Optimal Timing of Tax Policy in the Face of Projected Debt Increases

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Grant Scobie

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Abstract

This paper examines the optimal time path of the tax rate, in a model where an increasing ratio of government debt to GDP is projected in the absence of policy changes. Tax policy changes have feedback effects, as a result of incentives and other endogenous influences which impose constraints on the efficacy of those policies. Emphasis is given to the importance of uncertainty in devising an optimal policy. A welfare function is maximised, allowing for a range of variables, including the excess burden of taxation and a desired debt ratio.

JEL Classification: H63; H68; E62

Keywords: Tax policy; Stochastic projections; Debt ratio.

Executive Summary

This paper is concerned with fiscal policy when an increasing ratio of government debt to GDP is projected in the absence of any policy changes. The central question is given those circumstances, and assuming a simple proportional tax structure, what is the optimal time path of the tax rate? Focus on the income tax rate is warranted in view of the debate concerning the timing of tax changes; in particular whether some form of tax smoothing would be desirable. The approach taken here gives prominence to the importance of uncertainty in devising an optimal tax policy. This is considerable when dealing with projections over a medium to long-term future.

Policy makers will be faced with outcomes in every period of the planning horizon which deviate from projected levels. It is far from clear that a tax rate decided at the beginning of the planning period could in practice be left unchanged. For one thing, this would allow the debt ratio to move well outside the agreed target value. Faced with substantial uncertainty, there may be some value in waiting for some of the uncertainty to be resolved and modifying tax policy over time. An optimising strategy, in which expectations of future outcomes are revised as the uncertainty becomes resolved over time, is devised and its properties examined.

The paper computes optimal tax policies which maximise a social welfare function. This function is the present value of an annual welfare index whose arguments are, for each period: an indicator of tax progressivity incorporating welfare spending; real income; a measure of the excess burden of taxation; a cost of deviating from a specified debt target and; a cost of adjusting the tax rate. This is achieved in the context of a small aggregate model of the economy, incorporating key feedback effects. These include the disincentive effects of income tax and the effect of rising debt ratios on the country risk premium and hence interest rates. The model is calibrated in such a way as to duplicate, for the basic case of constant growth rates and no policy changes, the debt track projected by the New Zealand Treasury's Long Term Fiscal Model.

The optimal sequence of tax rates for a forty year planning horizon in the absence of uncertainty is first considered. It is found that the optimal tax rate falls slightly in the early years. This is because the significant effect of population ageing does not arise until later years. The optimal tax rate then rises gradually in each of the subsequent years. In contrast were a policy of tax smoothing adopted, there would be a substantial loss of welfare compared with the optimal policy.

For a policy of tax smoothing to be optimal, with its consequent budgetary surplus in the middle years of the planning period, a very special case of the welfare function must be adopted in which there is no cost of changing the tax rate in the first period, the cost of adjusting the tax rate in subsequent periods is high, and only the debt ratio in the final year matters.

Having introduced uncertainty, the optimal tax profile is found by allowing revisions in each year, based on the extent to which the actual randomly-generated outcome differs from that which was previously expected. Thus, the policy maker has the option for continuous review and adjustment. The characteristics of the resulting changing distribution of optimal tax rates over the planning period are thereby obtained. The probability that the optimal tax rate exceeds some specified value is computed. The time profile of the expected value of the distribution of the optimal rate is found to be slightly less smooth than in the deterministic version of the model where there is no uncertainty.

The type of analysis presented here is an exercise in welfare economics, investigating the implications of adopting a particular form of social welfare, or evaluation, function. This function represents the value judgements of a fictitious decision maker. The examination of alternative specifications shows the extent to which tax smoothing as an optimal policy requires a special set of assumptions in the deterministic case. In the stochastic case, actual future outcomes may make such a policy unsustainable.

With uncertainty, it is no longer possible to form a consistent plan to be followed over a long period. Where initial policy is based on reasonable expectations about the future, a process involving regular policy revisions, in the light of actual outcomes and revised expectations, manages to keep important policy variables within a reasonable range. This is particularly important where feedback effects and interdependencies are relevant. For example, allowing the debt ratio to increase to high levels (thereby raising the risk premium and debt service charges substantially) cannot easily be corrected by substantially increasing taxation given the associated adverse incentive effects and welfare costs.

As with all optimal tax models, no value-free policy recommendations can be made on the basis of the simulations reported here. Nevertheless, it is suggested that the modelling provides a useful framework in which alternative value judgements, and assumptions about central economic variables and relationships, can be examined.

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1 Introduction

It is widely recognised that in the face of demographic change, current fiscal policy settings in many countries could lead to unsustainable levels of public debt. One approach to prevent this arising is to raise taxation rates before debt exceeds manageable levels. However, given the uncertainty which inevitably surrounds long-run fiscal projections, the policy maker is faced with a difficult challenge of selecting the most appropriate time path for the tax rate. For example, should the tax rate be increased earlier rather than delaying, and if so by how much?

This paper examines the optimal time path of the tax rate in a simple proportional tax structure, where an increasing ratio of government debt to GDP is projected in the absence of any policy changes. A large number of policy responses to projected debt increases is clearly available.[1] These range from annual adjustments to various categories of social and other expenditure, to changes in a number of tax rates, and combinations of these. Such policies are likely to have feedback effects, as a result of incentives and other endogenous influences which impose constraints on the efficacy of those policies. An important constraint on policy choices is obviously imposed by the need to ensure fiscal sustainability over the longer term, since high and increasing debt ratios can impose large debt servicing costs as a result of a rising risk premium. Similarly, disincentive effects of higher tax rates constrain the government's ability to raise revenue.

The present paper analyses the optimal time path of the proportional income tax rate, although other adjustments are briefly considered. Focus on the income tax rate is warranted in view of the debate concerning the timing of tax changes and whether some form of tax smoothing is desired. Emphasis is given to the importance of uncertainty in devising an optimal policy.[2]

[1] Ostry et al. (2015) examine timing in the face of a possible exigency, using a normative model with a representative agent and no uncertainty.

[2] Diamond (2014, p. 6) suggested that, 'we need considerably more work to clarify the link between uncertainty in projections and the level of government savings, as well as the timing of preferred actions to increase savings'. CBO (2015) examined how varying its estimates of future mortality rates, productivity growth, interest rates on federal debt, and federal spending on Medicare and Medicaid would affect their projections. They commented (2015, p. 6) that, 'in deciding how quickly to carry out policies to put federal debt on a sustainable path – regardless of the chosen goal for debt – lawmakers would face difficult trade-offs'.

The question of whether tax smoothing is desirable is complex.[3] Given the non-linear (approximately quadratic) nature of the excess burden of taxation, it is sometimes assumed without further question that smoothing is always the optimal response. Tax smoothing has been compared with savings and the permanent income hypothesis. Tax rates respond to 'permanent changes' in the budget burden rather than transitory changes.[4] However, such a policy is likely to involve a period of budget surplus and, putting aside the concern that politicians may 'raid' such surpluses, the disincentive effects of initial high rates can affect income growth and therefore future tax bases.[5]

In addition, if the budget moves into surplus some distance away from what is regarded as a prudent or target debt ratio, this may imply missed opportunities for public investments that can yield productivity gains, reductions in tax rates, or other desirable tax-financed expenditure. Even if the future is assumed to be known with certainty, the optimal time path of adjustment depends not only on the starting position in relation to some desired debt ratio target, but also on the nature of the various feedback effects and trade-offs involved in policy decisions.

Decision making is further complicated by the existence of uncertainty, which is considerable when dealing with projections over a medium to long-term future.[6] It is perhaps tempting to compare this problem with that of investment in a multi-period project where the future returns are not known with certainty, there is a non-recoverable sunk cost of investing in the first period and there exists the option of waiting until later periods before making the investment: see, for example, Dixit and Pindyck (1994) and Pindyck (2008).[7] The sunk cost consists of any fixed costs which cannot be recovered (such as the difference between the cost of investing in specific equipment and its resale value) and the foregone value of waiting and obtaining more information (the option value). However, unlike

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3 An early modern discussion of tax smoothing is Barro (1979). Armstrong *et al*. (2007) also highlight the concave nature of the government's revenue function, arising from adverse incentive effects. Davis and Fabling (2002) stress the ability of the government during periods of surplus to obtain a rate of return in excess of the cost of borrowing, although this feature is not examined here.

4 See also Auerbach (2014) who argues that greater uncertainty leads to higher precautionary savings (subject to some conditions on utility functions).

5 The danger that a precautionary fund will be raided by a future government was stressed long ago by Ricardo (1893) in the context of the British Sinking Fund. Davis and Fabling (2002) model 'expenditure creep'and report that it can completely erode the efficiency gains from tax smoothing. They conclude that, 'strong fiscal institutions are a prerequisite for achieving the welfare gains from tax smoothing' (2002, p. 16).

6 Luo *et al*. (2014) modify the tax-smoothing model to allow for uncertainty about the model specification. They consider 'robust control', where the decision-maker designs a policy that can work well even if the model (the best approximation to the true model) is not the true model.

7 In the context of health and long-term care under demographic uncertainty, Lassila and Valkonen (2004, p. 637) find that the longer the time horizon, 'the virtues of using continuously updated demographic information to evaluate future expenditures become evident'.

the investment framework, the present context does not require an initial 'lumpy' amount of investment (such as the construction of a factory). Nevertheless, current actions have effects that may be non-reversible. For example, effects on individuals in one period, including excess burdens, cannot be reversed by subsequent tax rate adjustments.[8]

In the present context several central variables are subject to uncertainty, which (as explained in Section 5) is specified in a non-parametric manner. Although the probability distributions of the debt ratio and other endogenous variables in each year of the planning period can be obtained using Monte Carlo methods, these cannot be used in the standard approach used to obtain option values.[9] Nevertheless an optimising strategy, in which expectations of future outcomes are revised as the uncertainty becomes resolved over time, is devised and its properties examined.

In this paper, an independent judge maximises an evaluation function that reflects the judge's value judgements. This is usually referred to as a social welfare function (SWF), and is defined over a range of characteristics of the economy over a finite projection period. The analysis is thus characterised as welfare economics along 'what if' lines: that is, it considers the implications of adopting various value judgements. Section 2 describes the deterministic form of the projection model. A form of evaluation function is presented in Section 3, which also allows for redistributive effects of tax policy changes. Optimisation using the deterministic version of the model is examined in Section 4. Section 5 introduces uncertainty and discusses the general problem of decision making over time, when faced with uncertain projections. Simulation results using the stochastic version of the model are presented in Section 6 and conclusions follow in Section 7.

The analysis necessarily abstracts from some practical considerations that are sometimes discussed. For example, the question of how uncertainty is actually presented to policy makers is not considered here. Auerbach (2014, p. 38) comments that, 'the uncertainty associated with baseline projections and estimated effects of revenue and spending provisions tends to be suppressed not only in the presentation of projections but also in the legislative process itself'. Furthermore, Manski (2014, p. 35) argues that, 'we know essentially nothing about how decision making would change if statistical agencies were to communicate uncertainty regularly and transparently'.[10]

8 A further example of irriversibility is of investment in human capital via education expenditure.

9 Using a very simple tax and expenditure model, the role of option values was explored by Ball and Creedy (2014).

10 This aspect also relates to the problem of data inaccuracies, raised by Morgenstern (1950).

The welfare function allows for costs of adjusting taxes, but constraints on the flexibility of government policy which may rule out frequent small changes, are not considered here. Auerbach and Hassett (1998, 2002) suggested that, faced with uncertainty, the inability to have frequent policy changes suggests early action. However, inaction may be chosen because of the inability to reverse any adverse effects on particular groups. They concluded (1998, p. 23) that, 'the optimal policy response over time might best be characterized by great caution in general, but punctuated by occasional periods of apparent irresponsibility'.

2 The Projection Model

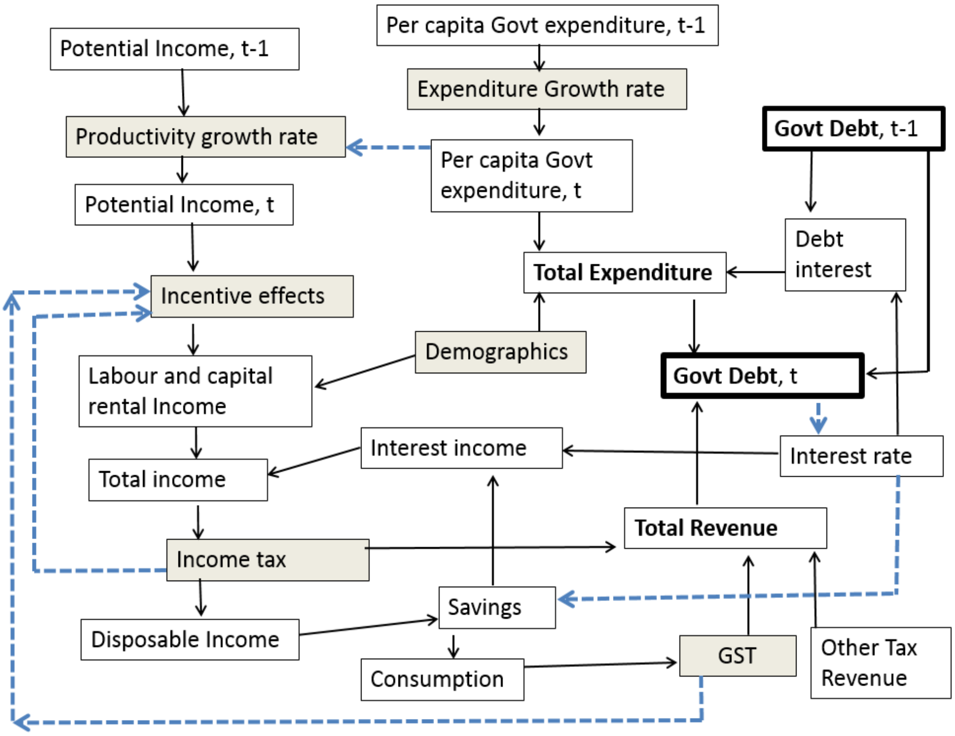
A requirement of the model is that it is capable of projecting the paths of government revenue and expenditure, and hence the public debt, under a range of assumptions and feedback effects. To make the model as transparent as possible, a high level of aggregation is used. It is clearly necessary to allow demographic variations in both population size and its age composition to influence government expenditure and revenue. In particular, distinctions are drawn between those of 'working age', 'retirement age' and those below working age.

2.1 Basic Structure of the Model

The basic structure of the model is described in Figure 1: further details are in Appendix A and Creedy and Scobie (2016), who examine deterministic projections in the absence of policy responses, and comparisons with selected policies designed to achieve the limited objective of a target debt ratio at the end of the projection period. The shaded boxes indicate components that influence other variables: these include the income tax and Goods and Services Tax (GST) structure, the incentive effect of taxes, the demographic structure of the population, and productivity and expenditure growth rates. The items in bold font are, along with the debt levels, the main aggregates of total government revenue and expenditure. Feedback effects are indicated by the dashed lines connecting boxes.

The generation of income changes from one year to the next is described down the left-hand side of the diagram which, in turn, leads to tax revenue. Capital income is modelled simply as interest income: there is therefore no attempt to treat the production side of the economy explicitly. The model contains no explicit wage rate, nor does it deal with labour and capital inputs into production. A 'base level'

Figure 1: Outline of the Model



of productivity is taken as exogenously given and, as explained below, productivity changes can arise from growth in public expenditure on health and education per person, which is considered to augment human capital.[11] Government expenditure is described down the central part of the diagram. Starting from per capita values for four expenditure components (welfare benefits for the retired; other welfare benefits per person; expenditure on health and education; other social spending), along with the relevant growth rates, the total value of expenditure is influenced by the age structure of the population. The generation of debt over time is then indicated on the right-hand side of the diagram. All variables are in real terms. The income tax is a simple proportional tax, with constant average and marginal rates, and the GST system is applied to all expenditure. The use of fiscal drag to increase tax revenues over time is thus not a possible policy option in the model; nor is the possibility of the government monetising the debt and inflating.

The four feedback effects are indicated by dashed lines. Taxes have adverse incentive effects which influence employment income. Expenditure on health and education has a (lagged) effect on productivity growth. The government debt ratio influences the interest risk premium. The assumption is made that the same interest rate applies to all debt, so that debt effectively involves the issue of one-period bonds. Finally, the interest rate, equal to the exogenous world rate

[11] In a more general model it may be desirable to allow for the effects on productivity of government infrastructure spending. Furthermore, it is recognised that different forms of taxation have different implications for growth rates.

plus a risk premium, affects the saving rate. There is little information about the precise form of this relationship. Here, debt ratios up to about 150 per cent of GDP are assumed to produce small (linear) increases in the risk premium. It is acknowledged that this may be regarded as an 'optimistic' specification. Beyond 150 per cent, the risk premium increases rapidly (and quadratically). In the stochastic projections reported below, the risk premium was in fact capped at 15 per cent to prevent results becoming explosive (and thereby distorting the mean and median in particular).

The calibration of the model involves setting a large number of initial variables and parameter values, obtained using an extensive range of New Zealand data (see Creedy and Scobie, 2016, Tables 2 to 9). An important feature of the model – when the feedback and uncertainty features are 'turned off' – is that it produces 'benchmark' forty-year projections of the government debt ratio that closely match those produced by the considerably more disaggregated Treasury Long Term Fiscal Model: see Bell and Rodway (2014). In the benchmark case, growth rates and other policy variables (such as tax rates) are held constant. An absence of feedback effects implies that the economy can be allowed to reach any debt ratio and then brought back to a target level by an appropriate tax and expenditure policy. However, the feedback effects considered here – particularly those affecting the risk premium and labour supply incentives – make recovery from very high debt ratios extremely difficult in view of the high debt servicing costs and the reduction in the tax base resulting from high tax rates.

3 An Evaluation Function

As mentioned in the introduction, the approach taken is to suppose that the time profile of the economy can be evaluated using an explicit function – referred to as a SWF – which reflects the judgements of a disinterested decision maker. The essence is to capture the willingness to make trade-offs among alternative outcomes. Clearly, a wide range of functions may be considered.

However, suppose each period is evaluated using some metric, referred to as the annual welfare index, W_t, which is considered to be a function of a number of outcomes. First, disposable income per person, $Y_{A,t}/N_t$, contributes positively to welfare. Second, the excess burden of taxation contributes negatively to welfare: this is considered to be a function involving the square of the effective rate, denoted τ_t^*: this is based on the well-known result that the excess burden is approximately proportional to the square of the tax rate. The loss arising from incentive effects is of course captured by the effect on income per capita. Let v_t and τ_t denote, respectively, the broad-based GST rate and the proportional income tax rate. If s_t is the constant proportional savings rate, the effective tax rate is shown in Appendix A to be given by $\tau^* = \tau + \frac{v_t(1-s_t)(1-\tau_t)}{1+v_t}$.

Third, suppose that the decision maker aims to achieve a debt target, DR^*, expressed in terms of the ratio of debt to income. However, instead of this being an absolute constraint on policy, there is a cost each period of not meeting the target, depending on the square of the difference, $D_t/Y_{A,t} - DR^* = DR_t - DR^*$. Fourth, there may also be a cost of adjusting the percentage tax rate, represented by the square of the difference, $\tau_t^* - \tau_{t-1}^*$.

So far the arguments of the welfare metric are expressed in aggregate terms, which is consistent with the highly aggregative nature of the projection model. Nevertheless, it is possible to obtain a progressivity index, P_t, which reflects the redistributive extent of the tax and transfer system. It depends on the effective tax rate and the welfare benefit per person: for derivation of this index see Appendix B. This is the fifth argument of the annual index, which is assumed to reflect a desire for redistribution. An increase in progressivity can be obtained by increasing welfare payments which, through the government's budget constraint, requires either an increase in the current taxation rate or an increase in debt (or reduction in surplus). An increase in the tax rate in turn has a disincentive effect, as well as creating higher excess burdens.

These five summary measures could be arguments in a wide range of functional forms. This paper concentrates on a welfare metric for each period which is expressed as the additive function:

$$W_t = \beta_1 P_t^{\eta} + \beta_2 \left(\frac{Y_{A,t}}{N_t}\right)^{\alpha} - \beta_3 \left(\tau_t^*\right)^2 - \beta_4 \left\{100\left(DR_t - DR^*\right)\right\}^2 - \beta_5 \left\{100\left(\tau_t^* - \tau_{t-1}^*\right)\right\}^2 \tag{1}$$

Since it is considered to represent the values of a disinterested decision maker, it cannot be 'defended' on objective grounds, except to argue that the nature of the trade-offs are 'sensible'. Total welfare over the T periods is the present value:

$$\Psi = \sum_{t=1}^{T} \left(\frac{1}{1+\xi}\right)^{t-1} W_t \tag{2}$$

where ξ is the time preference rate of the decision maker. An excess tax burden, missing the debt target ratio and adjusting the tax rate are 'bad', while income per person, expenditure on health and education and progressivity are 'good'. Welfare spending is included (positively) in the progressivity measure. Expenditure, $E_{I,t}^*$,

Table 1: Benchmark Parameter Values

Coefficient	Value	Coefficient	Value
β_1	100	α	0.7
β_2	0.1	DR^*	0.2
β_3	250	ξ	0.04
η	0.98	β_4	0.5
γ	0.7	β_5	50.0

also contributes to future values of $Y_{A,t}$, but has a positive independent effect too. However, when considering the optimal time profile of the income tax rate, this term is not relevant as it is constant for a given growth rate of E_I^*.

This welfare metric does not allow for intergenerational considerations, and abstracts from overlapping generations. One can effectively think of those alive at the start of the period as living through to the end. The fact that W_t appears linearly in Ψ implies risk neutrality on the part of the judge.[12] The nature of this welfare function also ensures that Strotz's (1956) condition for time-consistency is satisfied in the deterministic case; that is, where outcomes are the same as those expected at the beginning of the period, re-optimising throughout the planning period produces exactly the same policy in each year as when optimising from the beginning of the period. A feature of the function in (2) is that there is a fixed planning horizon of T periods, which may lead to 'end period' issues, although this is not a serious problem here. The results for the non-stochastic case, shown below, suggest that this affects the last few years slightly. This disappears if the cost of missing the debt target in any year is increased substantially. Experiments show that reducing the length of the planning period by several years has a negligible effect on optimal values for earlier periods.

Table 1 provides benchmark values for the parameters. The approach taken in setting these values was to set β_1 arbitrarily equal to 100, and then impose values of other parameters in turn such that the implicit trade-offs are approximately one-to-one (in terms of proportionate changes). For the simulations reported below, the target debt ratio was set at $DR^* = 0.2$. While in practice the desired debt ratio is subject to considerable debate, the main lessons obtained from the present analysis are not affected by the specific value used here, although the shapes of the optimal tax and debt profiles clearly differ depending on whether the

12 Positive constant relative risk aversion could easily be introduced by raising W_t to some exponent but, following Ball and Creedy (2014), this would be expected to have little effect on the results. The use of Epstein-Zin (1989) preferences would add unwarranted complexity in the present context.

target ratio is above or below the initial value.[13]

4 Optimal Tax Rates in The Deterministic Model

In the deterministic version of the model, the objective of the decision maker is to select the time sequence of tax rates, τ_t, over the forty-year period to maximise Ψ. That is, the plan is made at the beginning of the period, on the assumption that the time path of relevant variables is known with certainty, so that there will be no need to make revisions. The set of optimising rates can be obtained using numerical methods.[14] Adding further decision variables, such as the rate of growth of selected expenditure categories, considerably complicates the analysis. Hence, attention is restricted here to the income tax rate over time, given the attention paid to this variable in the tax-smoothing literature and policy debates. The time horizon of 40 years is dictated by the statutory requirement of the New Zealand Long Term Fiscal Statement.

Using the values in Table 1 produces an optimal path for the tax rate shown in Figure 2. The associated debt profile in shown in Figure 3. The welfare function attaches a loss to deviations from the debt target of 20 per cent of GDP in each projection year, and this causes the debt path to follow this ratio quite closely until the end of the period.[15] The resulting optimal tax rate profile is little changed from that shown above. Given the imposed cost of adjusting the income tax rate, the time-path is quite smooth. It initially falls slightly, since debt can be reduced in the early years in view of the fact that the population ageing effect does not cause rising debt until later in the projection period.

For the optimal tax profile to be noticeably flatter than that arising in the benchmark case, the cost of changing the tax rate, determined by the coefficient, β_5, needs to be substantially increased in order to allow the debt ratio to vary more over

13 Experiments using the non-stochastic version of the model showed that, for example, having a lower debt target of 10 per cent involves an initial increase in the tax rate over the first few years only (compared with the profile reported below), with the remaining profile looking very similar to the 20 per cent case. The debt ratio drops to 10 per cent by 2023 and is then steady (falling initially below 10 per cent), except for the 'end period' effect discussed above. The effects of a higher debt ratio target are, as expected, to lower the optimal tax rate in the first few years, as debt rises to the target ratio, after which the tax rate gradually rises.

14 The examples below were obtained using the add-on data analysis facility, 'Solver', in Excel.

15 This 'end period effect' can be avoided by raising the cost of missing the debt target in each period to $\beta_4 = 10$. The resulting optimal tax rate profile is little changed from that shown above.

Figure 2: Tax Rate Profile: Benchmark Case

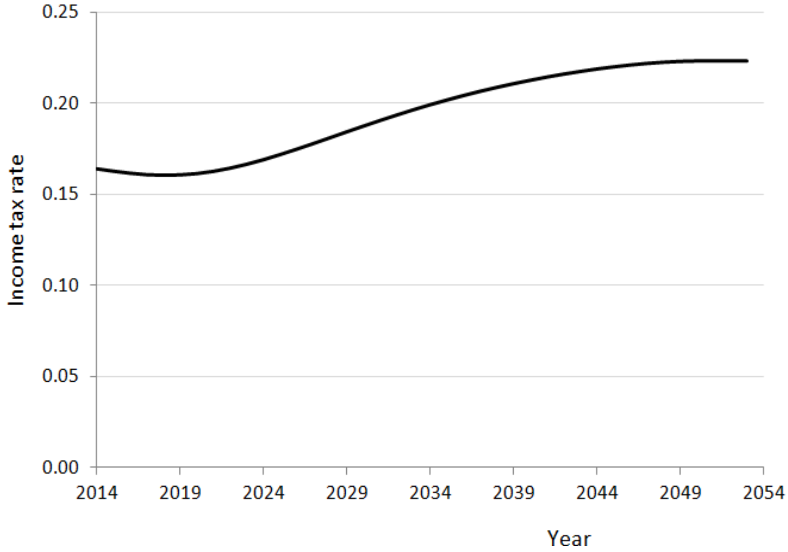
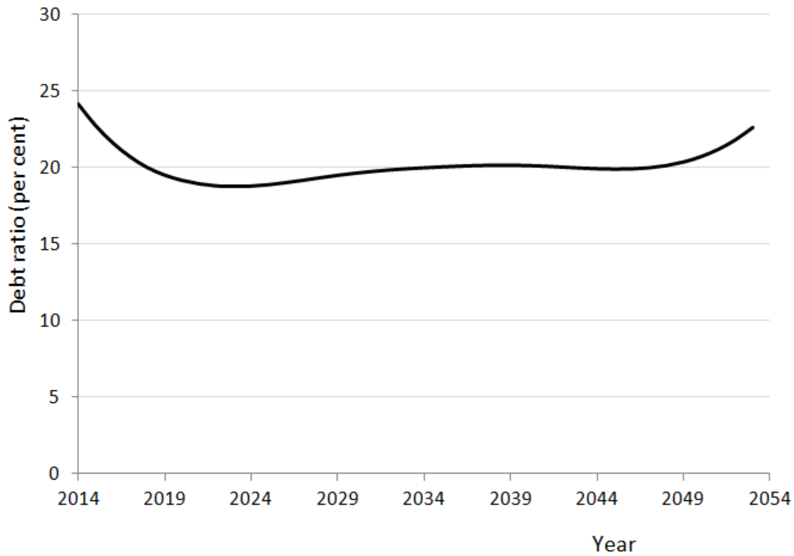


Figure 3: Debt Ratio Profile: Benchmark Case



the period. Furthermore, the cost associated with missing the debt target in intermediate years (determined by β_4) needs to be substantially reduced, while the simultaneous achievement of a 20 per cent debt ratio at the end of the period requires a high value of β_4 in the final couple of years.

The rising tax rate does not arise from discounting future values, since dropping ξ to zero has a negligible effect on these results. Attaching considerably more weight to progressivity, by increasing β_1, has a negligible effect. Furthermore, relatively large changes in β_3 and β_2 are needed to produce noticeable changes in the profiles.

Suppose the debt ratio has a negative effect on social welfare in any period only if it exceeds the target ratio, but has a much higher cost in those cases, and there is a much higher cost associated with changes in the tax rate. In this situation, the debt ratio falls to about 5 per cent of GDP after 20 years, before rising to the target level at the end of the planning period. The optimal tax rate increases steadily over time, to just over 20 per cent by 2053, in contrast with the case shown above where it falls in the early years.

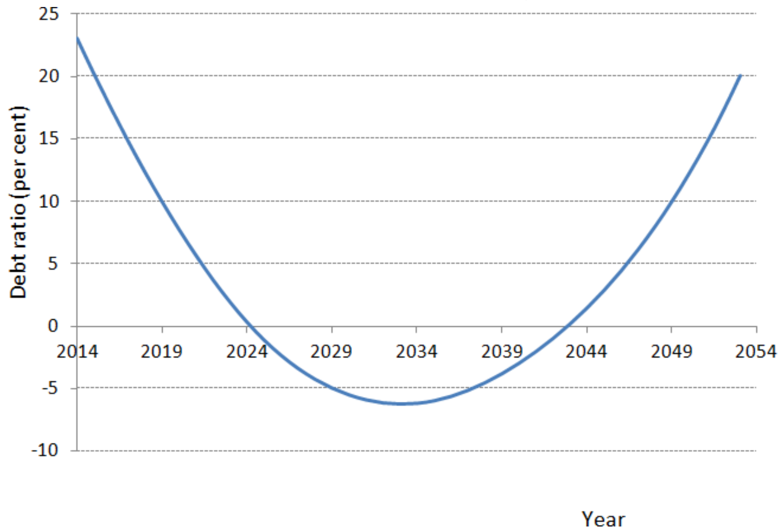
The extent to which tax smoothing is sub-optimal can be seen by comparing total welfare in the benchmark case above with that obtained by imposing a constant income tax rate of 18.5 per cent (which achieves a terminal debt ratio of approximately 20 per cent of GDP). Total welfare is found to drop from 6185 to 2111, a reduction of 66 per cent. As seen below, tax smoothing involves a surplus over part of the planning period, during which the annual welfare index is negative. The welfare function reflects the fact that the surplus involves foregone opportunities for productive investments. It does not include any precautionary benefits from having a surplus.

4.1 Tax Smoothing

In view of the attention given to tax smoothing, it is of interest to consider the form of the social welfare function under which smoothing is optimal in the present model, since these are very strong. It may be thought that the specification and benchmark calibration of the welfare function introduces a strong bias against tax smoothing and towards achieving a relatively smoother profile for the debt ratio. However, even with a considerable increase in the cost attached to changing the tax rate, leading to the optimal debt ratio to deviate from the target value over most of the planning period, the optimal tax rate follows a smooth increasing path. A more radical change needs to be made to the welfare function, as follows.

Suppose the judge is concerned only with the excess burden of taxation and the cost of changing the tax rate, while the achievement of a specified debt ratio is only relevant in the final projection year, a flatter tax rate profile is achieved, but it is far from a tax smoothing outcome. The optimal tax rate at the end of the planning period is just over 2 per cent while the minimum debt ratio is 5 per cent in the middle of the period.

Figure 4: Debt Ratio Path in the Tax Smoothing Case



However, suppose there is no cost (in terms of the SWF) to making an adjustment to the tax rate in the first year. This additional assumption allows for a sharp increase in the first period and leads to an 'approximate smoothing' result. The income tax rate, τ, jumps from 16.5 to 18.3 per cent in the first year and increases very slowly to a maximum of 19.1 per cent at the end of the planning period. Implications for the debt ratio are shown in Figure 4: these results are obtained for $\beta_3 = 1000$; $\beta_4 = 50$ in the final year and is zero otherwise; $\beta_5 = 100$; $\beta_1 = \beta_2 = 0$. There is a surplus during the years 2025 to 2042 inclusive. In the present context, the tax smoothing case is therefore seen to arise from a narrow form of evaluation function. The fact that a debt target is relevant only for the end of the planning period means that the optimal tax rate does not follow the debt ratio profile.

5 The Stochastic Model

This section introduces uncertainty into the basic model. Subsection 5.1 provides a brief description of the way in which stochastic projections are obtained: for further details see Ball *et al.* (2016). The question of how tax planning in the face of such uncertainty can be modelled is considered in Subsection 5.2.

5.1 Introducing Uncertainty

Although the model has a fairly high level of aggregation, there are many variables whose time path is uncertain. The paper is limited to considering just two of the four expenditure components, the world interest rate and the rate of productivity growth. These four variables were selected for examination on the grounds that they are important determinants of the time path of the debt ratio and, as shown in Ball *et al.* (2016), have been subject to considerable variability over time. The effect of allowing these four variables to be stochastic is nevertheless sufficient to demonstrate the importance of dealing explicitly with uncertainty.

The structure of the model itself – such as the nature of the basic relationships involved – is not considered to be subject to uncertainty. The parameters of the various reduced-form relationships governing the feedbacks and endogenous changes are assumed to be known and fixed. The limited nature of the uncertainty modelled therefore needs to be kept in mind. Despite this, there are substantial lessons obtained by departing from the deterministic approach that is characteristic of the majority of projection models.

One approach to dealing with uncertainty is to rely on a set of *a priori* assumptions about the distributions of the relevant variables. Assumptions about the form of the joint distribution of the variables may be based on considerations relating to the structure of the economy, policy settings and vulnerability to a range of exogenous shocks. Such assumptions may be more or less informed by past events. For example, the view may be taken that the past is not necessarily a reliable guide to future variability given a range of institutional and other changes. However, this is not the approach adopted here.

In the present context, the question raised is: what are the implications for the likely path of the debt ratio if future variability is thought to be similar to that observed in the past? In the absence of strong reasons for imposing other *a priori* specifications, past history – while obviously not precisely repeated – is regarded as a reasonable starting point.[16] The method used here does not explicitly model discrete catastrophic, or beneficial, events. However, to the extent that such events took place within the past years observed, their effects on the relevant variables are implied. The approach thus assumes that such events are no more or less likely than in the past.

[16] This is of course subject to the difficulty that past observations combine stochastic shocks and policy responses to those shocks.

5.2 Decision Making with Random Outcomes

Faced with considerable uncertainty, one possible strategy is simply to adopt some kind of rule of thumb, rather than attempting a 'full-blown dynamic optimisation'. For example, it may be decided to set an initial relatively high tax rate, with the aim of allowing government debt to absorb any subsequent shocks: some periods may need relatively higher debt while at other times there may be a precautionary surplus. Information about the relevant probability distributions, obtained for example using a simulation exercise, may be used to used to determine a rule of thumb bearing in mind the associated debt ratio probabilities over the period. Ball *et al*. (2016) report conditional probability distributions associated with several such basic policies. However, given the extent of uncertainty, it cannot be assumed – unlike the deterministic case – that any decision made at the beginning of the planning horizon will not need to be reviewed and possibly revised.

The way in which adjustments are made clearly depends on the formation of expectations regarding future debt and other profiles. Consider the case where tax policy is based on current and (conditional) expected values of annual welfare over the planning horizon. Each period, the policy is revised to make allowance for the fact that in any given year, the actual outcome will differ from that year's expected value (since the expectation was formed in the previous year). If expectations were correct, no change in the policy is required for that year.

In setting out an optimising strategy, involving a sequence of decisions, some specific notation is needed to distinguish values in different time periods, depending on when the plan is made. Define $\tau = [\tau_1, \tau_2, ...\tau_T]'$ as a vector of income tax rates over the planning period, $t = 1, ..., T$. Let τ^j denote the tax vector based on optimisation carried out at period, j. Hence the relevant elements of τ^j are $\tau_j, \tau_{j+1}, ..., \tau_T$. The policy choices concern income tax rates although the model also contains a broad-based Goods and Services tax: hence τ does not reflect the overall effective tax rate.

Let W_t denote annual welfare in period t: in view of the inherent uncertainty, this is a random variable. To simplify the discussion, consider discrete distributions and suppose there are S possible outcomes, giving W_t^s, for $s = 1, ..., S$, as the set of possible outcomes for period, t. But these values of W also depend on the tax rate. Hence define $W_t^s(\tau^j)$ as the outcome, s, in period $t > j$, for the tax rate selected by optimisation at period, j. In addition, $E[W_t(\tau^j)]$ is the expected value of W at time, t, using the tax vector, τ^j, where $t > j$. Letting $p_{s,t}^j$ denote the probability (evaluated at j) of outcome s in period $t > j$, expected welfare is given by:

$$E\left[W_t\left(\tau^j\right)\right] = \sum_{s=1}^{S} p_{s,t}^j W_t^s\left(\tau^j\right) \tag{3}$$

The values of $p_{s,t}^j$ could be evaluated using Monte Carlo methods, but in practice this involves a prohibitive amount of computation, especially given the need for recalculation of the profile each period.[17] The approach considered here is therefore to suppose that an approximation for $E\left[W_t\left(\tau^j\right)\right]$ can be obtained from the relevant deterministic projection model. Such projections are of course conditional on the tax rate profile, which changes as tax policy is revised in the face of new information. Hence $E\left[W_t\left(\tau^j\right)\right]$ is replaced by the deterministic value, denoted $W_t^D\left(\tau^j\right)$.

Thus, in any given period, t, a previous tax policy decision combines with stochastic shocks to produce an outcome, $W_t\left(\tau^{t-1}\right)$. In that period, tax policy is reviewed, and possibly changed, thereby affecting expectations of future flows. The latter are approximated by a conditional deterministic projection, $W_j^D\left(\tau^t\right)$, for $j = t+1, ..., T$.

Consider the beginning of the planning period, where an initial value, say $W_1\left(\tau^0\right)$, is given (depending on the inherited tax rate profile, τ_0). The optimal profile of tax rates over the planning period, based on the current value and expected future (conditional) values of welfare, is give by the solution to:

$$\tau^1 = \arg\max\left[W_1\left(\tau^0\right) + \sum_{t=2}^{T}\left(\frac{1}{1+\rho}\right)^{t-1} W_t^D\left(\tau^1\right)\right] \tag{4}$$

where ρ is the discount rate of the policy maker.

In the second period, the welfare, $W_2^h\left(\tau^1\right)$, arises from the particular set, h, of random outcomes. This is in general different from $W_2^D\left(\tau^1\right)$. Optimising again from period 2 gives the vector of subsequent optimal tax rates of:

$$\tau^2 = \arg\max\left[W_2^h\left(\tau^1\right) + \sum_{t=3}^{T}\left(\frac{1}{1+\rho}\right)^{t-2} W_t^D\left(\tau^2\right)\right] \tag{5}$$

In this case, discounting is back to period 2, the date when the plan is revised.

[17] In discussions of option values in the context of uncertain investments, it is usual to suppose that the relevant probabilities are known. Examples in (a highly simplified model of) the present context are discussed by Ball and Creedy (2014). The changing nature of the probabilities, combined with the fact that they are based on non-parametric specifications of the various expenditure and interest rate distributions, rules out the comparable examination of an option value of waiting.

This process can then be extended to a third and subsequent periods, until the end of the complete planning period. The resulting sequence of annual welfare, $W_t\left(\tau^{t-1}\right)$, and tax rates is thus conditional on the precise sequence of random outcomes from the S possibilities. The sequence of optimal tax rates takes the first element of the vector, τ^1, the second element of the vector, τ^2, and so on.

Figure 5: A Sequence of Values

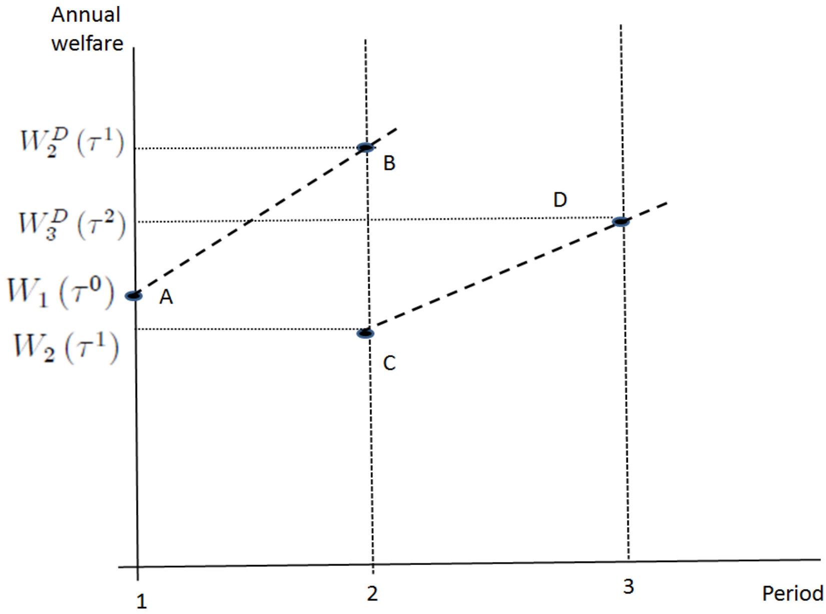


Figure 5 shows an example. In period 1, the known starting position is $W_1\left(\tau^0\right)$, at point A. A tax policy is formed in that period, giving rise to the vector, τ^1. Suppose the deterministic projection is shown by the profile AB. However, in period 2 the actual outcome is $W_2\left(\tau^1\right)$, at point C. This differs from the deterministic projection of $W_2^D\left(\tau^1\right)$, at point B. In the face of this new information (the outcome of the stochastic variables in the second period) a new tax policy, τ^2, is decided for future periods. This give rise to the new deterministic projection of $W_3^D\left(\tau^2\right)$, giving the deterministic profile starting from C, of CD. The process then continues until the end of the planning period. A time profile of tax rates is thus available for a single set of draws from the random variables.

In the present context this involves the following sequence of computations. In the first period, solve for 40 optimal tax rates, using a deterministic 40-year projection and allowing for tax-rate feedback effects on those projections (along with other feedback effects); evaluate the outcomes of random variables in the second period; using those values as ‘starting points’, solve for 39 optimal tax rates, using a 39-year deterministic projection (again allowing for feedback effects); evaluate the outcomes in the third period following random ‘draws’ from relevant distributions; solve for 38 optimal tax rates using a 38-year deterministic projection using the third-period outcomes as ‘initial values’, and so on. There are therefore 39 multi-dimensional optimisation problem to be solved, where the dimensions are gradually reduced from 40, to 39, to 38 and so on. This process gives a single sequence of optimal tax rates over the 40-year period.

The process can be repeated a specified number of times to carry out a Monte Carlo analysis. Each run or iteration is associated with a different sequence of actual outcomes arising from random values of the stochastic variables and thus a different sequence of optimal tax rates. The use of many repetitions gives rise to a distribution of optimal rates in each year of the planning period. The first period of the 40-year optimisation corresponds with the resulting vector of optimal rates obtained using the deterministic version of the model. Hence all sequences of optimal rates in the stochastic model begin from the same tax rate imposed in the first period. The distributions 'fan out' from that initial tax rate. Examples are given in the following section.

6 Optimal Tax Rates in The Stochastic Model

This section reports the results of applying the optimisation strategy outlined above to the 'benchmark case' of the social welfare function used in Section 4. The strategy was illustrated in a simple case using Figure 5. An example of its application to the present model is shown in Figure 6, which gives the path of the optimal tax rate for a single 'run' of the Monte Carlo analysis. The solid line shows the tax rate in each period – obtained by optimising over all subsequent periods, where the deterministic projection (starting from the relevant period) is used to form expectations. The sequence of optimal tax rates associated with such deterministic projections, depending on the draws of the random variables which determine the starting point for each period, is shown by the grey lines in the figure.

Figure 7 shows the profiles of selected quantiles of the distribution of optimal tax rates in each period, resulting from the Monte Carlo process. The distributions are initially symmetric, but gradually become positively skewed, as seen by the distances between median and higher quantiles, compared with the distances for the lower quantiles. The arithmetic mean optimal tax rate profile therefore starts by following the median profile and then rises slightly above it. A comparison of the mean profile with the optimal rate in the deterministic case is shown in Figure 8. These necessarily start at the same point, but the deterministic case gives a slightly smoother profile than the average rate in the stochastic case.

Figure 6: Example of a Sequence of Optimal Tax Rates

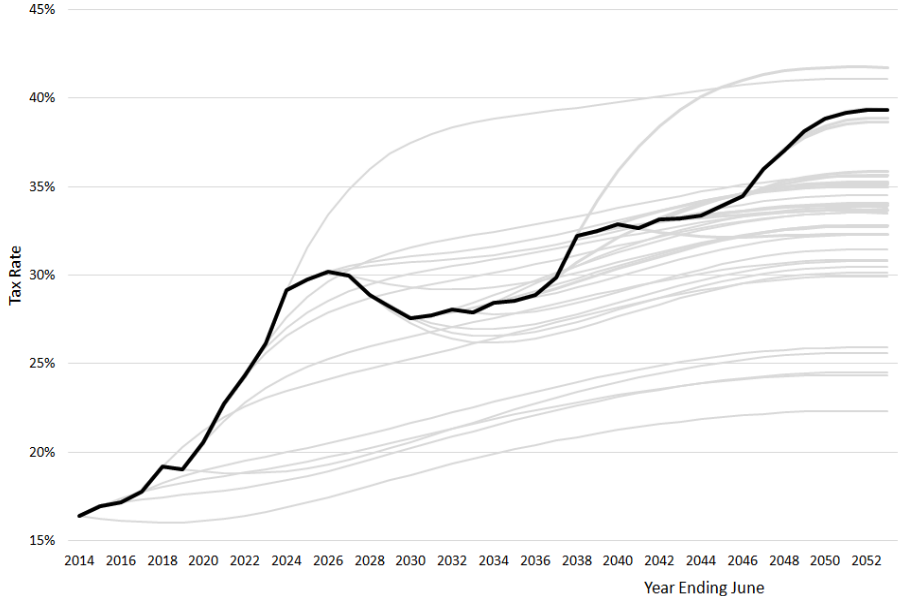


Figure 7: Profiles of Distribution of Optimal Tax Rates

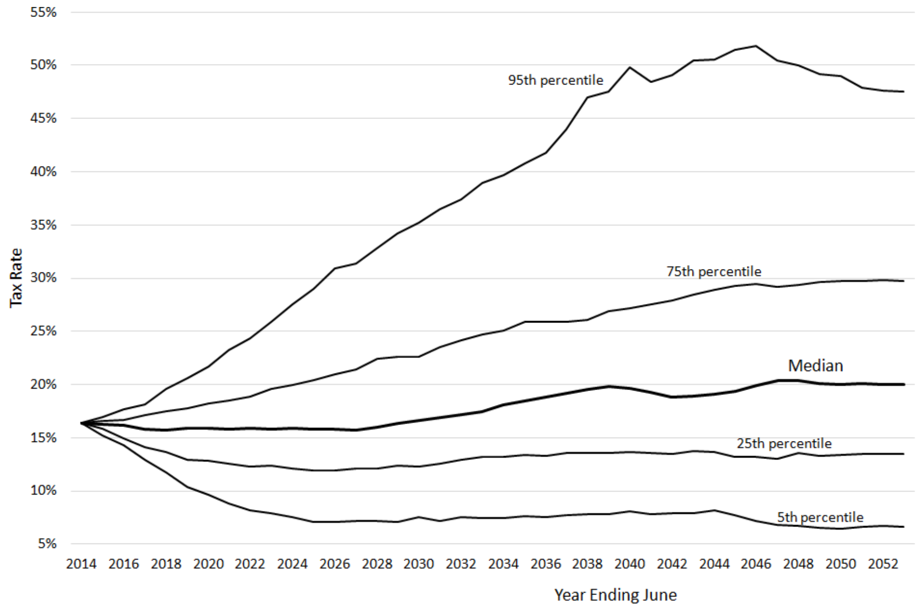
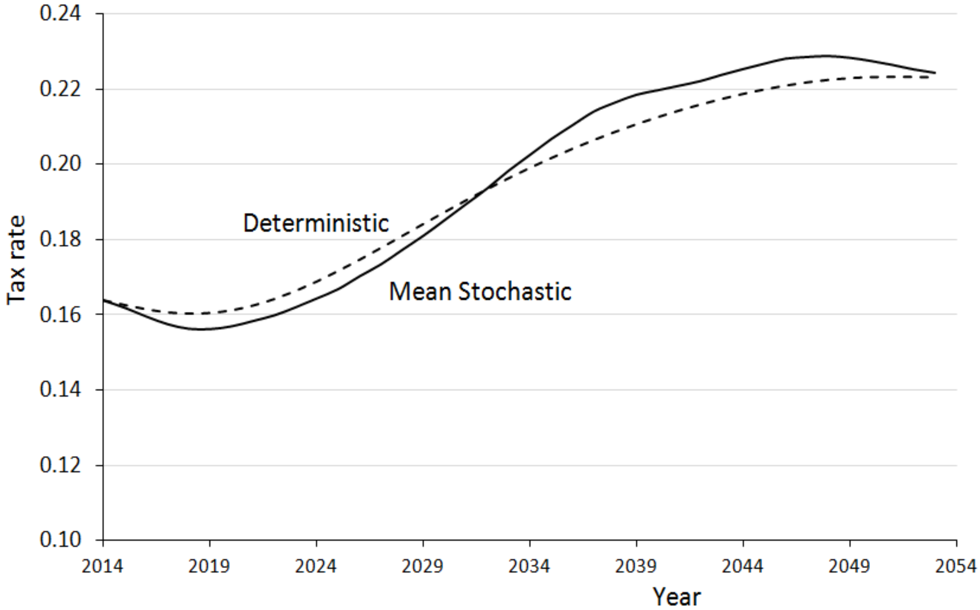
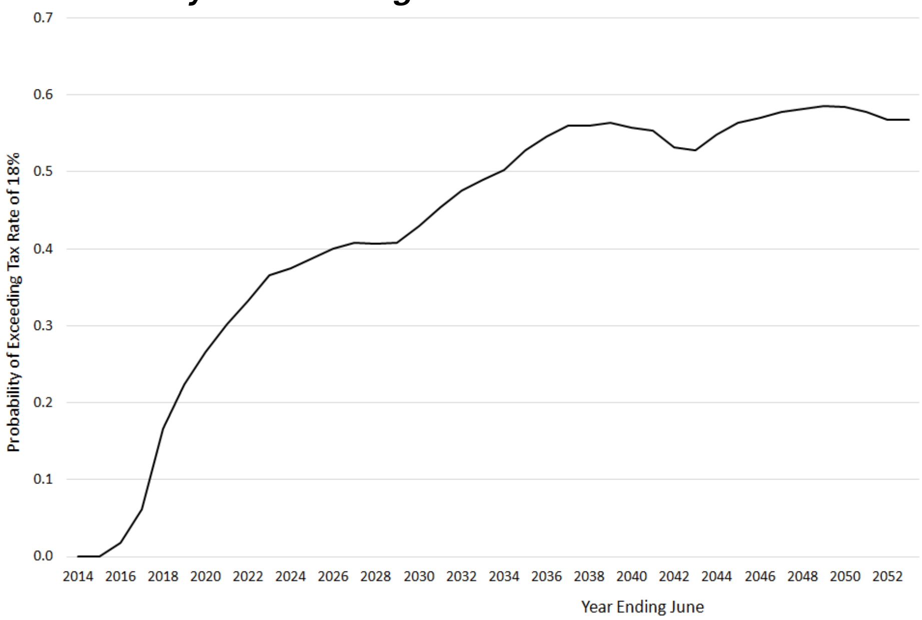


Figure 8: Optimal Tax Profiles: Deterministic and Mean Stochastic Optimal Rates



Given the distributions of optimal rates, it is possible to obtain the probability that the optimal rate exceeds a specified value in any year. For example, the probability that the optimal rate exceeds 18 per cent varies in any year is shown in Figure 9. Clearly this is zero initially, since the starting tax rate is 16.5 per cent. The probability exceeds 50 per cent about half way through the planning period.

Figure 9: Probability of Exceeding tax Rate of 18 Per Cent



Associated with the changing distribution of optimal tax rates over time is a corresponding set of debt ratio distributions. These are summarised in Figure **??**. The probability of being in surplus over the period, along with the probability of being in the high debt range where the risk premium responds sharply, is negligible. Indeed the inter-quartile range for the debt ratio is around 10 to 15 percentage points over most of the period. The debt loss function is symmetric around 20 per cent, and the probability of this ratio being exceeded in any year is shown in Figure 11. This probability begins at 1 because the initial debt ratio is above 20 per cent of GDP, but it falls rapidly before gradually rising and then 'hovering' around 50 per cent.

Figure 10: Distributions of Debt Ratio

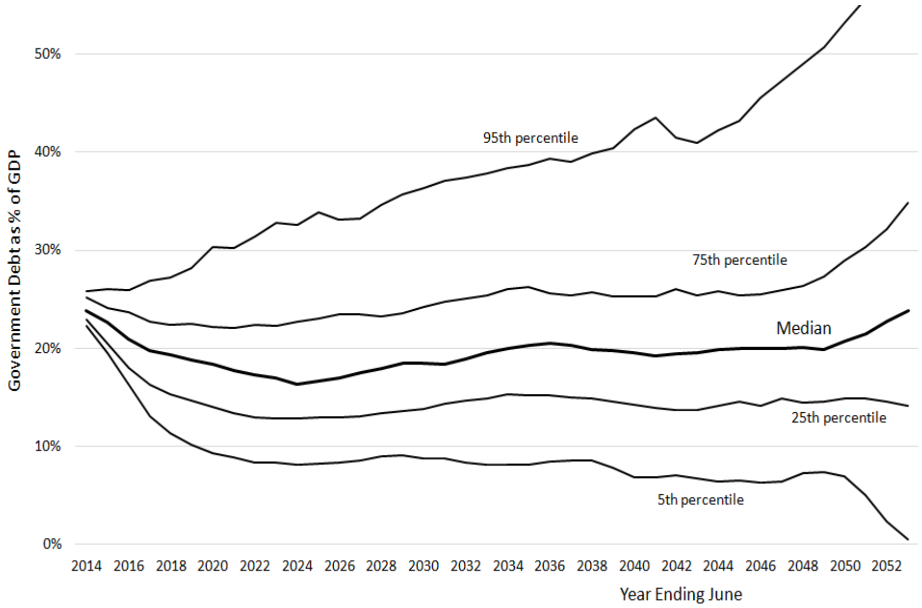
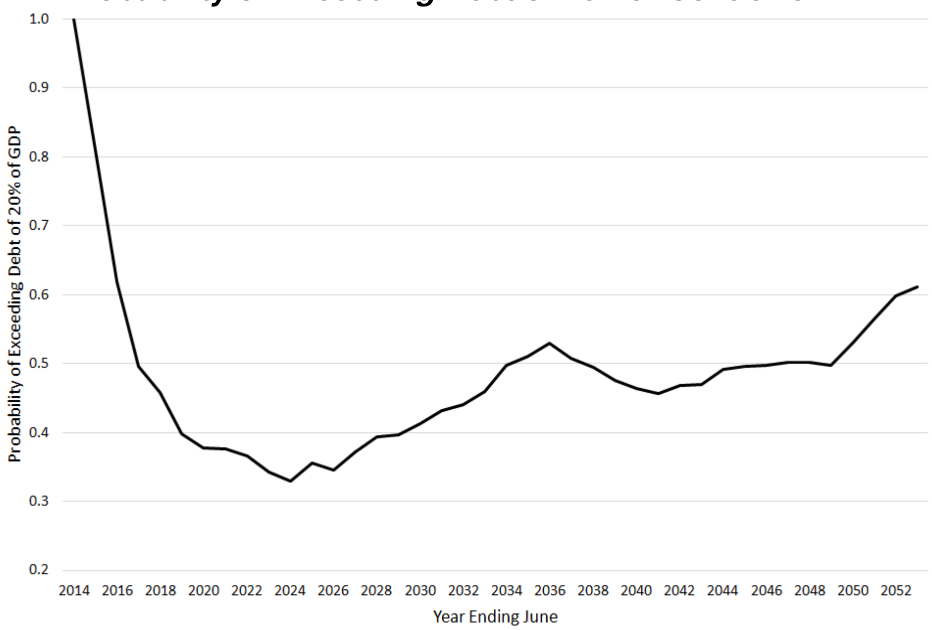


Figure 11: Probability of Exceeding Debt of 20 Per Cent of GDP



7 Conclusions

This paper has considered a difficult problem for those responsible for fiscal policy. Faced with population ageing and a consequent rising and unsustainable ratio of public debt to GDP over a planning period in the absence of any policy changes, some action must be taken to raise revenue and/or reduce expenditure (assuming inflation or default are not feasible strategies). One approach, prompted by the literature on tax smoothing is to immediately raise the tax rate to a level such that, for example, the debt will not increase above some target ratio over the planning horizon. This requires that both the future path of a range of variables and the specification of the underlying model are known with certainty.

In practice, uncertainty is pervasive. The policy maker will be faced with outcomes in every period of the planning horizon which deviate from projected levels. It is far from clear that a tax rate decided at the beginning of the planning period could in practice be left unchanged, allowing for the debt ratio to move well outside the agreed target value. Faced with substantial uncertainty, there may be some value in waiting for some of the uncertainty to be resolved and modifying tax policy over time.

To address these issues, the present paper has computed optimal tax policies which maximise a social welfare function. This function is the present value of an annual welfare index whose arguments are, for each period: an indicator of tax progressivity incorporating welfare spending; real income; a measure of the excess burden of taxation; a cost of deviating from a specified debt target and; a cost of adjusting the tax rate. This is achieved in the context of a small aggregate model of the economy, incorporating key feedback effects. These include the disincentive effects of income tax and the effect of rising debt ratios on the country risk premium and hence interest rates. The model is calibrated in such a way as to duplicate, for the basic case of constant growth rates and no policy changes, the debt track projected by the New Zealand Treasury's Long Term Fiscal Model.

The paper first derived the optimal sequence of tax rates for a forty year planning horizon in the absence of uncertainty. That sequence is one that maximises the value of the social welfare function. It was found that the optimal tax rate falls slightly in the early years: this is because the significant effect of population ageing does not arise until later. The optimal tax rate then rises gradually in each of the subsequent years. The resulting optimal debt never moves into surplus. If tax smoothing, designed to achieve only a final debt target, is imposed, it is found that the loss of welfare is substantial compared with the optimal policy.

For a policy of tax smoothing to be optimal, implying a surplus in the middle years of the planning period, a very special case of the welfare function must be adopted in which there is no cost of changing the tax rate in the first period, the cost of adjusting the tax rate in subsequent periods is high, and only the debt ratio in the final year matters.

With the introduction of uncertainty, a Monte Carlo analysis was carried out whereby, for each 'iteration', an optimal sequence of tax rates was evaluated using a modified decision-making process, since any policy decided at the beginning of the planning period will turn out to be sub-optimal once actual outcomes are experienced. The optimal tax profile for any iteration is computed by allowing revisions in each year, based on the extent to which the actual randomly-generated outcome differs from that which was previously expected. Thus, the policy maker has the option for continuous review and adjustment. The characteristics of the resulting changing distribution of optimal tax rates over the planning period were thereby obtained. The approach also allowed calculation, for each period, of the probability that the optimal tax rate, and resulting debt ratio, exceed some specified value. The time profile of the expected value of the distribution of the optimal rate was found to be slightly less smooth than in the deterministic version of the model where there is no uncertainty.

The type of analysis presented here is clearly an exercise in welfare economics. It has investigated the implications – in cases of certainty and uncertainty – of adopting a particular form of social welfare, or evaluation, function. This function represents the value judgements of a fictitious decision maker. The examination of alternative specifications has shown the extent to which tax smoothing as an optimal policy requires a special set of assumptions in the deterministic case. In the stochastic case, actual future outcomes may make such a policy unsustainable.

It has been seen that decision making is considerably complicated by the existence of uncertainty, even where there is no uncertainty about the model's ability to reflect the real world, and where information is available about relevant distributions. It is no longer possible to form a consistent plan to be followed over a long period. Where initial policy is based on reasonable expectations about the future, a process involving regular policy revisions, in the light of actual outcomes and revised expectations, manages to keep important policy variables within a reasonable range. This is particularly important where feedback effects and interdependencies are relevant. For example, allowing the debt ratio to increase to high levels (thereby raising the risk premium and debt service charges substantially) cannot easily be corrected by substantially increasing taxation (in view of the associated adverse incentive effects and welfare costs). As with all optimal tax models, no value-free policy recommendations can be made on the basis of the simulations reported here. Nevertheless, it is suggested that the modelling provides a useful framework in which alternative value judgements, and assumptions about central economic variables and relationships, can be examined.

Appendix A: Formal Statement of The Model

This Appendix provides a description of the main components of the model. Let D_t denote debt at the end of time period, t, for $t = 1, ..., T$, where D_0 is the debt inherited from the past. If r_t is the domestic interest rate at time t, equivalent to the government bond rate, then the debt servicing cost at time t, denoted d_t, is given by $d_t = r_t D_{t-1}$. The interest rate depends on the world interest rate, r_w, which is assumed to be constant, and a risk premium, $r_{p,t}$,so that $r_t = r_w + r_{p,t}$. Government expenditure includes welfare spending, W_t, which consists of untaxed transfer payments of $W_{B,t}$, received by non-pensioners, and aggregate (untaxed) superannuation benefits of $W_{S,t}$. In practice New Zealand Superannuation is taxable, but allowance for this is made in the calibration of the model. The levels per person are denoted $W^*_{S,t}$ and $W^*_{B,t}$, so that if $N_{S,t}$ and $N_{B,t}$ denote the number in receipt of the pension and welfare benefits respectively, $W_{S,t} = N_{S,t} W^*_{S,t}$ and $W_{B,t} = N_{B,t} W^*_{B,t}$.

All other spending at t is denoted by E_t. This is composed of spending on publicly-provided goods such as health and education, $E_{I,t}$, and other expenditure, $E_{O,t}$, so that $E_t = E_{I,t} + E_{O,t}$. The former may be considered as investment in human capital, while the other expenditure has no direct impact on individuals. As explained below, $E_{O,t}$ is assumed to have no direct impact on the labour supply, and thus incomes, of individuals. While $E_{I,t}$ does not have a direct impact, it influences income via its effect on productivity growth. Variations in these spending categories are produced by variations in per capita amounts, $E^*_{I,t}$ and $E^*_{O,t}$ and variations in the total population, N_t: hence $E_t = N_t \left(E^*_{I,t} + E^*_{O,t} \right)$.

Total government expenditure, G_t, is thus:

$$G_t = W_t + E_t + r_t D_{t-1} \tag{A.1}$$

Define R_t as total tax revenue from direct and indirect taxes, so that debt in period t is:

$$D_t = D_{t-1} + G_t - R_t \tag{A.2}$$

Define $Y_{P,t}$ as total 'potential income' in period t, from labour and rental income. To allow for productivity growth at the rate ρ_t, write:

$$Y_{P,t} = (1 + \rho_t) \, Y_{P,t-1} \tag{A.3}$$

Let L_t indicate the ratio of actual to potential income, so that aggregate income can be written as $Y_t = L_t Y_{P,t}$.

Assume that all forms of income are taxed at the same rate. Then if S_t denotes aggregate savings at time, t, as defined above, these are all assumed to be invested at the going rate, r_t. Capital, K_t, is thus $K_t = K_{t-1} + S_{t-1}$. As this refers to the accumulation of savings, no depreciation is applied. As discussed above, the production side of the economy, including investment and capital accumulation, is not modelled explicitly. Hence aggregate income is:

$$Y_{A,t} = Y_t + r_t K_{t-1} \tag{A.4}$$

For simplicity, this assumes that the borrowing and lending rates are equal, and the same both for the government and individuals. The above specification does not allow for population growth. A simple adjustment is made by raising $Y_{A,t}$ by a proportion that depends on the growth rate, from period $t-1$ to t, of the population above working age.

Suppose that income tax is simply a constant proportion, τ_t, of taxable income. Revenue is also obtained from indirect taxes. Define V_t as indirect tax revenue at t, from a GST/VAT type of system, where v_t is the tax-exclusive rate applied to all expenditure. However, indirect taxes applied to E_t are ignored here since these are netted out in the government's budget constraint. The tax-inclusive indirect tax rate is $v_t/\left(1+v_t\right)$.

Savings, S_t, are made from net income. Assume that all transfer payments, W_t, are consumed. Then if savings are a constant proportion, s_t, of post-tax income, $S_t = s_t\left(1-\tau_t\right)Y_{A,t}$ and indirect tax revenue is:

$$V_t = \left[\frac{v_t\left(1-s_t\right)\left(1-\tau_t\right)}{1+v_t}\right]Y_{A,t} + \left[\frac{v_t}{1+v_t}\right]W_t \tag{A.5}$$

Total tax revenue, R_t, consists of income tax, plus V_t, plus other revenue, $R_{O,t}$. The latter is specified as an amount per capita, $R^*_{O,t}$, which is subject to an exogenous growth rate, along with growth arising from the increase each period in the population above working age. Substituting for V_t in $R_t = \tau_t Y_{A,t} + V_t + R_{O,t}$ gives:

$$R_t = \tau^*_t Y_{A,t} + \left[\frac{v_t}{1+v_t}\right]W_t + R_{O,t} \tag{A.6}$$

where τ^*_t is the overall effective income tax rate, given by:

$$\tau^*_t = \tau_t + v_t\frac{\left(1-s_t\right)\left(1-\tau_t\right)}{\left(1+v_t\right)} \tag{A.7}$$

Hence τ_t^* reflects the combined effect of the income and consumption tax rates.

The risk premium at time t is considered to be a function of $D_{t-1}/Y_{A,t-1} = DR_{t-1}$. Evidence suggests that the risk premium increases only slowly for relatively small values of this ratio, but increases rapidly once it exceeds a value of DR^*; for example, see Ostry *et al.* (2010). The response of the risk premium to debt ratios is also discussed by Fookes (2011) in the New Zealand context. For $DR_{t-1} > DR^*$, suppose:

$$r_{p,t} = \theta_1 + \theta_2 DR_t + \theta_3 \left(DR_t\right)^2 \quad \text{(A.8)}$$

and for $DR_{t-1} \leq DR^*$ the premium increases linearly:

$$r_{p,t} = \theta_1 + \theta_2 DR^* + \theta_3 \left(DR^*\right)^2 - \theta_0 \left(DR^* - DR_{t-1}\right) \quad \text{(A.9)}$$

Suppose the saving rate, s_t, depends on the interest rate. In principle this effect is ambiguous, but it is assumed here that the interest-elasticity of savings is positive. This is reflected in a reduced-form relationship:

$$s_t = \theta_{11} + \theta_{12} r_t \quad \text{(A.10)}$$

with $\theta_{12} > 0$. Furthermore, the savings rate enters into the determination of the effective tax rate, τ_t^*, as shown in (A.7).

To capture adverse incentive effects of the tax and transfer system, suppose the variable, L_t, is a function of the tax rate, so that $L_t = L\left(\tau_t^*\right)$, with $dL_t/d\tau_t^* < 0$. Suppose the elasticity of taxable income, defined with respect to the effective net-of-tax rate, $1 - \tau_t^*$, is constant. Then:

$$L\left(\tau_t^*\right) = \theta_8 \left(1 - \tau_t^*\right)^{\theta_9} \quad \text{(A.11)}$$

Investments in the quality of human capital through both health and education enhance productivity. Suppose changes in ρ depends on previous growth of the per capita public expenditure component, $E_{I,t}^*$. The change in ρ depends on the change ℓ years previously, that is in $E_{I,t-\ell}^*$. If a dot above a variable indicates a proportionate change:

$$\dot{\rho}_t = \frac{\theta_4}{1 + \theta_5 \theta_6^{\dot{E}_{I,t-\ell}^*}} \quad \text{(A.12)}$$

This logistic form captures decreasing returns. If ρ_B is a 'base level' of productivity change:

$$\rho_t = \rho_B \left(1 + \dot{\rho}_t\right) \quad \text{(A.13)}$$

Despite the simplicity of the model, suitable orders of magnitude of many of the variables can be obtained from National Income data and demographic projections. The data sources and values are set out in detail in Creedy and Scobie (2016). Parameter values used for the various functions are listed in Table 2.

Table 2: Benchmark Parameter Values for Functions

Risk premium: For $DR_{t-1} > DR^*$, $r_{p,t} = \theta_1 + \theta_2 DR_{t-1} + \theta_3 \left(DR_{t-1}\right)^2$	
For $DR_{t-1} \leq DR^*$, $r_{p,t} = \theta_1 + \theta_2 DR^* + \theta_3 \left(DR^*\right)^2 - \theta_0 \left(DR^* - DR_{t-1}\right)$	
θ_1	0.026
θ_2	-0.03
θ_3	0.015
θ_0	0.0015
DR^*	1.0
Productivity growth changes: $\dot{\rho}_t = \theta_4 / \left(1 + \theta_5 \theta_6^{\dot{E}^*_{I,t-\ell}}\right)$	
θ_4	0.6
θ_5	35
θ_6	0.00005
ρ_B	0.015
ℓ	5
Incentive effects of taxation: $L\left(\tau_t\right) = \theta_8 \left(1 - \tau_t^*\right)^{\theta_9}$	
θ_8	1.0
θ_9	0.5
Saving rate: $s_t = \theta_{11} + \theta_{12} r_t$	
θ_{11}	0.03
θ_{12}	0.0833

Appendix B: Redistribution

The model has been specified in aggregate (or per capita) terms, with no reference to any form of income distribution. However, a measure of the redistributive effect of the tax and transfer structure can be obtained in terms of the reduction in inequality when moving from pre-tax income to net (after tax and transfer) income. There is clearly some redistribution between periods of the life cycle, because of the tax-financed superannuation. The limited perspective imposed by using a single-period evaluation of redistribution should be borne in mind.

Concentrating on workers, the net (after-tax) income per person, $Y_{N,t}/N_{W,t}$ is given by gross income, $\left(W_{B,t} + Y_t\right)/N_{W,t}$, less income and consumption tax. Hence, since $N_{W,t}$ cancels:

$$Y_{N,t} = (W_{B,t} + Y_t) - \left(\tau_t^* Y_t + \frac{v_t W_{B,t}}{1 + v_t}\right) \tag{B.1}$$

This can be rearranged to give:

$$Y_{N,t} = \left[1 - \frac{v_t}{1 + v_t}\right] W_{B,t} + (1 - \tau_t^*)\, Y_t \tag{B.2}$$

The first term, $\left[1 - \frac{v_t}{1+v_t}\right] W_{B,t} = W_{B,t}/\,(1 + v_t)$, is simply the post-GST value of the transfer payment.

The expression in (B.2) describes a simple linear relationship between net and gross income. From the point of view of individuals, the term, $\left[1 - \frac{v_t}{1+v_t}\right] W_{B,t}$, is fixed. To simplify, consider in general the linear transformation from gross income, y, to net income, z, given by:

$$z = \alpha + (1 - \tau)\, y \tag{B.3}$$

where α is a basic income (transfer payment) and τ is the proportional tax rate. This 'basic income–flat tax' structure is clearly a progressive system. Taking means gives $\bar{z} = \alpha + (1 - \tau)\,\bar{y}$ and taking variances gives $V(z) = (1 - \tau)^2\, V(y)$. Hence, the coefficients of variation of after-tax and before-tax income, η_z and η_y respectively, are related as:

$$\eta_z = \frac{V(z)^{1/2}}{\bar{z}} = \left(\frac{1 - \tau}{\alpha/\bar{y} + 1 - \tau}\right) \eta_y \tag{B.4}$$

or:

$$\eta_z = \left(1 + \frac{\alpha/\bar{y}}{1 - \tau}\right)^{-1} \eta_y \tag{B.5}$$

Letting $P = 1 + \frac{\alpha/\bar{y}}{1-\tau}$, then P represents the degree of progressivity of the system (that is, the extent to which the tax and transfer system is inequality reducing). Higher values of P give rise to lower values of the coefficient of variation of net income. Clearly, this increases as both α and τ increase.

In the context of the present model:

$$P_t = 1 + \frac{W_{B,t}/\,(1 + v_t)}{Y_t\,(1 - \tau_t^*)} \tag{B.6}$$

The second term is the net of GST value of the transfer payment divided by the net of tax value of income, using the overall effective income tax rate, $\tau_t^* = \tau_t + v_t\,(1 - s)\,(1 - \tau_t)\,/\,(1 + v_t)$. Hence, although the model is specified in aggregative (per capita) terms, the progressivity index, P_t, can be considered as a measure of the redistributive effect of the tax and transfer system. The higher is the ratio of the per capita value of welfare spending (transfer payment) to the per capita aggregate gross income from labour and capital, the more redistributive is the system. Substituting for τ_t^* from (A.7) gives:

$$P_t = 1 + \left(\frac{W_{B,t}}{Y_t}\right) \frac{1}{(1-\tau_t)(1+s_t v_t)} \tag{B.7}$$

Hence $\partial P_t / \partial \tau_t > 0$ and $\partial P_t / \partial v_t < 0$. An increase in the uniform indirect tax therefore reduces the redistributive effect of the tax and transfer system in this framework, because of the effect of savings. If $s_t = 0$, then v_t has no effect on P_t, which would be more appropriate with a longer-period income concept (allowing the stock of savings to be consumed). Furthermore, an increase in the saving rate, s_t, also reduces the redistributive effect in this single-period context.

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