, and hence real GNP, fluctuations. Warrants and convertible debt have more complex effects.

Necessarily, the effect of capital structure is removed in arriving at a firm's asset beta, by use of a degearing formula. In addition, differences in operating leverage and market weights can be corrected by use of appropriate formulas. This leaves dimensions (1) ... (6), for which satisfactory formulas are lacking. A possible response to this is to engage in subjective adjustments. However, we strongly recommend against this (see Appendix 4 for discussion of the reasons for this). The industry believed to be the best comparator in respect of (1) ... (6) should be selected. In doing so, the weighting of each dimension, and its degree of similarity to the situation of the Crown entity or SOE, will be a matter of judgement. Having so selected an industry, its constituent firm's asset betas should then be averaged. If material and possible, adjustment for differences in operating leverage and market weights should be undertaken.

It is possible that this process may fail to provide suitable comparators or a sufficient number of comparators to offer confidence in the average value. As a general rule, results from less than 10 comparators should be viewed with extreme caution. Furthermore, any average which is less than the low risk default (discussed below) should be rejected. A sceptical view should also be taken of any average comparator asset beta which is more than 70% over the market average (currently about .7). Where comparators are unavailable or are not acceptable, either of two default asset betas could be used:

- treat the Crown entity or SOE as average risk and use the market asset beta. The market equity beta is always 1.0. When this is converted to an asset beta to reflect current market leverage in New Zealand, the asset beta is approximately 0.7; or
- treat the Crown entity or SOE as very low risk, and use the asset beta of that industry whose asset beta is the lowest of those industries with a sufficient number of firms to offer confidence in the estimate. Overseas market data suggests this lower bound should be set at 0.3. However this number should be subject to ongoing scrutiny.

Betas that have been adjusted in some proprietary manner for particular purposes, such as BARRA, BLUME or VASICEK betas, should also be avoided. However, if possible, correction for variations in firm and market leverage over the beta estimation period should be performed.

Treasury is developing a database to provide a source of comparator firms and their equity and asset betas. For those using different data, the formula for calculating asset beta from equity beta is set out below.

Converting the comparator firm's equity beta to an asset beta

The following formula is recommended to convert an equity beta β_e to its corresponding asset beta β_a:

$$\beta_a = \beta_e\left(\frac{S}{V}\right)$$

where

β_a	=	an asset beta
β_e	=	an equity beta
S	=	the comparator firm's equity at market value
V	=	the sum of the comparator firm's market values of equity and debt

Incorporating international leverage differences

Some, and possibly all, of these comparator firms may be in foreign economies and their betas are therefore measured against a foreign market index. In this case, an adjustment may be needed to allow for any difference between leverage in the foreign market and leverage in the New Zealand market. The formula to convert a foreign asset beta (β_{aF}), as calculated above, to its New Zealand equivalent (β_a) is:

$$\beta_a = \beta_{aF} \frac{[1 + (B_{MF} / S_{MF})]}{[1 + (B_M / S_M)]}$$

where

B_M/S_M = New Zealand market ratio of debt to equity at market value

B_{MF}/S_{MF} = Foreign market ratio of debt to equity at market value

To calculate the required asset beta by averaging the comparator's asset betas

Unless there is a compelling reason for other averaging methods or weightings, an arithmetic mean of the comparator's asset betas should be taken to arrive at an overall comparator beta, subject to any adjustments for operating leverage, market weights, and market leverage for foreign betas.

This process should be repeated for each line of business, with the Crown entity or SOE asset beta being a weighted average of the individual business line asset betas, weighted with respect to their respective capital requirements.

This is the last step in determining an asset beta for a particular Crown entity or SOE. It is also the last step in determining all the variables in the basic formula (1).

2.2.5 *Other variables*

The formulae (2) to (4) incorporate the additional variables of:

- **i**, the expected inflation rate proxied by the expected change in the consumer price index
- $\mathbf{T_c}$, the company tax rate, set at 33% to reflect current tax law.

2.3 Calculating the capital charge

The capital charge is calculated by applying the cost of capital rate to the invested capital of the reporting entity.

Invested capital is defined in the Economic Reporting Protocol as “the total capital (equity, debt and their equivalents) employed by the entity”. The Protocol does not set out in detail how to measure invested capital, but the Protocol does require the reporting entity to disclose its reporting policies, and to reconcile reported invested capital with the reporting entity’s equity and debt capital as reported in its Statement of Financial position.

Issues likely to arise in determining invested capital include:

- the inclusion of all forms of capital, such as equity, long and short term debt, and capital implicit in, for example, lease arrangements (or equivalently the inclusion of all assets)
- the valuation of assets
- the treatment of changes to invested capital through time by, for example, capital injections, dividend payments and earnings.

There also needs to be consistency between the choice of either a nominal or real cost of capital rate, and the manner in which invested capital is measured. Appendix 5 discusses this in some detail. Broadly, three options are available:

- apply a nominal rate to the depreciated historic cost of assets
- apply a nominal rate to revalued assets, and include any revaluation amounts in the income (or NOPAT[2]) measure
- apply a real rate to revalued assets, but do not include any revaluation amounts in the income (or NOPAT) measure.

Each reporting entity will need to determine how it is to measure and report invested capital. This Handbook offers no recommendation about this.

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[2] Net Operating Profit After Tax (NOPAT).

3

Application to Capital Budgeting

Capital budgeting typically includes assessing the present value of projects by discounting their expected future stream of net cash flows. The discount rate should be consistent with the cost of capital rate used for value based reporting purposes, but modifications of the rate may be required:

- the asset beta used to calculate the cost of capital rate for a proposed project needs to reflect the systematic risk of the project. Where the project is a new line of business for the reporting entity, or the entity has a range of businesses with differing risk characteristics, the project beta may differ from the beta used by the entity for value based reporting purposes. A project specific beta can be identified by applying the process described above for value based reporting.
- there needs to be consistency between the choice of either a nominal or real cost of capital rate, and the net cash flows being discounted. If the project's cash flows are expressed in nominal terms, then the cost of capital rate needs to be nominal. Conversely, if the project's cash flows are in real terms, a real cost of capital rate is required. Equations (1) to (4) above set out how to derive nominal and real rates.
- in capital budgeting, the riskless rate need not be the same in all years. If these rates differ significantly by term-to-maturity, then separate riskless rates (and hence costs of capital) may be applied to each year.

4

Application to Setting Output Prices in Non-contestable Product Markets

Crown entities and SOEs that operate in non-contestable product markets often need to support their pricing decisions by providing stakeholders with cost information, including the cost of capital. In general the method recommended for estimating the cost of capital for value based reporting can be used for this purpose. However, there are issues that may arise in this situation:

- consistency between nominal or real capital charge rates, and the manner in which assets are treated
- whether actual interest costs or current interest costs are incorporated in output prices
- the presumed tax scenario.

Decisions on these issues will need to be made by the entity in the context of disclosing information and setting prices. This Handbook does not recommend a preferred approach, but outlines how each issue can be approached.

Formulae (1) and (2) above calculate the capital charge after company tax. When setting output prices, a pre-tax capital charge may be required. The pre-tax capital charge can be calculated by using formulae (3) or (4) above.

4.1 Nominal or real rates

As with value based reporting, the choice of either nominal or real capital charge rates needs to be consistent with how the capital base (invested capital) is measured. Regardless of whether a nominal or real capital charge rate is used, all other costs need to be expressed in nominal terms.

4.2 Treatment of interest costs

The recommended formulae (1) to (4) for estimating the cost of capital rate for value based reporting reflect the assumption that the cost of capital is independent of the proportions of debt and equity. This implies that the cost of capital of the entity is made up of only:

- the current risk free rate
- an allowance for systematic risk and taxes.

Thus, there are no “default” or “liquidity” premiums included in the cost of debt, and the cost of debt is at current rates. In regard to the assumption of no default or liquidity premiums, these would collectively add up to 1% to the cost of debt. With a company tax rate of 33% and 50:50 debt:equity capital, the effect would be to raise the cost of capital by up to .33%. For purposes of value based reporting this is not considered significant. However, it might be considered significant for output price setting. In this case a weighted average cost of capital (WACC) formula is required, and this is discussed below.

In regard to the assumption that the cost of debt is at current rates, an entity may have raised debt in the past that is at a rate significantly different to the current rate. Where this is the case the issue arises as to who should bear these differences in financing costs, i.e., should they be incorporated in output prices, or be borne by the equity holders. In the former case, the WACC formula is required, as discussed below. Furthermore, risk to equity holders has been reduced, and this should be reflected in a lower asset beta.

The WACC formula requires the costs of equity and debt to be calculated separately, and then combined with appropriate weightings to provide an overall cost of capital (**k**). The WACC formula is:

$$\mathbf{k = k_e(1 - L) + k_d\,(1 - T_c)L} \qquad \mathbf{(5)}$$

where

k_e	=	cost of equity
	=	$R_f(1 - T) + \varnothing\beta_e$ **(6)**
β_e	=	$\beta_a(1 + (B/S))$
$\varnothing$	=	tax adjusted market risk premium
β_a	=	asset beta of the Crown Company
B/S	=	ratio of company's debt to equity at market value (the book values of equity and of debt may need to be used as proxies in calculating this ratio)
k_d	=	actual interest rate on debt
L	=	the leverage ratio using book values of debt and equity
T_c	=	corporate tax rate, set at 33%
T	=	personal tax rate on interest , set at 33%
R_f	=	riskless rate, proxied by the current five year government stock rate at the beginning of the reporting period

4.3 The presumed tax scenario

Equation (6) above, and equations (1) to (4), include the assumption of tax neutrality between debt and equity financing. This implies that all dividends are fully imputed and that all investors can fully use the imputation credits.

If these assumptions are not appropriate, then formula (6) above becomes:

$$k_e = R_f(1 - T) + \varnothing\beta_e + DT_d \qquad (7)$$

where

D = dividend yield

= dividend divided by the market value of equity

$\mathbf{T_d}$ = tax rate on dividends net of any imputation effect. This rate needs to reflect the relevant tax circumstances.

If $\mathbf{T_d}$ is not zero, then the entity's cost of capital is influenced by its dividend yield (**D**), and this is a variable determined by the entity. This raises the question of whether output prices should reflect the entity's dividend policy. If this is considered appropriate then formula (7) applies. Otherwise formula (6) applies.

5

Application to Valuation

A business valuation typically requires discounting the business' future expected net cash flows to determine a present value. This is similar to a capital budgeting exercise, but includes the whole business rather than just a particular project.

In the case of a valuation, and particularly where the valuation is for the sale of the business, the need for precision is likely to be more important than the general acceptability of the methodology, and the seller and purchaser may take different views on some of the variables in the cost of capital formula. This section outlines how to work through some of these possible differences in view.

Where the only variable in contention is the beta, that is the systematic risk of the business, then formula (1) can be used for nominal rates, or formula (2) if a real rate is required. Differing beta can be simply introduced into the formula. From above, these two formulae are:

$$\mathbf{k} = \mathbf{R_f(1-T)+\varnothing\beta_a} \qquad \mathbf{(1)}$$

$$\mathbf{k_r} = \frac{\mathbf{1+R_f(1-T)+\varnothing\beta_a}}{\mathbf{1+i}} - \mathbf{1} \qquad \mathbf{(2)}$$

Where other variables are in contention, for example where tax neutrality is not considered to hold, or where the current cost of debt is recognised as embodying default and/or liquidity premiums, equation (8) provides greater flexibility to explore the implications of these differences. This formula is:

$$\mathbf{k} = \mathbf{k_e(1-L) + k_d(1-T_c)L} \qquad \mathbf{(8)}$$

where

L = leverage ratio specified by the valuer in market value terms

$\mathbf{k_e}$ = cost of equity

$$= \mathbf{R_f (1 - T) + \varnothing\beta_e + DT_d}$$

$\mathbf{R_f}$	=	riskless rate, proxied by the current five year government stock rate at the beginning of the reporting period
$\mathbf{T}$	=	personal tax rate on interest, set at 33%
$\varnothing$	=	tax adjusted market risk premium
$\boldsymbol{\beta_e}$	=	$\boldsymbol{\beta_a} \mathbf{(1 + (B/S))}$
$\boldsymbol{\beta_a}$	=	asset beta of the Crown Company
B/S	=	ratio of company's debt to equity implied by the leverage ratio above
D	=	dividend yield
	=	dividend divided by the market value of equity
$\mathbf{T_d}$	=	personal tax rate on dividends net of any imputation effect. This rate needs to reflect the relevant tax circumstances
$\mathbf{k_d}$	=	interest rate on debt (at current rates)

A complication can arise with this formula when $\mathbf{T_d}$ is not equal to zero, that is where the personal tax rate on dividends, net of any imputation effect, is not zero. The equation for **D**, the dividend yield, includes the market value for equity. However, when undertaking a valuation, this value is unknown until the cost of capital rate is determined and the valuation is completed.

This complication can be overcome through iteration, by estimating the value of equity, deriving the dividend yield, and then completing the valuation. If the final equity valuation is significantly different from the estimate, the equity value will need to be altered, and the valuation re-run. This process is repeated until the estimated and resulting equity values are equal.

An alternative to this iteration is to algebraically extract the dividend yield term from the denominator of the discounted cash flow. This approach is set out in Appendix 2.

Appendices

Appendix 1 - Choosing an Appropriate Methodology

Current practice internationally in the choice of public sector discount rates for Capital Charging and Capital Budgeting exhibits considerable variation. However the use of private sector methodology is widespread. The British government in its recent application of Capital Charging to the public health system uses a risk free rate [see Kemp (1990)], as does the US government's Central Budgeting Office [see Hartman (1990)]. These amount to a private sector rate with no risk allowance. By contrast the Australian government's Department of Finance (1987, 1991) supports the use of a private sector rate with a risk allowance. Closer to home, the capital charge regime for New Zealand government departments uses a private sector rate with risk allowance and correction for corporate tax differences between departments and private sector firms.

Practice aside, the theoretical question of applying private sector methodology to the public sector has enjoyed a long and controversial history in the Economics / Public Finance literature. Lally (1995a, pp 17-18) reviews critiques of the application of private sector methodology to the public sector. Three significant arguments are offered.

The first is the Social Rate of Time Preference (SRTP) argument, the SRTP being the rate at which society (i.e., a representative citizen) trades off present for future consumption. Disregarding company taxes for the present, the SRTP is lower than the private sector cost of capital due to personal taxes (i.e., personal taxes are not a cost to society, and hence are not reflected in the SRTP, but are a cost to the suppliers of capital to private sector projects). If government investment does not displace private investment, then the appropriate discount rate for project selection (and hence for Value Based Reporting) is the SRTP.

There are at least three objections to the this line of argument. **Firstly**, some government investment may displace private investment, in which case consistency with the private sector is essential and this can only be achieved by government use of the private sector rate (again leaving aside company taxes). This points to some weighted average of the private sector rate and the SRTP, but with obvious difficulties in determining the correct weights. **Secondly**, calculation of the SRTP typically proceeds by subtracting some rate of personal taxation from the private sector rate. This naturally raises questions about the appropriate rate. As we will see, presumptions about the personal tax scenario also underlie determination of the private sector rate. However an incremental difficulty with the SRTP (calculated in this way) is the problem that personal taxes may not raise the private sector rate above the SRTP by the full amount of the tax (in the same way that a 20% tariff may not increase prices by 20%). Alternative methods exist to estimate the SRTP, such as inferring rates of time preference from consumption behaviour, but these would seem to involve even graver estimation errors. **Thirdly**, because the SRTP is lower than the private sector rate, and the correct choice is not known , then use of the SRTP (or some average of it and the private sector rate) risks over investment rather than under investment in the public sector. Over investment would seem to be the more serious, because under investment might be made up by the private sector (examples include private provisions for health and education) whereas public sector over investment will not have any compensating effects on private investment.

The second significant argument for deviating from the private sector model (i.e., a different valuation if the same project is in the private or the public sector) arises from company taxes. As with the personal tax issue, if government investment displaces private investment, then the appropriate valuation is that of the private sector. Objections to this argument largely parallel the discussion about personal taxes and the SRTP, and the same conclusion is reached i.e., consistency with the private sector, in the valuation sense, is desired. Moreover the government has signalled its desire for such consistency by imposing corporate taxes on Crown companies. So the private sector cost of capital model should be used for Crown companies. Conversely if a Crown entity is exempt from these taxes, then consistency in project valuation demands raising the cost of capital to compensate for the absence of these taxes [and this has been done with the capital charge regime for New Zealand government departments [refer Lally (1995a, p14-15)].

The third significant argument for deviating from private sector methodology is that government investment involves externalities other than taxation revenue (as does private sector investment), that government (as an agent of society) should consider these effects, and that these effects should be impounded into the discount rate. As a debatable example, government investment is said to stimulate private investment [see Arrow (1982)]. However any evidence in general of these positive effects is scant, and negative effects clearly exist; for example, the deadweight costs of the taxation required to fund government investment. Furthermore, the size of the discount rate adjustment to reflect any net positive externalities of public sector investment will be an extremely subjective estimate. Finally, incorporation of the effect via the discount rate (rather than in the benefit stream) presumes a uniform effect over all projects, and such uniformity seems doubtful.

These conceptual arguments and the desire to minimise contentious estimation problems, so as to meet the "general acceptability" test, point to a preference for the private sector rate, with an adjustment if the Crown entity is not subject to tax. Such an adjustment is made in the capital charge regime for New Zealand government departments.

Appendix 2 - Versions of the Private Sector Model and the Relevance of Leverage

Private sector cost of capital methodology embraces a large set of possibilities. At the most basic level these are:

(1) a weighted average of the costs of debt (after corporate tax) and equity (WACC), where the weights are the proportions present in the Crown company's financial structure

(2) WACC where the weights are "target" or "long run" proportions for the Crown company

(3) WACC where the weights are the proportions present in the financial structure of comparator private sector companies used in the process of estimating the cost of equity

(4) the cost of equity prevailing in the absence of debt.

The capital charge regime for New Zealand government departments is an example of the methodology in (3).

Clearly, if WACC is not affected by changes in the proportions of debt and equity, then all four methods will provide the same results. Conversely if WACC does depend on leverage then the choice must lie between (1), (2) and (3). Method (1) accords most closely in principle to the concept of the firm's WACC. However Method (1) requires that the market value of the company's equity be observable (since market value weights are required) and this is not possible for Crown companies, whose equity is not traded. Proxies are available, such as multiplying book value equity by some market to book ratio drawn from comparator firms whose market value for equity is observable. However the resulting error is likely to be so large that (1) would appear to offer no advantage over (2), whilst requiring periodic recalculations (with leverage "changes") whose accuracy would be largely spurious.

By contrast method (2), once a target ratio was selected, would preserve this target whilst attempting to steer the firm's financial structure towards it (mindful of the substantial measurement error for equity).

Method (3) would apply by default where a government "entity" had no independent authority to borrow and any borrowing on their behalf by the Crown could not be traced. For example, government departments obtain financing from a pool supplied from both debt and taxation and this pool is used for both investment and current expenditure [for this reason the Capital Charge regime for New Zealand government departments employs Method (3)].

If WACC is believed to be unaffected by changes in the proportions of debt and equity (or approximately so) then simplicity and the avoidance of some controversies argues for use of (4). Method (4) avoids:

- having to estimate equity value in (1)
- having to specify a target leverage in (2)
- for (3) having to estimate a cost of debt for the Crown company consistent with the comparator's leverage.

To summarise, if leverage does not matter choose (4). Otherwise,

- use (1) if equity is traded
- use (2) if equity is not traded but borrowing is traceable
- use (3) if borrowing is not traceable.

A clear consequence of using the unlevered cost of equity is that debt costs become irrelevant in estimating the cost of capital.

Turning to the question of whether or not WACC is unaffected by changes in the proportions of debt and equity, by definition:

$$\mathbf{WACC = k_e \frac{S}{V} + k_d(1 - T_c)\frac{B}{V}}$$

$\mathbf{k_e}$ = expected rate of return on equity just compensating for risk and the time value of money

$\mathbf{k_d}$ = cost of debt

$\mathbf{T_c}$ = company tax rate

$\mathbf{S}$ = equity value

$\mathbf{B}$ = debt value

$\mathbf{V}$ = $\mathbf{S} + \mathbf{B}$ (so that $\frac{\mathbf{B}}{\mathbf{V}}$ = leverage ratio)

The first point to note here is that the cost of debt is a promised rate of return rather than an expected rate and, in the presence of default risk, the promised rate of return will be larger. To take a simple example, suppose a prospective corporate bond promises principal plus 12% in one year, and the chance of default is 1% (whereupon the investor receives neither interest nor principal). In that event the expected return per $1 lent is:

$$\mathbf{\frac{Expected\ Payoff - \$1}{\$1} = \frac{.99(\$1.12) - \$1}{\$1} = .11}$$

So part of the promised yield of 12% (1%) is “illusory” and is called a default premium ($\mathbf{P_1}$). Furthermore, the expected return on the bond ($\mathbf{E_d}$) may comprise compensation not just for risk and the time value of money but also for inferior liquidity relative to the benchmark government bond (conceivably the cost of equity could also include this but conventional models exclude it). Thus $\mathbf{E_d}$ comprises compensation for risk and the time value of money ($\mathbf{E'_d}$) plus a liquidity premium ($\mathbf{P_2}$). Thus:

$$\mathbf{k_d = E'_d + P_1 + P_2}$$

and hence

$$\mathbf{WACC = k_e \frac{S}{V} + E'_d(1 - T_c)\frac{B}{V} + (P_1 + P_2)(1 - T_c)\frac{B}{V}} \quad \mathbf{(1)}$$

To develop further we need a model that specifies $\mathbf{k_e}$ and $\mathbf{E'_d}$. The only credible candidates are some version of the Capital Asset Pricing Model (CAPM) or the Arbitrage Pricing Theory (APT) [Ross (1976)]. However estimation problems involved in the APT are even more substantial than in the CAPM. Furthermore, and presumably for this reason, the APT has not yet been employed (to the writer's knowledge) in cost of capital estimation in New Zealand. In view of all this and the "general acceptability" requirement then some version of the CAPM must be employed.

The standard version of the CAPM [Sharpe (1964), Lintner (1965)] is the most commonly employed world-wide, and is used locally for the Capital Charging regime for New Zealand government departments. However the most widely used version in New Zealand is a tax adjusted CAPM which recognises differential tax treatment for capital gains, dividends and interest [The first version of a tax adjusted CAPM was provided by Brennan (1970) whilst Lally (1992) amongst others has adjusted it for dividend imputation]. The impetus for adoption of the tax adjusted CAPM would appear to have been the introduction of dividend imputation in New Zealand, although other aspects of the tax system (particularly the capital gains tax situation) would have justified it. Both the standard CAPM and the tax adjusted CAPM versions assume internationally segregated capital markets. This assumption is particularly difficult to sustain in New Zealand's case, and accordingly CAPM versions without this restriction are appealing [the first such model is due to Solnik (1974) whilst Lally (1996) has adapted it for personal taxes and dividend imputation]. However, like the APT, the CAPM without this restriction involves additional estimation problems and has not yet been employed in New Zealand. So, again bearing in mind the “general acceptability” test, a domestic CAPM is selected.

To avoid prejudging the tax issue at this stage the following discussion uses a domestic CAPM which is consistent with a wide range of tax scenarios. Following Lally (1992) the expected returns on equity and debt which compensate for time and risk are:

$$\mathbf{k_e = R_f(1 - T) + DT_d + \varnothing\beta_e} \qquad \textbf{(2)}$$

$$\mathbf{E'_d = R_f(1 - T) + (Expected\ Cash\ Yield)T + \varnothing\beta_d}$$

$$\mathbf{= R_f(1 - T) + E_dT + \varnothing\beta_d}$$

$$\mathbf{= R_f(1 - T) + (E'_d + P_2)T + \varnothing\beta_d}$$

hence

$$\mathbf{E'_d = R_f + \frac{\varnothing\beta_d}{(1-T)} + \frac{P_2T}{(1-T)}} \qquad \textbf{(3)}$$

$\mathbf{R_f}$ = riskless rate of interest

T = weighted average over investors (i) of $(\mathbf{T_i} - \mathbf{T_{gi}})/(1 - \mathbf{T_{gi}})$ where $\mathbf{T_i}$ is investor i’s tax rate on interest and $\mathbf{T_{gi}}$ their tax rate on capital gains

D = expected dividend yield on the share

$\mathbf{T_d}$ = as in **T** except $\mathbf{T_i}$ is replaced by i’s tax rate on cash dividends

∅ = market risk premium. This parameter is common to all assets and is elaborated on in Appendix 3

$\boldsymbol{\beta_e}$ = equity beta (risk measure for equity)

$\boldsymbol{\beta_d}$ = debt beta (risk measure for debt)

$\mathbf{P_2}$ = liquidity premium

Substituting (2) and (3) into (1) yields:

$$\mathbf{WACC} = [R_f(1-T) + DT_d + \varnothing\beta_e]\frac{S}{V}$$

$$+ [R_f + \frac{\varnothing}{(1-T)}\beta_d + \frac{P_2 T}{(1-T)}](1-T_c)\frac{B}{V}$$

$$+ (P_1 + P_2)(1-T_c)\frac{B}{V} \qquad (4)$$

The equity beta β_e can additionally be expressed as a function of the asset beta β_a [when leverage is zero] and the debt beta β_d. Following Conine (1980):

$$\beta_e = \beta_a[1 + \frac{B}{S}(1-\delta)] - \beta_d\frac{B}{S}(1-\delta)$$

where δ is a parameter reflecting the tax scenario. In a classical tax world (where dividends / interest / capital gains are identically taxed at the personal level and the interest expense earns a corporate tax deduction) $\delta = T_c$. By contrast, in a tax neutral world (debt has no tax advantage over equity after considering both corporate and personal taxes) $\delta = 0$. Substituting this last equation above into (4) yields:

$$\mathbf{WACC} = R_f(1-T)\frac{S}{V} + DT_d\frac{S}{V} + \varnothing\beta_a - \varnothing\beta_a\delta\frac{B}{V}$$

$$- \varnothing\beta_d(1-\delta)\frac{B}{V} + R_f(1-T_c)\frac{B}{V}$$

$$+ \varnothing\beta_d\left(\frac{1-Tc}{1-T}\right)\frac{B}{V} + [P_1 + \frac{P_2}{(1-T)}](1-T_c)\frac{B}{V}$$

Noting that:

$$\frac{S}{V} = 1 - \frac{B}{V} \text{ and } 1 - T_c = 1 - T + T_c - T$$

then

$$\mathbf{WACC} = \mathbf{R_f(1-T) + DT_d + \varnothing\beta_a + \frac{B}{V}\Bigg[R_f(T_c - T)}$$

$$\mathbf{- \ DT_d - \varnothing\beta_a\delta + \varnothing\beta_d\left\{\frac{1-T_c}{1-T} - (1-\delta)\right\}}$$

$$\mathbf{+ \{P_1 + \frac{P_2}{(1-T)}\}(1-T_c)\Bigg]} \qquad \mathbf{(5)}$$

So for WACC to be unaffected by the proportions of debt and equity the term [·] must be zero. The term $\mathbf{R_f}$ is independent of any other term inside [·]. So $\mathbf{R_f(T_c–T)}$ must be zero i.e., offset elsewhere in [·] is not possible except by chance. Since $\mathbf{R_f}$ is positive then $\mathbf{T_c = T}$. So [·] becomes:

$$\mathbf{- DT_d - \varnothing\delta(\beta_a - \beta_d) + \{P_1 + \frac{P_2}{(1-T)}\}(1-T_c)}$$

Similarly the dividend yield **D** is independent of any other term. So $\mathbf{DT_d}$ must be **0**. Thus $\mathbf{T_d} = \mathbf{0}$. By similar reasoning $\mathbf{\varnothing\delta(\beta_a - \beta_d)} = \mathbf{0}$. But since $\varnothing > \mathbf{0}$ and $\beta_a > \beta_d$ then $\delta = \mathbf{0}$. Finally:

$$\mathbf{P_1 + \frac{P_2}{(1-T)} = 0}$$

Since neither $\mathbf{P_1}$ nor $\mathbf{P_2}$ can be negative then $\mathbf{P_1 = P_2 = 0}$.

Summarising, baring freak coincidences, the following conditions are necessary and sufficient for WACC to be independent of leverage:

(a) $\mathbf{T_c = T}$

(b) $\mathbf{T_d} = \mathbf{0}$

(c) $\delta = \mathbf{0}$ i.e., tax neutrality

(d) $\mathbf{P_1} = \mathbf{P_2} = \mathbf{0}$ i.e., no default or liquidity premiums

The first two conditions are fully consistent with tax neutrality. So the necessary and sufficient conditions for WACC to be unaffected by changes in the proportions of debt and equity are tax neutrality and no default or liquidity premiums.

The question is now whether these two conditions are met, or approximately met.

Condition 1 - tax neutrality

Turning to the first point - tax neutrality - the “general acceptability” requirement argues for minimisation of contentious parameter estimation questions. Accordingly one is obliged to choose between one of only two competing tax scenarios - the neutrality scenario described earlier (giving debt no tax advantage over equity) and the classical model in which capital gains / interest / dividends are identically taxed and interest earns a corporate tax saving at the corporate tax rate. Prima facie the neutrality model is a better description of the New Zealand situation where there is imputation, the exemption of some investors from capital gains tax and unlimited deferral opportunities for the rest. So we consider the tax neutrality requirement to be met.

Condition 2 - no default or liquidity premiums

In equation (1), the effect of these liquidity and default premiums on WACC is less than 1:1 due to $\mathbf{B} < \mathbf{V}$ and $\mathbf{T_c} > \mathbf{0}$. For example, if $\mathbf{k_d}$ exceeds the riskless rate by 1.5% at a leverage of 50% (a generous estimate for Crown companies) and 1% is attributed to these two premiums, then their effect on WACC is to raise it by at most:

$$\mathbf{1\%(.67)(.5) = .33\%}$$

For purposes of value based reporting, this is hardly a substantial sum. So WACC is not materially affected. The consequence of ignoring these liquidity and default premiums is to underestimate rather than overestimate the cost of capital.

So the two conditions for WACC invariance are met (approximately). Accordingly method (4) is chosen - the cost of capital is set equal to the unlevered cost of equity - and accordingly debt costs become irrelevant. The cost of capital is then given by equation (5) with [·] = 0 and $\mathbf{T_d} = \mathbf{0}$. So the cost of capital (**k**) is:

$$\mathbf{k = R_f(1 - T) + \varnothing\beta_a}$$

$\mathbf{R_f}$ = riskless interest rate

T = tax rate on interest (set at the corporate tax rate of 33% to be consistent with tax neutrality)

$\boldsymbol{\varnothing}$ = market risk premium

$\boldsymbol{\beta_a}$ = asset beta

Application to valuation

If it is considered that WACC is invariant to leverage then value now ($\mathbf{V_0}$) is given by:

$$\mathbf{V_0 = \sum_t \frac{E(X_t^u) - N_t}{(1+k)^t}} \qquad \mathbf{(6)}$$

$\mathbf{k}$ = $\mathbf{R_f(1 - T) + \varnothing\beta_a}$

$\mathbf{E(X_t^u)}$ = expected nominal cash flow in year **t** from existing investment, after company tax but as if there were no leverage

$\mathbf{N_t}$ = new investment in year **t**

To take an example suppose:

	Year 1	Year 2	Year 3 (and onwards)
$\mathbf{E(X_t^u)}$	\$1m	\$1.1m	\$1.2m
$\mathbf{N_t}$	\$.8m	\$.6m	0

Also suppose $\mathbf{R_f}$ = .08, **T** = .33, ∅ = .09 and $\boldsymbol{\beta_a}$ = .5. Then the discount rate is:

$$\mathbf{k = .08(1 - .33) + .09(.5) = .0986}$$

So:

$$\mathbf{V_0 = \frac{\$1m - \$.8m}{1.0986} + \frac{\$1.1m - \$.6m}{(1.0986)^2} + \frac{\$1.2m}{(1.0986)^3} + \frac{\$1.2m}{(1.0986)^4} + \ldots}$$

$$\mathbf{= \frac{\$.2m}{1.0986} + \frac{\$.5m}{(1.0986)^2} + \frac{\$1.2m + \left\{\frac{\$1.2m}{.0986}\right\}}{(1.0986)^3}}$$

$$\mathbf{= \$10.680m}$$

If by contrast WACC is considered to be sensitive to leverage, due for example to the assumption of tax neutrality between debt and equity not holding, then **k** in equation (6) above must be the WACC formula:

$$\mathbf{k = k_e(1 - L) + k_d(1 - T_c)L}$$

$\mathbf{k_d}$ = cost of debt (at current rates)

L = leverage specified exogenously by the valuer

$\mathbf{k_e}$ = $\mathbf{R_f(1 - T) + \emptyset\beta_e + DT_d}$

$\boldsymbol{\beta_e}$ = $\boldsymbol{\beta_a(1 + \frac{B}{S})}$

$\mathbf{\frac{B}{S}}$ = debt to equity ratio implied by **L**

D = expected dividend yield

$\mathbf{T_d}$ = tax rate on dividends, net of the imputation effect

However this cost of capital formula cannot be used directly to discount cash flows, because the resulting valuation implies a dividend yield and yet this appears in the cost of capital formula. One solution to this which has been employed is iteration i.e., guess at a dividend yield, then conduct the valuation. If the resulting dividend yield is consistent with the initial guess, then stop. If not adjust the initial dividend yield and repeat the valuation. Continue until the initial guess matches the outcome from the valuation. An alternative to this iteration is to algebraically extract the dividend yield term from the denominator of the discounted cash flow. This leads to the following formula:

$$V_0 = \sum_t \frac{\{E(X_t^u) + UE(IC_t^u)\}(1-T) - N_t}{(1+d)^t} \qquad (7)$$

$$d = (1-L)[R_f(1-T) + \varnothing\beta_e] + Lk_d[1 - T_c(1-U)](1-T)$$

$E(IC_t^u)$ = expected imputation credits attached to dividends in year t, if there was no leverage. Typically this is equal to company tax in year t, if there was no leverage

$0<U<1$ = coefficient reflecting the ability of investors in aggregate to utilise these imputation credits to lower their personal tax on dividends. Because some investors (foreigners for example) cannot use these credits or use them fully, we expect $U < 1$

Consider the previous example and suppose that unlevered company tax was:

	Year 1	Year 2	Year 3 (and onwards)
	$.4m	$.42m	$.45m

Also suppose $U = .75$, $k_d = .088$, $T_c = .33$ and $L = .4$. Then

$$\beta_e = .5[1 + \frac{4}{6}] = .833$$

The discount rate in (7) is then:

$$d = .6[.08(1-.33)+.09(.833)] + .4(.088)[1-.33(1-.75)](1-.33)$$
$$= .0988$$

So value from (7) is:

$$V_0 = \frac{[\$1m+.75(\$.4m\}.67-\$.8m}{1.0988} + \frac{[\$1.1m+.75(\$.42m)].67-\$.6m}{(1.0988)^2}$$

$$+ \frac{[\$1.2m+.75(\$.45m)].67}{(1.0988)^3} + \frac{[\$1.2m+.75(\$.45m)].67}{(1.0988)^4} + \ldots$$

$$= \frac{\$.071}{1.0988} + \frac{\$.348m}{(1.0988)^2} + \frac{\$1.03m + \left\{\frac{\$1.03m}{.0988}\right\}}{(1.0988)^3}$$

$$= \$9m$$

This value is noticeably below the previous value of $10.68m, principally due to **U** < **1** and the unlevered expected company tax being less than that required to annul shareholders dividend tax. If **U** = **1** and unlevered expected company tax was sufficiently large i.e.,

$$\textbf{Expected Company Tax} = \frac{Tc}{1-Tc} E(X_t^u)$$

$$= \begin{cases} \$.4925m \ \ (yr\ 1) \\ \$.542m \ \ (yr\ 2) \\ \$.591m \ \ (yr\ 3)\ \text{and beyond} \end{cases}$$

then the discount rate (with **U** = **1**) in (7) becomes .10 and the cash flows now equal those in (6). So:

$$V_0 = \frac{\$.2m}{1.1} + \frac{\$.5m}{(1.1)^2} + \frac{\$1.2m + \left\{\frac{\$1.2m}{0.1}\right\}}{(1.1)^3}$$

$$= \$10.55m$$

This is slightly below the $10.68m in the first calculation because the discount rate (10%) is slightly larger than in the first calculation (9.86%), and this is because the cost of debt is larger than the risk free rate (8.8% v 8%) i.e., default and / or liquidity premiums exist.

Further details and the proof for the valuation formula (7) appears in Lally (1995f).

Appendix 3 - The Riskless Rate and the Market Risk Premium

The riskless rate

Clearly domestic government debt offers the best proxy for a riskless rate of return. However the rate generally varies with term (i.e., the term structure is not flat) and the CAPM does not indicate which should be used. However value based reporting must be consistent with ex ante project valuation. In the presence of a term structure which is not flat, project valuation requires a different discount rate for each year's cash flow, due to the differing riskless rates. This is equivalent to using a single riskless rate, which averages over the various annual rates, with weights equal to the proportion of total project value represented by that year's cash flow. Thus the resulting average depends upon the duration of the project. For value based reporting to be consistent with this, the single riskless rate used must be an average over the project specific rates just described, with weights equal to the proportion of total entity value represented by each project, i.e., an average over all annual riskless rates, with weights equal to the proportion of the firm's total value represented by cash flows of that duration. Since the term structure tends to be monotonic [continually increasing with term or decreasing or flat] this average riskless rate should be approximately the riskless rate corresponding to the average duration of the firm's cash flows. This suggests a rate in the 5-10 year span. Assuming little difference in the choices here, a five year rate is recommended.

In addition to these term structure variations in rate at a given point in time, the rate also varies over the reporting period (year). This gives rise to the question of which point in time is used to set the rate. Again the guiding principle is consistency with ex ante project valuation. Projects entered into part way through the year should be evaluated ex ante using the riskless rate at that time. For ex post reporting to be consistent with this, the theoretical ideal is to partition the year into a large number of very short sub-periods and compute the $ cost of capital as rate x base (with rate reflecting the riskless rate at the beginning of that sub-period). Adding these $ cost of capitals for each sub-period produces the annual $ cost of capital.

However, the following factors exist:

- additional data is required
- the variation in the riskless rate over the course of a year is modest
- cost of capital determination involves numerous parameter estimation problems with potentially far more impact than this issue of calculating the riskless rate.

In light of these factors, a pragmatic solution is recommended: use the rate at the beginning of the reporting period. Computation of the asset base should however reflect injections or withdrawals of capital during the year and the timing of those changes. To ensure that the rate choice is not sensitive to daily variation, the rate set should be the average of daily rates over the one month period immediately preceding the reporting period.

To summarise this discussion of the riskless rate, choose the five year rate prevailing at the beginning of the reporting period (i.e., the average of daily rates over the one month immediately preceding the reporting period). Each reporting period then uses a potentially different rate.

The Market Risk Premium

We turn now to the market risk premium (**Ø**). In the tax adjusted version of the CAPM this is:

$$\text{Ø} = E_m - D_m T_m - R_f(1 - T)$$

$\mathbf{E_m}$ = expected rate of return on the market portfolio (**m**), proxied by a share index

$\mathbf{D_m}$ = dividend yield on the share index

$\mathbf{T_m}$ = tax parameter applicable to the share index. Given the tax neutrality scenario already argued for then $\mathbf{T_m}$ currently must be 0. However, pre imputation, it would be the representative investor tax rate on dividends

Estimates of **Ø** have been arrived at by arithmetic averaging over the annual ex post counterparts for each year since 1958, with New Zealand data. The result is an estimate of about 9%. The estimation process parallels in principle that for the market risk premium in the standard CAPM, where most work of this type has been done. However considerable statistical uncertainty exists about this 9% estimate - using a standard deviation for annual market return of 20%, then the 95% confidence interval around it is almost ±7%. This is sufficiently wide as to invite considerable scepticism about the accuracy of the point estimate. Accordingly one enquires into estimates from other countries and longer periods. Unfortunately all other such arithmetic averaging estimates known to the writer relate to the market risk premium in the standard CAPM, denoted $\mathbf{Ø_s}$ and with formula:

$$\mathbf{Ø_s = E_m - R_f}$$

To convert the 9% estimate for **Ø** to an estimate of $\mathbf{Ø_s}$, the appropriate process is as follows. Currently:

$$\mathbf{Ø = E_m - R_f(1 - T)}$$
$$\mathbf{= Ø_s + R_f T}$$

With **T** = .33 and a current $\mathbf{R_f}$ of about 8% then:

$$\mathbf{Ø = Ø_s + .026}$$

So 9% for **Ø** converts to 6.4% for $\mathbf{Ø_s}$. An alternative conversion process is to estimate $\mathbf{Ø_s}$ directly from the same data set and this yields 8.5%. This is less satisfactory because tax changes over time impart less intertemporal stability to

$Ø_s$ than **Ø** and therefore more bias to a direct estimate of $Ø_s$ than **Ø**. However it permits comparisons more readily with other researchers: Ibbotson (1991) obtains 8.4% for the US using data from 1926, Siegel (1992) 4.7% for the US (1802→), Dimson and Marsh (1982) 8.7% for the UK (1919→), Officer (1989) 7.9% for Australia (1882→), and Chay et al (1993) 6.2% for New Zealand (1931→). Additionally Merton (1980), using the Ibbotson data set obtains 10.3% after making a correction for heteroscedasticity (the correction has no appreciable impact on the New Zealand estimate). In short the 9% estimate for **Ø** is empirically equivalent to 8.5% using conventional technology and this lies within the range of other estimates for $Ø_s$. Furthermore the 9% estimate is theoretically equivalent (currently) to 6.4% for $Ø_s$ and this is broadly consistent with employed estimates of $Ø_s$ [Interestingly the 6.4% value noted above corresponds almost exactly to the 6.5% estimate used in the Capital Change regime for New Zealand government departments, which was arrived at by taking a mid-point from a set of studies]. All this leaves us with rather more confidence in the 9% estimate than suggested by its 95% confidence interval.

Notwithstanding this comfort, these additional estimates still have substantial 95% confidence intervals - the lowest of them (Siegel) is ±3% and the most widely used one (Ibbotson) is ±5%. Furthermore the simple averaging processes imply that the population mean is unchanged over the estimation process. This in turn implies that risk aversion, risk (i.e., market volatility) and personal taxation (the latter only for the $Ø_s$ estimates) have not changed over that period, since the mean reflects these factors. However the risk and personal taxation implications are clearly empirically false and the first (risk aversion) is at least suspect. The personal tax issue is dealt with by direct estimation of a tax adjusted market risk premium, as has been done. However the other two factors are more difficult to deal with. Regarding risk aversion, a number of papers [see for example Fama and

French (1989)] reveal that a set of time varying variables which could be regarded as proxies for risk aversion (such as the default spread) have some power to explain market risk premium changes. However the statistical reliability of the relationships is weak.

Regarding market risk, measurement of the current value itself presents no great problems [and changes have been substantial - see Merton (1980)]. However there are problems in extrapolation, and even graver problems in determining the relationship between risk and the market risk premium. French et al (1987) conclude that the relationship is positive whilst Glosten et al (1993) reach the opposite conclusion. Furthermore, sign aside, the variability in returns is so large that the functional form of the relationship is impossible to estimate. For example, French et al (1987) could not statistically distinguish between the premium being proportional to standard deviation and to variance, and Merton (1980) shows that the current estimates of the premium from these two functional forms are markedly different.

First NZ Capital (1997) have modified Merton's methodology to deal with certain biases, and generate an estimate for **Ø** of .07. Furthermore, this estimate is virtually invariant to the "functional form" problem. However, the extrapolation issue remains, i.e., in so far as **Ø** so estimated exhibits long-term mean reversion, the current estimate may not be appropriate for discounting long-term future cash flows.

Lally (1995c) generates an estimate for $Ø_s$ which presumes that its unlevered counterpart is constant over time. Accordingly the unlevered counterpart is estimated from unlevered market returns, and then levered up to reflect current market leverage. The unlevered premium is estimated from US data, over approximately the Ibbotson time span, as .066. With New Zealand market leverage at 30%, this translates into an estimate for $Ø_s$ of .094. This is broadly consistent with direct estimates of $\mathbf{Ø_s}$. However, significant future changes in market leverage might invalidate this conclusion.

As if this were not enough, even more fundamental concerns can be raised. Lally (1995d) shows that the act of proxying the market portfolio by a share index **may** induce serious biases in cost of capital estimation (via non-offsetting biases in estimating the market risk premium and beta). Furthermore Brown et al (1995) show that because estimates are drawn exclusively from surviving stock markets (i.e., no breaks for several decades) then average realised returns are likely to overestimate expected returns.

These statistical concerns and potential biases leave considerable room for doubt about the accuracy of the 9% estimate (and indeed all other such estimates, locally and internationally). However a number must be selected and the choice must be both plausible and generally acceptable. The 9% estimate meets both of these requirements and accordingly is recommended.

Two further issues deserve brief mention. Firstly, a variety of complaints can be raised about the quality of the data used to estimate the market risk premium in New Zealand. One is that, post 1984, the economy is fundamentally different. However, while this might cause actual returns to exceed expected returns in response to this new information, there seems no reason to suppose that it changes risk or other fundamentals relevant to expected return but not recognised as time varying in the model. A second complaint is that 1987 is a freak year and should be excluded. However the nature of probability distributions is that, occasionally, rare events happen. Use of a long time series hopefully leads to their frequency matching their probability, and we have no reason to suppose otherwise here. A third complaint is that risk free rates pre-1984 were government controlled downwards and therefore bias upwards the MRP estimate from pre-1984 data. The suggestion is then to ignore pre-

1984 data. However this would leave a data series so short that the MRP estimate would have no statistical reliability. The process employed here - to utilise the pre-1984 data with some unknown degree of imperfection and to check it against MRP estimates from other economies free of such regulation - is considered superior.

The second and final issue relates to the question of the statistical averaging technique. As noted previously the estimate of 9% is an arithmetic rather than a geometric average, as this accords with the expectation **E**, i.e., arithmetic, but not geometric, averages are unbiased estimators of **E**. Sometimes however one sees use of geometric averages, and the usual justification is as follows: suppose annual returns are equally likely to be -50% and +100%. After two years the most likely outcome from investing $1 is a terminal wealth of $1. Moreover this is consistent with the geometric mean return per year of 0, i.e.,

$$\$1\,(1 + \text{geometric mean})^2 = \$1(1 + 0)^2 = \$1$$

By contrast the arithmetic mean return per year of 25% is not consistent with terminal wealth of $1, i.e.,

$$\$1(1.25)^2 = \$1.5625$$

This argument involves a confusion between expectations and most likely outcomes. The full distribution of possible outcomes after 2 years (and probabilities) is:

$$\$1(1+1)^2 = \$4\ (.25)$$

$$\$1(1+1)(1-.5) = \$1\ (.25)$$

$$\$1(1-.5)(1+1) = \$1\ (.25)$$

$$\$1(1-.5)^2 = \$.25\ (.25)$$

So the most likely terminal wealth is indeed $1, and this is consistent with the geometric mean return per year of 0. However, the expected wealth after 2 years is $1.5625 because the $4 outcome is further from the most likely outcome of $1 than the $.25 outcome. As noted this $1.5625 is consistent with an expected return per year of 25%, and use of this is demanded by the CAPM. Geometric means may be consistent with some other asset pricing theory, but they are not consistent with the CAPM, and should not be coupled with it.

Appendix 4 - Beta

The beta of an investment is defined as the covariance between its return and market return, divided by the variance of market return. Under some standard assumptions the beta of an investment coincides with the slope coefficient (**β**) in a regression of the investment's return (**R**) on market return (**R_m**) i.e.,

$$\mathbf{R = \alpha + \beta R_m + e}$$

where **α** is the intercept term and **e** a mean zero residual. Thus if **R_m** changes by 1% and the expected change in **R** is 1.2%, then **β** would be 1.2. The betas of interest to us for value based reporting of Crown entities and SOE's are asset betas, i.e., betas of firms with no debt. Since virtually all firms possess some debt, then asset betas are arrived at by estimating equity betas from the regression process described above, and then employing a theoretical model to strip out the effect of debt. This gives rise to three problems:

(a) Since the equity of Crown companies is not traded, then equity returns cannot be measured. Furthermore accounting betas have been found to be unsatisfactory proxies, i.e., at the individual security level, accounting betas explain only 20% of the cross-sectional variance of market betas - see Beaver and Manegold (1975). So one has to search out comparator companies, estimate their asset betas, and then in the face of many apparently equally good comparators, engage in some averaging process.

(b) Regression estimates are subject to statistical error, and these are substantial at the individual firm level - a typical standard error is .25 so that the 95% confidence interval around an estimate is ± .5, a range very substantial in relation to that of the true values (approximately .3 $\rightarrow$ 1.7). This leads one to average over a number of companies, so as to reduce that confidence interval, and therefore reinforces the averaging required in (a). Thus large sets of comparators ("industries") are desired. Furthermore additional statistical correction processes may be employed, and these are discussed below.

(c) Determining the correct theoretical model for converting estimated equity betas into asset betas i.e., the degearing formula.

Choosing an appropriate comparator

Addressing the first problem - what is an appropriate comparator - requires some knowledge of what underlies betas. Since beta measures systematic risk, it arises from the firm's exposure to macro-economic events ("factors") affecting the returns of most firms in the economy. Chen, Roll and Ross (1986) suggest these factors are real GNP growth, inflation, change in market risk aversion and change in the long term real interest rate. Formally, and following Dybvig and Ross (1985), since:

$$\beta = \frac{Cov(R, R_m)}{Var(R_m)}$$

and

$$R - E = b_1F_1 + b_2F_2 + ... + b_kF_k + e$$

where **R** is rate of return, **E** its expectation, F_i the (random) value of factor i = 1, 2 .. k (independent) and b_i the coefficient of the asset's return on factor **i**, then:

$$\beta = b_1\left[\frac{Cov(F_1, R_m)}{Var(R_m)}\right] + ... + b_k\left[\frac{Cov(F_k, R_m)}{Var(R_m)}\right]$$

So beta is a linear function of the factor coefficients b_1 ... b_k and thus betas would differ according to differences in their factor coefficients. Amongst equities we expect the responsiveness of returns to the last three factors - inflation, market risk aversion and the long term real interest rate - to be similar. However responsiveness to the first factor will vary significantly over equities. Thus we expect differences in equity betas to arise largely from differences in responsiveness to real GNP.

The next question is that of what governs these sensitivities of equity returns to real GNP. We expect the following variables to be material:

(1) nature of the output - the returns of firms producing necessities should have lower sensitivity to real GNP than for firms producing luxuries, because the demand for their product is less sensitive to real GNP

(2) nature of the customer - the returns of firms delivering an output to government should have lower sensitivity to real GNP than for firms whose output is delivered to the private sector, because the demand by their customer (government) should be less sensitive to real GNP

(3) nature of contracts with suppliers and customers - the returns of firms with, for example, long term fixed price contracts covering both inputs and outputs should have lower sensitivity to real GNP than for firms without these, because prices are not adjusted in response to real GNP shocks

(4) degree of regulation - the return of a firm subject to, for example, rate of return regulation should exhibit less sensitivity to real GNP, because demand changes arising from real GNP changes will be countered by price changes in the opposite direction

(5) degree of monopoly - assuming monopolists react to demand shocks by varying the "cushion" provided by suboptimal price setting and cost control more than do non-monopolists, then their returns will exhibit less sensitivity to demand, and hence real GNP, fluctuations

(6) the nature of the firm's real options - to adopt a new project, close-down an existing one, proceed to the next stage of a multi-stage project, and to expand / contract

existing operations. Dixit and Pindyck (1994) examine these real options possessed by firms while Black and Scholes (1973) show that the sensitivity of an option value (and therefore of a firm possessing one) to the underlying variable will vary, inter alia, according to how close the option is to being "at the money" and the term to maturity of these options. The existence of options involving expansions of the firm will tend to increase the firm's beta, as option values are even more sensitive to underlying variables than the firm value exclusive of the option element, and these two value elements are positively correlated. By contrast, the existence of options involving contractions of the firm will tend to reduce the firm's beta, because the option value and firm value exclusive of such are negatively correlated

(7) operating leverage - the returns of firms with higher operating leverage (i.e., higher fixed operating costs to total operating costs) should have higher sensitivity to real GNP, because cash flows are more sensitive to demand, and hence real GNP, fluctuations. Rhee (1986) has modelled this

(8) index weight - firms with a non-trivial weighting in the market index against which betas are calculated will have their beta drawn towards the market average of 1. Lally (1997) has modelled this. Even for a market weighting of 5%, the effect on beta can be substantial

(9) capital structure - the principal issue here is the proportion of capital represented by "straight" debt, i.e., the level of financial leverage. The (equity) returns of firms with higher financial leverage should have higher sensitivity to real GNP, because cash flows (to equity) are more sensitive to demand, and hence real GNP, fluctuations. Hamada (1972) has modelled the relationship between firm leverage and beta, whilst Ehrhardt and Shrieves (1995) extend this to convertible

debt and warrants. Lally (1994a) shows that firm leverage matters only in relation to market leverage. Thus, ceterus parabus, firms in different markets with different market leverages will have different betas.

Necessarily, the effect of capital structure is removed in arriving at a firm's asset beta, by use of a degearing formula. In addition, differences in market weights and operating leverage may be corrected by use of appropriate formulas [see Lally (1997) and Rhee (1986) respectively]. This leaves dimensions (1) ... (6), for which satisfactory formulas are lacking. A natural response would be to attempt subjective adjustments. However in view of the sheer complexity of adjustment (betas reflecting sensitivities to several rather than one macro variable) and the inability to properly audit any subjective adjustments which are made (because true betas are unobservable) we strongly recommend against such adjustment. The industry believed to be the best comparator in respect of (1) ... (6) should be selected and that industry's constituent firm's asset betas averaged. Where possible, and material, adjustments should also be made for operating leverage and index weighting. Also, as discussed later, if a comparator firm is foreign, then correction for country differences in market leverage should be done.

It is possible that this process may fail to provide suitable comparators or a sufficient number of comparators to offer confidence in the average value. As a general rule, results from less than 10 comparators should be viewed with extreme caution. Furthermore, any average which is less than the low risk default (discussed below) should be rejected. A sceptical view should also be taken of any average comparator asset beta which is more than 70% over the market average (currently about .7). Where comparators are unavailable or are not acceptable, either of two default asset betas could be used (and these were employed in the capital charge regime for New Zealand government departments):

(i) treat the firm as average risk and hence use the market asset beta : the market equity beta of 1 is then reduced to reflect market leverage - at current market leverage in New Zealand this would be about .7

(ii) treat the firm as very low risk and hence use the asset beta of that industry whose asset beta is lowest amongst industries with a sufficient number of firms to offer considerable confidence in the estimate - corrected for country index differences this is about .22 for US electric utilities, and .44 for UK electric utilities. A compromise has been used by Transpower and Airways Corporation of .3, and this would seem reasonable. However, this number should be subject to ongoing scrutiny.

Whichever is judged the better proxy should then be used, i.e., is the firm of concern believed to be closer to average risk or very low risk. Limitation of the set of default positions to these two reflects the belief that, in the absence of comparators, the best one can hope to do is to distinguish between very low risk activities and all others, i.e., no finer partition is possible. This “conservative” view is consistent with the view expressed above that it is inappropriate to make subjective adjustments to statistically estimated comparator betas.

Statistical Errors

Turning now to the second problem of statistical errors and processes for minimising these, we have already noted the desirability of averaging over large numbers of comparator firms. This alone argues against use of New Zealand firms, since the number in any industry with measurable betas (i.e., publicly listed) is very small. For the reverse reason US data is preferred (although it might be supplemented with data from other countries). The principal US suppliers of such estimates that the writer is aware of are Value Line, Bloomberg and BARRA. Each of these suppliers engages in some form of adjustment to the time series based regression estimates referred to earlier ("raw" betas). Both Value Line and Bloomberg supplement these "raw" estimates with "Blume" betas [Blume (1975)], in which "raw" estimates are corrected so as to draw them in towards 1. Lally (1994b) argues that such corrections are inappropriate for cost of capital estimation, although they may be desirable for other purposes.

BARRA by contrast generates estimates by correcting the raw values by a set of adjustments, based on cross-sectional regressions which have located the best explanatory variables (such as financial leverage, growth and size) [see Rosenberg and Guy (1976)]. The disadvantages of this process are data mining and the possibility of statistical errors which compound rather than mitigate those in the raw betas. The advantage is to reflect in the beta estimate the current values of explanatory variables rather than their average values over the estimation period (which is typically five years). However it is not clear whether the trade-off is favourable - see Harrington (1983). Conservatism then leads to favouring raw estimates. However Lally (1995e) provides a theoretical process (as opposed to BARRA's empirics) for correcting raw estimates to reflect current firm and market leverages. However, leverage data over the beta estimation period is required to implement it.

A final correction process worthy of mention is that of Vasicek (1971), in which "raw" betas are drawn towards an "industry" average. If the "industry" is the set of all firms in the market, as in the betas of London Business School's Risk Measurement Service, then they are much like Blume betas [see Lally (1994b)], and therefore should not be used for cost of capital estimation.

Only if the "industry" used in the Vasicek adjustment corresponds to the set of comparators chosen by us will the Vasicek process be appropriate for cost of capital estimation. Even here though, the effect of averaging over the comparators will ensure that there is little difference between using raw and Vasicek betas. Accordingly the use of raw betas is still recommended.

The degearing formula

Turning now to the model for converting a firm's equity beta (β_e) to its asset beta (β_a), the most widely used formula internationally is that of Hamada (1972), which assumes:

(a) classical tax world

(b) passive debt policy, i.e., debt is fixed in absolute $'s rather than as a constant proportion of equity

(c) debt returns are free of systematic risk

(d) only straight debt and ordinary shares are present in a firm's capital structure.

The formula converts β_e to β_a as follows:

$$\beta_a = \beta_e / [1 + \frac{B}{S}(1 - T_c)] \qquad (1)$$

where T_c is the corporate tax rate and B/S is the ratio of the firm's debt to equity in market value terms.

By contrast with (a) the tax scenario assumed in this paper is that of tax neutrality. Under this assumption, formula (1) becomes instead:

$$\beta_a = \beta_e / [1 + \frac{B}{S}] = \beta_e \left(\frac{S}{V}\right) \qquad (2)$$

where V is the sum of $B + S$ [see Lally (1994a)]. This formula (2) is standard practice in New Zealand.

Even without tax neutrality, if debt policy is active [debt is a constant proportion of equity, an assumption which is more realistic than passive debt policy and which is implicit in the use of a WACC for discounting over more than one period] then formula (2) above is still valid to a close approximation [see Miles and Ezzell (1985)].

Each of the above two degearing formulas assume that debt returns are free of systematic risk. Thus the beta of debt (β_d) is assumed to be zero. Clearly this is not the case. A generalisation of (2) which recognises this is:

$$\beta_a = \beta_e\left(\frac{S}{V}\right) + \beta_d\left(\frac{B}{V}\right) \qquad (3)$$

[see Conine (1980)]. However, implementation of this formula requires an estimate of the debt beta for the company, and this poses problems. Regression based estimates, paralleling those for β_e, are subject to lack of returns data for corporate debt, and will be biased up (down) according to whether the firm did (did not) default over the estimation period.

Some analysts have attempted to estimate a company's debt beta by injecting the promised yield on the company's debt (k_d) into the CAPM and then solving for β_d. For the tax adjusted version of the CAPM used in this paper [see Lally (1992)] this is:

$$k_d = R_f(1 - T) + k_d T + Ø\beta_d \qquad (4)$$

However, as discussed in Appendix 2, the CAPM does not incorporate default and liquidity premiums, which are part of the promised yield k_d. By ignoring these two premiums, and solving equation (4) for β_d, the effect would be to overstate β_d. Whether the overstatement would be less severe than the understatement arising from treating $\beta_d = 0$ is not clear, and will depend upon the true (but unknown) values for β_d and the two premiums. Conservatism then leads to choosing $\beta_d = 0$.

The effect of treating β_d as zero when it is positive is to understate the asset beta β_a [see equation (3)] and hence **underestimate** the cost of capital. However, the effect on the cost of capital is not large. For example, even if the comparator company's β_d is .2 [which seems generous in view of the low default option for β_a of .3] and **B/V** is .25

[which is approximately the average for US firms] then β_a is understated by .05 at most, and therefore the cost of capital by **Ø**(.05) = .09(.05) = .0045, i.e., less than half of 1%.

Finally turning to point (d), the Hamada formula, as well as formula (2), assume that only straight debt and ordinary shares are present in a firm's capital structure. If warrants or convertible debt are present then corrections are necessary [see Ehrhardt and Shrieves (1995)]. Typically, however, any correction is small.

Incorporating International Leverage Differences

Some, and possibly all of the comparator firms may be in foreign economies and their betas are therefore measured against a foreign market index. In this case, an adjustment may be needed to allow for any difference between leverage in the foreign market and leverage in the New Zealand market. The formula to convert a foreign asset beta (β_{aF}), as calculated above, to its New Zealand equivalent (β_a) is:

$$\beta_a = \beta_{aF} \frac{[1 + (B_{MF} / S_{MF})]}{[1 + (B_M / S_M)]}$$

where:

B_M/S_M = New Zealand market ratio of debt to equity at market value

B_{MF}/S_{MF} = Foreign market ratio of debt to equity at market value

[See Lally (1994a)].

Summary

Summarising the beta question then, the following is recommended:

(i) seek comparators from large industry sets, which typically implies the use of US data

(ii) if material and possible, correct for differences in operating leverage and market weights

(iii) avoid subjective adjustments to regression estimates of beta

(iv) if comparators are not apparent, use the better of the two default positions - average risk or very low risk

(v) use raw rather than Blume, BARRA or Vasicek betas. However, if possible, correct for variation in firm and market leverage over the beta estimation period

(vi) In the de-gearing process, use formula (2)

(vii) if betas are foreign, then correct for international differences in market leverage.

There still remains considerable room for differences of opinion. Short of offering estimates of asset betas for each Crown company, it is still possible to offer some warnings about the following situations from the writer's experience:

(a) Calculating an average from sets A and B, finding them to be different and then attaching some significance to this or offering an explanation. However unless differences are statistically significant, no concern is warranted.

(b) Calculating an average from firms in country 1 and also from Country 2, then taking a simple average of the two. However if one of these two sets has considerably more elements, then a simple average is inappropriate.

(c) Observing that a comparator has certain risks not present in the firm of concern, and then concluding that the comparator's beta would be too high. However these additional risks may be largely unsystematic (diversified away in a large portfolio) and are therefore irrelevant to beta. Examples are R&D failure risks, and bankruptcy experiences for US regulated utilities (the bankruptcies are presumably due to cost overruns in plant construction which the regulators will not allow recovery of via pricing).

(d) Observing that a revenue stream does not materially fluctuate with real GNP, and concluding that beta is close to zero. However this disregards the fact that beta is governed by sensitivities to macro factors other than real GNP (see the earlier discussion). Moreover US electric utilities have revenue streams with de facto protection against real GNP fluctuations and yet their betas are clearly positive.

Appendix 5 - Choosing an Appropriate Measure of the Asset Base

Value Based Reporting requires a cost of capital in $ terms. So the cost of capital in rate terms (or some derivative of it) must be applied to an asset base. A number of possible measures of the asset base present themselves. The primary level of classification is according to whether the asset base against which cost of capital is applied is historic cost (net of depreciation) or current value (capable of calculation in various ways). Consistent with this depreciation must then be calculated from historic cost or current value respectively. We consider first the use of historic cost.

Asset base at historic cost

Given the use of historic cost net of depreciation, a wide range of depreciation methods are found in practice. Since the choice influences the asset base then it influences the $ cost of capital. We illustrate this using the straight line method of depreciation. Consider a project costing $3.82m with expected cash flow in one year of $1.03m, and this is expected to inflate at 3% p.a. for a further four years. Also suppose the nominal cost of capital is 13%. Then the present value of the cash flows, discounted at 13%, is $3.82m. So the NPV is zero i.e., the project is economically neutral. Using straight line depreciation ($.764m p.a.) the expected income stream is as follows:

	1	2	3	4	5
Cash Flow	1.030	1.061	1.092	1.126	1.159
- Depreciation	.764	.764	.764	.764	.764
- Cost of Capital @ 13%	.496	.397	.298	.199	.099
Income	**-.23**	**-.10**	**.03**	**.163**	**.296**

The present value of the expected income stream is equal to the project NPV (of zero). Furthermore this desirable property holds for any depreciation method applied to the historic cost of the asset [see Ohlson (1995)]. However the usual result, as shown above, from an "accounting" depreciation method such as straight line is that the expected income pattern for an economically neutral project is initially negative, then positive. Many NPV positive projects will have this property. This has obvious disincentive effects for management. Furthermore, income in any year reflects not just management performance and unavoidable forecast errors but the "depreciation effect". Thus, before any conclusions can be drawn from income results about management performance, allowance would have to be made for the depreciation effect.

However there exists one depreciation method applied to historic cost which yields expected income in every year with the same sign as the project NPV (positive in all years for an NPV positive project, etc.). No correction would then be needed for the depreciation effect. This depreciation method is called Compound Interest [see Kaplan (1982),p 528-534)]. Under it, depreciation in year **t** is:

$$\mathbf{DEP_t = CF_t - IROR.BV_{t-1}}$$

where $\mathbf{CF_t}$ is the cash flow initially expected in year **t**, **IROR** is the internal rate of return on the project and $\mathbf{BV_{t-1}}$ is the asset book value at the beginning of year **t** [for year 1, the beginning of year book value is the asset cost].

Consider the example previously of an economically neutral project. The IROR on it is 13%. So the expected income stream is zero in each year as follows:

	1	2	3	4	5
Cash Flow	1.030	1.061	1.092	1.126	1.159
- Depreciation	.533	.634	.748	.878	1.026
- Cost of Capital @ 13%	.496	.427	.345	.248	.133
Income	**0**	**0**	**0**	**0**	**0**

Should cash flow turn out to be more (or less) than expected then income will be positive (or negative) in that year. However this depreciation method has not apparently enjoyed any success outside of the academic literature. The fact that economic rather than accounting arguments underlie it, and the tendency for depreciation arising from it to increase over asset life (especially under inflationary conditions) presumably explain this lack of success.

Asset base at current values

The historic cost methods discussed above imply that depreciation and $ cost of capital over the entire life of a project are fully determined at the moment of asset acquisition. An alternative approach is to determine these two costs in light of current asset values. The underlying rationale is to measure managerial performance in light of current values rather than historic costs. Furthermore the “inheritance effect” is minimised, i.e., present management’s evaluation is minimally dependent upon prior decisions made by themselves or their predecessors. Current asset value might be determined from replacement cost, exit value or Discounted Cash Flow. In the latter case, the appropriate discount rate would be as previously described. Whichever of these valuation methods is used, two distinctive approaches are possible.

The first of these is to convert the nominal cost of capital (in rate terms) to a real rate and then apply it to the revalued asset base. Furthermore asset revaluations are **excluded** from income. Specifically the real rate is applied to the asset base at the beginning of the year subject to it being revalued to reflect expected price movement over the year. In addition depreciation is based on the revalued asset base. Providing the inflation rate used to convert the nominal cost of capital to a real rate equals the expected rate of appreciation in (new) asset values then the Ohlson property is preserved, i.e., the present value of the income stream equals the NPV of the project. However, as described before, any depreciation method except Compound Interest yields the undesirable "depreciation effect". To illustrate, consider the previous example and assume that the value of a new asset is expected to inflate at 3% p.a. Using the same rate to determine the real cost of capital, this is:

$$\frac{\mathbf{1.13}}{\mathbf{1.03}} - \mathbf{1} = \mathbf{.0971}$$

Also using the straight line method as before, expected depreciation in year **t** is:

$$\mathbf{DEP_t = 3.82(1.03)^t(.20)}$$

and the $ cost of capital is expected to be:

Year 1 : $3.82(1.03)(.0971) = .382$

Year 2 : $3.82(1.03)^2(.8)(.0971) = .315$ etc.

So the expected income stream is:

	1	2	3	4	5
Cash Flow	1.03	1.061	1.092	1.126	1.159
- Depreciation	.787	.811	.835	.860	.886
- Cost of Capital @ 9.71%	.382	.315	.243	.167	.086
Income	**-.139**	**-.065**	**.014**	**.099**	**.187**

The present value of this expected income stream is equal to the project NPV (of zero). We also still see a negative, then positive expected income pattern, due to straight line depreciation. However, compared to straight line depreciation on historic cost, this pattern is considerably less pronounced. Moreover, as the asset life tends to infinity, then providing the asset is expected to increase in value at the same rate as the cash flows inflate, the expected income pattern on an economically neutral project will tend to zero in all years. Thus the expected income pattern converges on that for Compound Interest Depreciation applied to Historic

Cost. However unlike Historic Cost methods, the **actual** charges for depreciation and $ cost of capital in a year will reflect actual asset value at that time.

The second of these current cost methods involves applying the nominal cost of capital to the revalued asset base at year beginning and including the revaluations in income (this inclusion compensates for applying a nominal rather than a real rate). The revaluation is defined as the asset value change net of depreciation. So the sum of revaluation and depreciation equals the asset value change. Thus the choice of depreciation method is irrelevant. Providing the asset value is expected to change over time in line with the evolution of expected cash flows, then an economically neutral project will have expected income of zero in all years - the same desirable property as Compound Interest Depreciation applied to Historic Cost. Using the earlier example, the succession of yearly asset values consistent with the expected cash flow is:

$3.82, $3.286m, $2.652m, $1.904m, $1.026m, 0

Thus the expected income stream is zero in all years as follows:

	1	2	3	4	5
Cash Flow	1.03	1.061	1.092	1.126	1.159
- Depreciation & Revaluation	.533	.634	.748	.878	1.026
- Cost of Capital @ 13%	.496	.427	.345	.248	.133
Income	**0**	**0**	**0**	**0**	**0**

Again however, and unlike historic cost methods, the actual charges for depreciation (net of revaluation) and $ cost of capital in a year will reflect actual asset values at that time.

We now ask what are the relative merits of the last two (current value) methods. We have already revealed one difference - the latter (nominal rate) method is not exposed to the arbitrary choice of an accounting depreciation method, and therefore does not suffer the “depreciation effect” in interpreting income results [for example, a neutral project having a negative then positive income stream]. However the nominal rate method is highly exposed to fluctuations in asset values.

To illustrate this point, consider an infinite life asset (so depreciation disappears). It has a cost of $10.3m with expected cash flow in one year of $1.03m, and this is expected to inflate at 3% p.a. forever. Also the nominal cost of capital is 13%. Then the present value of the cash flow is equal to $10.3m. So NPV is zero i.e., the project is economically neutral. If the asset value is expected (reasonably) to grow at 3% p.a. then the expected income patterns of the two methods will each be zeros in all years.

Now however suppose that the asset value rises in the first year by 10%, falls in the second year by 3.6% (so that the cumulative value change over the two years is equivalent to a 3% increase per year), and then continues to grow forever at 3% p.a. Furthermore, project cash flows are as expected. The resulting income streams for the two methods will then be:

Real:	1	2	3	....
Cash Flow	1.03	1.061	1.092	....
- Cost of Capital @ 9.71%	1.03	1.133	1.092	....
	0	**(.072)**	**0**	**....**
Nominal:				
Cash Flow	1.03	1.061	1.092	....
+ Revaluation	1.03	(.408)	.328	....
- Cost of Capital @ 13%	1.339	1.473	1.42	....
	.721	**(.82)**	**0**	**....**

So income under the nominal, but not the real, method is highly exposed to asset value fluctuations. Arguably the inclusion of such effects in income is undesirable because they are beyond managerial control and therefore weaken the income measure as an indication of management performance.

Before seeking to summarise this evaluation of the four methods, we note some estimation issues arising from use of a real cost of capital. Firstly, a real rate (**r**) is related to a nominal rate (**R**) by:

$$\mathbf{r} = \frac{\mathbf{1+R}}{\mathbf{1+i}} - \mathbf{1}$$

where **i** is the inflation prediction embedded in the nominal rate. To achieve the Ohlson result that the present value of the expected income stream equals the NPV of the project, **i** should be the expected rate of inflation in the asset price. However a firm generally constitutes a portfolio of projects. Furthermore individual asset price increases are difficult to estimate. So, pragmatically, we choose the CPI index. Furthermore the period of concern must be that to which the risk free rate relates, and this is five years. However good inflation forecasts are available for only one year at best. So again pragmatically we choose the one year forecast, i.e., an average of respected forecasters views on CPI inflation over the next year. All of this might seem unduly subjective. However any error in estimating inflation must be considered in light of the consequences of using one of the nominal methodologies described.

We can then summarise the discussion as follows. If assets are not revalued then depreciation and $ cost of capital reflect historic costs rather than current values. For any depreciation method, the Ohlson property holds. However any depreciation method except compound interest implies that income results may have a time pattern (negative then positive for example) which is purely an product of the depreciation method chosen (the "depreciation effect"), and this complicates the assessment of management performance.

If assets are revalued then managers are evaluated according to current values rather than past costs. Thus “inheritance effects” are minimised. However this process is costly. Also, since revaluations are subjective and capable of being influenced by management, they are liable to be biased. Aside from choices for current value, the choice of real versus nominal methodology arises. Both methods preserve the Ohlson property, at least approximately. Real methodology stills leaves one exposed to the “depreciation effect” (unless compound interest methodology is used). Additionally it requires annual inflation forecasts. However it largely avoids exposure to asset value fluctuations, which are arguably irrelevant to any assessment of management. Nominal methodology has the inverse characteristics, i.e., it is not exposed to the depreciation effect, and it does not require inflation forecasts, but it is highly exposed to asset value fluctuations.

A final technical point concerns variations in the asset base over the course of the reporting period (year). This arises due to capital injections or withdrawals during the year, and is dealt with by adding injections (or deducting withdrawals) in proportion to the fraction of the year for which those changes were in effect. Thus if an injection of $1m occurred nine months into the year, then the injection was in effect for .25 of the year and thus the asset base would be:

Beginning of year level + $1m (.25)

If instead there was a withdrawal of $1m after nine months then the asset base would be:

Beginning of year level – $1m (.25)

Appendix 6 - Comparison with the Capital Charge Regime for Government Departments

The distinctions between the capital charge regime methodology for New Zealand government departments and the model recommended for value based reporting by Crown companies are as follows:

	Value Based Reporting	Capital Charge regime
Basic Model	Unlevered cost of capital	WACC
Grossing up for company tax	Not required if the entity is subject to tax	Yes
Cost of Equity model	Tax Adjusted CAPM	Standard CAPM
Estimate of MRP	9% (= 6.4% for standard CAPM)	6.5%
Riskless Rate	5 year govt stock rate	5 year govt stock rate
Beta Degearing formula	Consistent with tax neutrality	Not required
Real or Nominal	Choice	Real

This reveals that the only fundamental distinctions between the two models lies in the grossing up for company taxes and the assumed tax scenario. Grossing up for taxes simply reflects the absence of corporate tax for departments. Regarding the latter distinction, the methodology recommended here for value based reporting reflects tax neutrality, whilst that for the capital charge regime for New Zealand government departments reflects a classical tax regime. Given that the capital charge regime was initiated only shortly after the introduction of dividend imputation (when the classical tax scenario was still implicit in cost of capital estimation) and a general acceptability test also applied there, then the underlying tax assumption for the capital charge regime was natural. Since then, cost of capital estimation in New Zealand has (properly) shifted away from the classical scenario towards the tax neutrality one, and this shift underlies the methodology recommended here for value based reporting. So the distinction in methodology is historical in nature and should be eliminated by altering Capital Charge regime methodology to that recommended here.

Regarding the question of grossing up for company taxes, this is still appropriate for the Capital Charge regime for New Zealand government departments. However, if an unlevered cost of capital rather than a WACC is used, then the grossing up formula will be different. The purpose of grossing up is to produce the same valuation of projects by untaxed government entities as would be achieved by taxed entities in either the private or public sector. Lally (1995a, p21-22) shows that there is no grossing up formula that will give "correct" answers for all patterns of cash flows in projects. The grossing up formula used in capital charging produces the "correct" answer for a nominal perpetuity (constant expected nominal cash flow per year to infinity), and so we consider that case first.

Let:

X = expected nominal cash flow per year before interest and tax

$\mathbf{T_e}$ = effective tax rate for taxed entities before consideration of leverage

k = nominal cost of capital for a taxed entity (as specified previously)

Then the value of this project for a taxed entity is:

$$V = \frac{X(1 - T_e)}{k}$$

Letting $\mathbf{k_t}$ be the nominal cost of capital for an untaxed entity, value there is:

$$\mathbf{V_t = \frac{X}{k_t}}$$

To equalise the two values then:

$$\mathbf{k_t = \frac{k}{1-T_e}} \qquad \textbf{(1)}$$

i.e., the nominal cost of capital for an untaxed entity is that for a taxed entity, divided by $1 - \mathbf{T_e}$. So if $\mathbf{k}$ = 10% and $\mathbf{T_e}$ = .33 then $\mathbf{k_t}$ = 14.9%. This grossing up formula for the nominal rate is conceptually identical to that used for Capital Charging, even though the later applied grossing up separately to the costs of debt and equity in the WACC.

If it is desired to convert this rate $\mathbf{k_t}$ to a real one, as used in Capital Charging, then the real counterpart to $\mathbf{k_t}$ is:

$$\mathbf{k_{rt} = \frac{1+k_t}{1 + i} - 1}$$

$$\mathbf{= \frac{1+\left[\frac{k}{1-T_e}\right]}{1 + i} - 1} \qquad \textbf{(2)}$$

where $\mathbf{i}$ is the expected inflation rate.

It might be argued however that in the presence of inflation a better “reference project” is a perpetuity in real rather than nominal terms, i.e., choose $\mathbf{k_t}$ so that the valuation of a real perpetuity by an untaxed entity is the same as that for taxed entities. Letting **X** be the constant expected **real** cash flow per year, then value for a taxed entity is:

$$\mathbf{V = \frac{X(1+i)(1-T_e)}{1+k} + \frac{X(1+i)^2(1-T_e)}{(1+k)^2} + \ldots}$$

$$\mathbf{= \frac{X(1-T_e)}{\frac{1+k}{1+i} - 1}}$$

$$\mathbf{= \frac{X(1-T_e)}{k_r}}$$

where $\mathbf{k_r}$ is the real cost of capital for a taxed entity.

For an untaxed entity, value is similarly determined to be:

$$\mathbf{V_t} = \frac{\mathbf{X}}{\frac{\mathbf{1+k_t}}{\mathbf{1+i}} - \mathbf{1}}$$

$$= \frac{\mathbf{X}}{\mathbf{k_{rt}}}$$

So to equalise values here:

$$\mathbf{k_{rt}} = \frac{\mathbf{k_r}}{\mathbf{1-T_e}}$$

$$= \frac{\frac{\mathbf{1+k}}{\mathbf{1+i}} - \mathbf{1}}{\mathbf{1-T_e}} \qquad (3)$$

i.e., the real cost of capital for an untaxed entity is that for a taxed entity divided by $1–T_e$. To convert $\mathbf{k_{rt}}$ to nominal terms ($\mathbf{k_t}$), if a nominal cost of capital is to be used, then:

$$\mathbf{k_t} = \mathbf{(1 + k_{rt})(1 + i) – 1}$$

$$= \left[\frac{\frac{\mathbf{1+k}}{\mathbf{1+i}} - \mathbf{1}}{\mathbf{1-T_e}} + \mathbf{1}\right]\mathbf{(1 + i) – 1} \quad \text{by (3) above}$$

$$= \frac{\mathbf{k}}{\mathbf{1-T_e}} + \frac{\mathbf{iT_e}}{\mathbf{1-T_e}} \qquad (4)$$

In summary, if we choose a grossing up formula to equalise the values of nominal perpetuities for taxed and untaxed entities then formula (1) [nominal] or (2) [real] applies. If by contrast we seek to equalise the values of real perpetuities for taxed and untaxed entities, then formula (3) [real] or (4) [nominal] applies. However the pair (1),(2) is not consistent with the pair (3), (4). This is very clear from comparing the two formulas for $\mathbf{k_t}$, i.e., (1) and (4). It is also reflected in the two formulas for $\mathbf{k_{rt}}$, i.e., (2) and (3). The difference between the two pairs is up to half the inflation rate (i). So if $\mathbf{i}$ = 2% then the difference is up to 1%. Which pair then is better?

Under inflation a real perpetuity is more realistic than a nominal one, and hence formulas (3), (4) are more appropriate. However few projects are perpetuities, and extensive simulations would be required on finite life projects to resolve the issue. Until then we simply note that the difference is not substantial at low inflation rates.

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