

The Role of Value Judgements in Measuring Inequality

John Creedy and Jesse Eedrah

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The Role of Value Judgements in Measuring Inequality

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Abstract

When measuring inequality, value judgements are imposed, often implicitly, at several stages. In particular, the choice of 'welfare metric', adult equivalence scale, unit of analysis and the inequality measure itself cannot avoid value judgements. This paper illustrates the effects of using different distributions and summary measures, using New Zealand data for the period 2007 to 2011. Using an annual accounting period, alternative welfare metrics and units of analysis are investigated. In addition, the sensitivity to assumptions about economies of scale within households is examined, and changes in inequality are decomposed into those arising from population and tax structure changes. When considering the period 2007 to 2010 all measures agree that inequality fell, although the extent of the reduction varies. For the period 2007 to 2011 (after the tax reforms of 2010) the answer to the question of whether inequality in New Zealand has risen or fallen depends crucially on the combination of welfare metric, income unit, adult equivalent scale and inequality measure used. In empirical studies it is therefore important to explore a wide range of alternative approaches, providing information for readers to make their own judgements.

Executive Summary

When measuring income inequality, value judgements are imposed, often implicitly, at several stages. Judgements about income distribution changes may well depend on mobility characteristics and thus income measured over a longer period, but even when using annual measures difficult choices must be made regarding precisely what is to be measured and the unit of analysis. The former choice concerns what is often referred to as the ‘welfare metric’. For example, this may be pre-tax incomes, wage rates, or a measure of expenditure or consumption. The unit of analysis could be the family, the household, or the individual. Both the welfare metric and the income unit could be artificial measures, designed to allow for differences in the composition of households and using adult equivalence scales.

The main argument of the present paper is that it is important for empirical studies to provide a range of clearly described alternative results, thereby allowing readers to make their own judgements. The importance of examining alternative distributions and measures is illustrated by considering recent changes in inequality in New Zealand and the contribution of the tax and transfer system in reducing inequality. The analysis makes use of the Treasury’s microsimulation model, Taxwell, which is based on Household Economic Survey data. The accounting period is thus necessarily a year.

The implications of alternative inequality measures of inequality are illustrated using Gini and Atkinson measures, where the latter also allow for a range of degrees of aversion to inequality. These measures imply very different value judgements regarding income inequality. In addition, adult equivalence scales involve value judgements, although these are often given cursory treatment in empirical studies. A measure of income per adult equivalent is often used in combination with the individual as unit, although this can imply a preference for more inequality if household size is strongly positively related to income and there are substantial economies of scale in consumption within households. The use of the ‘equivalent adult’ as the income unit is thus also investigated. The sensitivity of results to assumptions about economies of scale within households is examined.

When considering the period 2007 to 2010 all measures agree that inequality fell, although the extent of the reduction varies. For the period 2007 to 2011 (after the tax reforms of 2010) the answer to the question of whether inequality in New Zealand has risen or fallen depends crucially on the combination of welfare metric, income unit, adult equivalent scale and inequality measure used.

When decomposing changes in inequality into tax and population components, it is found that for all disposable income distributions and inequality measures, the effect of the tax and transfer changes between 2007 and 2011 was to increase inequality of disposable incomes slightly. However, the population structure changes had the effect, in all cases, of reducing measured inequality. The overall effect on inequality (depending on whether the population component outweighed the tax change effect) is found to depend on the inequality measure used.

A complex phenomenon such as inequality within a heterogeneous population does not allow simple unambiguous comparisons. It is all too easy for researchers, often implicitly, to impose their own value judgements or not to be aware of some of the complexities involved. For many years the theoretical literature on inequality measurement has attempted to untangle the many complexities and implications of alternative value judgements, yet too often empirical studies have ignored the subtleties. An aim of the present paper is thus to persuade empirical researchers and policy analysts to investigate and report results for a wider range of distributions and comparisons.

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The Role of Value Judgements in Measuring Inequality

1 Introduction

When measuring income inequality, value judgements are imposed, often implicitly, at several stages. The vast majority of studies use an annual accounting period, although judgements about income distribution changes may well depend on mobility characteristics and thus income measured over a longer period. The present paper is limited to annual measures, but difficult choices must then be made regarding precisely what is to be measured and the unit of analysis. The former choice concerns what is often referred to as the 'welfare metric'. For example, this may be pre-tax incomes, wage rates, or a measure of expenditure or consumption. Here no attempt is made to allow for the value of leisure.¹ The unit of analysis could be the family, the household, or the individual. Both the welfare metric and the income unit could be artificial measures, designed to allow for differences in the composition of households and using adult equivalence scales. It is argued that it is important for empirical studies to provide a range of clearly described alternatives results, thereby allowing readers to make their own judgements.

The importance of examining alternative distributions and measures is illustrated by considering recent changes in inequality in New Zealand and the contribution of the tax and transfer system in reducing inequality. The analysis makes use of the Treasury's microsimulation model, Taxwell, which is based on Household Economic Survey data.

Section 2 describes the range of distributions examined, distinguished by welfare metric and income unit. The value judgements involved in choosing alternative units are discussed. Section 3 describes the inequality measures used, namely the Atkinson and Gini measures, paying particular attention to the value judgements that are explicit, and in some cases implicit, in the use of these measures. The implications of including zero values in the distributions are also examined briefly. The data and construction of alternative distributions are explained in Section 4. Inequality measures for NZ in 2007, 2010 and 2011 are compared in Section 5. The period from 2007 to 2010 covers years which may be thought to be substantially affected by the global financial crises. However, there were few changes in the tax structure. Major reforms took place in 2010, so comparisons involving 2011 are of interest. The sensitivity of results to the assumption regarding economies of scale

¹ Furthermore, no attempt is made to allow for changes over time, such as the introduction of new commodities, or relative price changes which may have differential impacts on different income groups.

within households is also examined in Section 5. A decomposition of inequality changes into population and tax structure changes is presented in Section 6. In view of the timing of the tax reforms, the empirical decompositions are examined for the period 2007 to 2011. Brief conclusions are in Section 7.

2 Alternative Distributions

This section describes a range of possible distributions, depending on the welfare metric and unit of analysis chosen.

2.1 Thirteen Distributions

The distributions examined here are listed in Table 1, using distinctions between the welfare metric, the unit of analysis and the use of intra-household sharing rules. The final column of the table gives the ‘population’ size, where H is the number of households, N is the total number of individuals, N_E is the number of ‘adult equivalent’ persons, and N_W is the total number of employed individuals. In the table, the first eight distributions listed relate to a welfare metric based on some kind of household income measure. Five distributions with metrics based on individual incomes are then listed. The sequence by which the distributions are constructed is described at the end of this subsection.

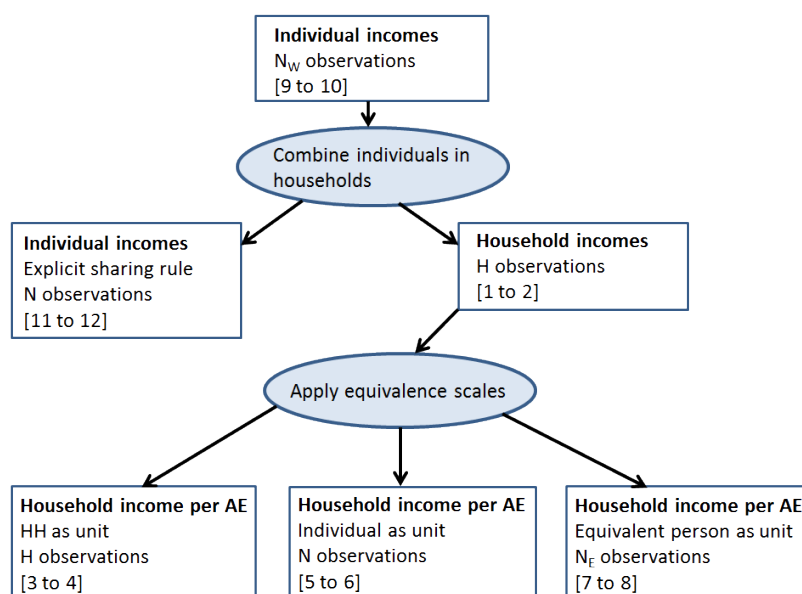
Table 1: Alternative Distributions

No.	Welfare metric	Unit	Sharing	Zeros	No.
1	HH market income	Household	NA	Yes	H
2	HH disposable income	Household	NA	No	H
3	HH market income per AE	Household	NA	Yes	H
4	HH disposable income per AE	Household	NA	No	H
5	HH market income per AE	Individual	Equal	Yes	N
6	HH disposable income per AE	Individual	Equal	No	N
7	HH market income per AE	Equiv indiv	Equal	Yes	N_E
8	HH disposable income per AE	Equiv indiv	Equal	No	N_E
9	Individual market income	Individual	No	No	N_W
10	Individual disposable income	Individual	No	No	N_W
11	Individual market income	Individual	Yes	Yes	N
12	Individual disposable income	Individual	Yes	No	N
13	Individual final income	Individual	Yes	No	N

Considering households, the simplest cases are distributions 1 and 2 in Table 1 which refer to total household market and disposable income respectively, for each of the H households. While some households may have no market income, the income transfer system ensures that all disposable incomes are positive. The simplest case involving the distribution of individual market incomes is number 9, where the population consists only of those N_W individuals participating in paid employment or profitable self-employment, or who receive income from other sources such as rental, interest and capital income.² This distribution contains no zero values. It may be compared with distribution number 10, that of disposable income for the same population of individuals. In comparing individuals, comparisons of inequality measures usually assume (that is, take the value judgement) that there are no non-income differences that are relevant: the units are homogeneous.³ Given that standard inequality measures are designed to deal with homogeneous units, no special problems arise.

However, distributions 1 and 2 involve units which are heterogeneous. The nature of households, their composition and the way resources are shared among members, need to be considered explicitly. There is a complex relationship between the distribution of market incomes and inequality in the distribution of resources as more widely perceived. A challenge is to construct distributions which in some way transform household incomes so that suitable comparisons can be made. This is where difficulties arise, not only because of the role of value judgements but because comparisons can involve artificial income concepts (such as income per adult equivalent person) and artificial units (such as the adult equivalent income unit). These aspects are examined in the following two subsections.

Figure 1: The Sequence of Distributions



² Of course, distributions of these sources may be considered separately, but are combined here, as discussed below.

³ As mentioned earlier, in common with most studies no allowance is made here for utility from leisure.

Figure 1 illustrates the sequence for compiling the various distributions, moving from the distribution of individual market incomes to household incomes and their transformation. The numbers in square brackets within each box refer to the distribution numbers in Table 1. For example, [1 to 2] indicates that comparisons involve the movement from pre-tax household income to post tax and transfer household incomes.

2.2 Adult Equivalence Scales

A common method of dealing with the heterogeneity of households, given only market and disposable incomes, is to make comparisons on the basis, not of actual incomes, but of an artificial income construct which reflects the differences in the demographic structure of the households. A simple way to obtain an individual-level income is clearly to assume an equal division within the household and divide total income by the number of individuals in the household. But the view is widely taken that not all members of the household have the same consumption needs. Furthermore, there may be economies of scale within a household. The latter can arise because some goods (including some durables and goods like heating and lighting) may be ‘public goods’ within the household and can therefore be consumed simultaneously by several people. In addition, there may be economies from purchasing larger quantities of some goods. Instead of dividing total household income by the number of people in the household (irrespective of their ages or gender), a measure of adult equivalent household size is obtained using a set of adult equivalence scales. However, this approach continues to assume equal sharing, but among adult equivalents.

Some people may object to the use of such scales. Those who take this view may, for example, object to treating children in terms merely of a cost or burden faced by parents, rather than as a desired benefit or advantage. They may consider household structure, fertility decisions, household production and market income as jointly determined.

A simple but flexible adult equivalence scale is the following.⁴ Let n_a and n_c denote respectively the number of adults and children in the household, and let m denote the adult equivalent size of the household. Then:

$$m = (n_a + \theta n_c)^\alpha \quad (1)$$

where θ and α are parameters reflecting the relative ‘cost’ of a child and economies of scale respectively. This form was introduced by Cutler and Katz (1992) and investigated by, for example, Banks and Johnson (1994) and Jenkins and Cowell (1994). Creedy and Sleeman (2005) found that, despite its simplicity, it provided a close fit to 29 alternative sets of equivalence scales. Having obtained the adult equivalent size of each household, it is a simple matter to calculate the total income per adult equivalent person.

⁴ This parametric form was introduced by Cutler and Katz (1992) and investigated by, for example, Banks and Johnson (1994) and Jenkins and Cowell (1994). Creedy and Sleeman (2005) found that it provided a close fit to 29 alternative sets of equivalence scales.

2.3 The Unit of Analysis

Given the artificial welfare metric of income per adult equivalent, comparisons then depend on the choice of unit of analysis in combination with this metric. The choice is not as straightforward as has often been assumed. In fact, three further pairs of distributions may be considered. First, comparisons can be made using the household as the unit of analysis: this gives distributions 3 and 4 in Table 1. Second, perhaps the simplest and most natural choice is to make comparisons using the individual as the unit of analysis: this gives distributions 5 and 6. Third, as first suggested by Ebert (1997), the unit of analysis could be the 'equivalent adult', giving distributions 7 and 8 in Table 1.

When using the individual as the unit of analysis, each person 'counts for one' irrespective of the household to which they belong. Inequality remains unchanged when one person is replaced by another person with the same metric (income per adult equivalent) but belonging to a different type of household. It thereby satisfies an 'anonymity principle'. However, it does not necessarily satisfy the 'principle of transfers'. This principle is the inequality-disliking value judgement which takes the view (in the context of homogeneous individuals) that an income transfer from a richer to a poorer individual (which leaves the relative rank of the two people unchanged) reduces inequality and represent an improvement. But if the existence of large economies of scale means that rich large households are highly efficient at generating welfare (in terms of the choice of this metric), it is possible, when using the individual as unit, for evaluations to be inequality-preferring. This was first shown by Glewwe (1991) and the welfare aspects are examined by Shorrocks (2004).

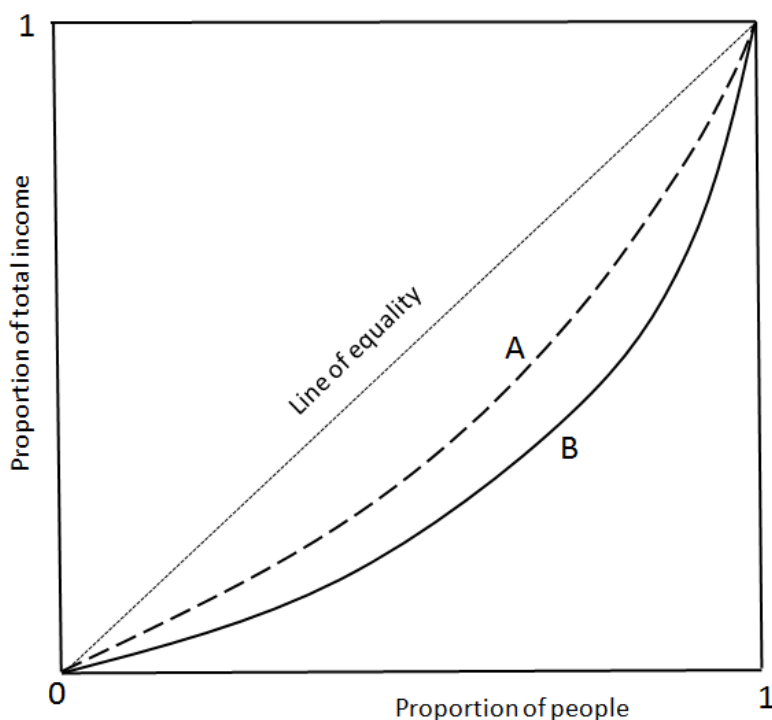
The third possibility uses the equivalent adult as the income unit. This artificial income unit is thus combined with its corresponding artificial income measure, income per adult equivalent. In this case there are not necessarily integer numbers of equivalent adults (except for single-adult households). Thus the distributions cannot be written simply as vectors. The equivalent adult size must be treated as a household weight in obtaining inequality or other measures.

The use of the artificial equivalent adult as the unit of analysis means that the income unit and the income concept are treated consistently. Each individual's contribution to inequality depends on the demographic structure of the household to which that individual belongs. Thus an adult in a one-person household 'counts for one'. But an adult counts for 'less than one' (has a weight less than 1) when placed in a multi-person household.

The use of this income unit is consistent with the principle of transfers. This can be useful because there are general results linking this value judgement to Lorenz curves. This widely-used graphical device, having ordered all incomes from lowest to highest, plots the cumulative proportion of people against the corresponding cumulative proportion of total income. Consider the two Lorenz curves shown in Figure 2. The two distributions are assumed to have the same arithmetic mean income. Suppose the curve of one distribution, say A, lies everywhere inside that of the other distribution, say B: that is, A's curve is closer to the upward sloping

diagonal line of equality which arises if all incomes are equal. Distribution A is said to 'Lorenz dominate' distribution B. The intuitive judgement that distribution A is unequivocally less unequal than distribution B is consistent with the value judgement expressed by the principle of transfers, defined above. That is, it is possible to move from distribution B to distribution A by a series of transfers satisfying the 'principle of transfers'.

Figure 2: Two Lorenz Curves



One way that distributions can be evaluated – that is, the explicit statement of value judgements imposed in making comparisons – is as follows. For a distribution x_i , for $i = 1, \dots, n$, suppose the evaluation function – representing the value judgements of an independent judge – takes the form, $\sum_{i=1}^n U(x_i)$, where $U(x_i)$ is a function representing the contribution of individual i 's income to W .⁵ The basic value judgements shared by all judges whose W functions take this form are that evaluations are individualistic (it depends on the incomes of all individuals), additive (can be expressed as a 'weighted' sum over all individuals) and Paretean (such that an improvement for any one unit, with no units being worse off, is judged to increase W). Suppose further that $U(x)$ is concave, so that the slope of the function gets smaller as x increases, such that for $x_k > x_j$, $dU(x_k)/dx_k < dU(x_j)/dx_j$. This additional assumption reflects the further value judgement of adherence to the principle of transfers (a transfer from k to j must increase W), where the degree of concavity reflects the extent of aversion to inequality. It has been established that all functions of this general kind would

⁵ It is tempting for economists to think of $U(x)$ as representing a (cardinal) utility function, assumed to be the same for all individuals. The case where $U(x) = x$ (implying no aversion to inequality) thus corresponds to the 'Classical utilitarian' case. However, it is best to think of $U(x)$ as simply representing the contribution of x to W , reflecting the independent judges' views. However, in the context, for example, of optimal income taxation, x_i is viewed as individual i 's (cardinal) utility from net income and leisure.

judge distribution A to be ‘better’ than B in that it gives a higher value of W . This result is true irrespective of the precise extent of aversion to inequality; for details and elaborations, see Lambert (2001). If Lorenz ‘dominance’ is established, then clearly all judges who have these basic value judgements would agree about which distribution is preferred to the other, irrespective of their precise aversion to inequality.⁶

Importantly, reliance on these strong results can only be made in the case of comparisons using the equivalent person as the unit of analysis. It cannot be assumed that comparisons are insensitive to the choice of income unit. Indeed, it is quite possible for a tax reform to be judged differently, changing inequality and welfare comparisons in opposite directions, when using the individual and the equivalent adult as income units, as shown below. Further examples are given in Decoster and Ooghe (2003) and Creedy and Scutella (2004).

2.4 The Use of Allocation Rules

In the previous subsection the welfare metric was based on an assumption of equal sharing among equivalent adults within the household. Further distinctions were then made depending on the choice of income unit. Yet another approach is to begin by using an explicit sharing rule to allocate income to individual members of each household. Instead of taking total market income in a household, or individual market income for those with positive values, household income is considered to be shared among all those in the household.⁷ The particular sharing rule used may be based on special surveys which provide information about income sharing, or it may be rather more *ad hoc*. Suppose that the allocation rule is based on an additive household size, s , defined as:⁸

$$s = 1 + 0.5(n_a - 1) + 0.3n_c \quad (2)$$

Hence the first adult is given a weight of 1, while all other adults are given a weight of 0.5 and all children are given a weight of 0.3. This type of explicit income-sharing rule is naturally associated with the use of the individual as the income unit. The use of this sharing rule gives rise to distributions 11 and 12 of Table 1.⁹

In addition to comparisons involving market and disposable incomes, ‘fiscal incidence’ studies go further and attempt to allocate some components of government

⁶ If the arithmetic means of the two distributions differ, the same result applies instead to the concept of the Generalised Lorenz curve: this plots the product of the proportion of total income and the arithmetic mean income against the corresponding proportion of people. Thus the vertical axis of the Lorenz curve is ‘stretched’ by an amount depending on the arithmetic mean.

⁷ In the empirical analysis reported below, when this sharing rule is applied, sharing is actually restricted to family members within a household.

⁸ This formulation actually corresponds to the modified OECD equivalent scale, which does not allow for economies of scale within households.

⁹ Distribution 11 shares market income. An additional alternative distribution would be to consider individual market incomes as in distribution 9 but with an additional $N - N_W$ zero values. Comparisons with distribution 12 would then combine the effects of sharing and taxes and transfers.

expenditure to individuals. In particular, health expenditure can be allocated based on age, gender and summary information about individuals' use of publicly financed health services. Similarly primary, secondary and tertiary education expenditure can be allocated to individuals based on age.¹⁰ This gives distribution 13 in Table 1.

The discussion has so far been in terms of distributions of market and disposable incomes. Some household surveys contain detailed information about household expenditures, and this can be used to compute an additional metric, that of disposable income after the deduction of indirect taxes. However, if – as in New Zealand – there is a broad-based goods and services tax applied at a uniform rate, combined with limited excises (for example, on tobacco, alcohol and petrol), the allocation is straightforward and involves an approximately proportional change. Hence, indirect taxes are ignored in the comparisons reported here.¹¹

3 Inequality Measures

This section briefly defines the inequality measures used below, the Atkinson and Gini measures, and describes the value judgements associated with each measure. It is also necessary to pay attention to the fact that some of the distributions described in the previous section have zero incomes. The formulae given are for unweighted distributions, but it is necessary to use weights to deal with both the sample weights (for aggregation to population values) and the cases where the welfare metric is household income per adult equivalent person and the income unit is either the individual or the equivalent adult. This presents no special problems for Atkinson measures, but care is needed when dealing with Gini measures, where the rank order of units is important; see the Appendix for treatment of the use of weights with the Gini.¹²

The Atkinson measure, for a relative inequality aversion parameter of ε , is defined as the proportional difference between the arithmetic mean and the 'equally distributed equivalent' income. The measure is based on a 'social welfare function', representing the value judgements of an independent observer, of the form:¹³

$$W = \frac{1}{n} \sum_{i=1}^n \frac{x_i^{1-\varepsilon}}{1-\varepsilon} \quad (3)$$

¹⁰ For details of an attribution process, see Aziz *et al.* (2013, Appendix C).

¹¹ However, when the incidence of education and health is examined in distribution number 13, Goods and Services Tax is deducted before those components are added.

¹² Even with integer number of individuals (where adjustments can easily be made to the ranks used), it is appropriate to treat the number of income units as weights. Indeed, weighted measures are also needed to deal with sample weights provided in cross-sectional surveys to enable population level values to be obtained.

¹³ Reference is sometimes made inappropriately to 'society's aversion to inequality'.

for $\varepsilon \neq 1$, and incomes of x_i , for $i = 1, \dots, n$. If $\varepsilon = 1$, then $W = \frac{1}{n} \sum_{i=1}^n \log x_i$.¹⁴ The equally distributed equivalent, x_{ede} , is that income level which, if obtained by every unit, gives the same ‘total welfare’ as the actual distribution; hence x_{ede} is the power mean:

$$x_{ede} = \left(\frac{1}{n} \sum_{i=1}^n x_i^{1-\varepsilon} \right)^{1/(1-\varepsilon)} \quad (4)$$

Then for arithmetic mean of \bar{x} , the Atkinson measure, A_ε , is:

$$A_\varepsilon = 1 - \frac{x_{ede}}{\bar{x}} \quad (5)$$

From the form in (3), it is clear that this is a member of the broad class of welfare functions that are individualistic, additive, Paretean, and satisfy the principle of transfers. It is obviously possible to modify the form of $U(x)$ to allow, for example, for constant absolute inequality aversion rather than constant relative aversion, but for convenience the latter specification is used here. From (5), $x_{ede} = \bar{x}(1 - A_\varepsilon)$, which expresses the equally distributed equivalent income in terms of \bar{x} and A_ε . Hence the value of W corresponding to the distribution can be written as $x_{ede}^{1-\varepsilon} / (1 - \varepsilon) = \{\bar{x}(1 - A_\varepsilon)\}^{1-\varepsilon} / (1 - \varepsilon)$. This reflects exactly the same ‘trade-off’ between equality, $(1 - A_\varepsilon)$, and mean income as x_{ede} itself. Hence, the welfare function associated with the Atkinson measure can be expressed in ‘abbreviated’ form as $W = \bar{x}(1 - I_x)$. Instead of writing W , as in (3), in terms of all individual incomes, the abbreviated form is expressed in terms of summary measures of the distribution; see Lambert (2001). The nature of the trade-off is an important implication of the basic value judgements underlying the use of the Atkinson measure.

The column headed ‘zeros’ in Table 1 indicates whether it is possible to have any incomes equal to zero in the relevant distribution. For example, any distribution that is concerned with market income and includes non-workers (and those without other income sources) can have income units with zero income. In these cases, care must be taken in using and interpreting Atkinson inequality measures. To illustrate a difficulty in the presence of zero values, suppose there are n individuals with incomes of $[0, 1, 1, \dots, 1]$, so there is only one unit with a zero value and the rest have equal incomes of 1 unit, and let $\varepsilon = 0.5$. The equally distributed equivalent income is thus $x_{ede} = \left(\frac{n-1}{n}\right)^2$. The arithmetic mean is $\bar{x} = \frac{n-1}{n}$, so that $A_{0.5} = 1/n$. Hence for large n inequality is zero. However, for all $\varepsilon \geq 1$, inequality is 1.

These properties may be compared with the Gini inequality measure. Geometrically, this can be regarded as a ‘distance measure’ of the difference between the Lorenz curve of the distribution from the line of equality in Figure 2. A commonly used expression for the Gini inequality measure, G , for $x_1 < x_2 < x_3 < \dots < x_n$, is:

$$G = 1 + \frac{1}{n} - \frac{2}{n^2 \bar{x}} \sum_{i=1}^n (n+1-i)x_i \quad (6)$$

Clearly for the distribution, $[0, 1, 1, \dots, 1]$, G tends to zero as n increases, and a Gini value of 1 results from $[0, 0, 0, \dots, 0, 1]$. The value judgements associated with

¹⁴ The expression in (3) is usually used though strictly the numerator is $x_i^{1-\varepsilon} - 1$, for continuity with the case where $\varepsilon \rightarrow 1$.

the use of the Gini measure are very different from those underlying the Atkinson measure. Indeed, it can be shown that the Gini measure cannot be consistent with any individualistic, additive, Paretean welfare function, W . However, an interpretation in terms of value judgements is discussed below. First, further insight into the Gini can be obtained by defining \bar{x}_R as an ‘reverse-order-rank-weighted mean’ of x_i , given by:

$$\bar{x}_R = \frac{\sum_{i=1}^n (n+1-i) x_i}{\sum_{i=1}^n (n+1-i)} \quad (7)$$

That is, each value is given a weight given by its ‘reverse rank’ (that is, its rank when in descending order – ordered from rich to poor, rather than poor to rich). Using $\sum_{i=1}^n i = n(n+1)/2$, it can be seen that:

$$G = \frac{1+n}{n} - \frac{n(n+1)}{n^2} \left(\frac{\bar{x}_R}{\bar{x}} \right) \quad (8)$$

For large samples this reduces to:

$$G = 1 - \frac{\bar{x}_R}{\bar{x}} \quad (9)$$

Hence the Gini has some superficial similarities with the Atkinson measure: both measures can be expressed as the proportional difference between the arithmetic mean income and another type of average, or measure of location. A further similarity arises when the value judgements underlying the Gini measure are considered, although they are very different from those associated with the Atkinson measure. Suppose the contribution to W , the evaluation function of the independent judge, from any pair of individual incomes is equal to the smallest income of the pair. Hence the values implicit in the use of the Gini measure contain a ‘maximin’ kind of idea – that only the lowest income matters in all pairwise comparisons. It can be shown that the average welfare across all pairs of individuals is $\bar{x}(1-G)$, which is of course a welfare function expressed in abbreviated form.

The similarity in terms of abbreviated welfare functions means that, for both Atkinson and Gini measures, the form of the trade-off between average income and its inequality is similar for each case, although of course the magnitudes can differ substantially. Consider a ‘social indifference curve’, showing combinations of \bar{x} and inequality, I_x , for which W is constant. By differentiating the abbreviated forms, the slope of such an indifference curve is given for each measure by:

$$\left. \frac{d\bar{x}/\bar{x}}{dI_x/I_x} \right|_W = \frac{I_x}{1-I_x} \quad (10)$$

This shows that, implicit in the values behind the use of these measures, a proportional change in inequality of $\Delta I_x/I_x$ is viewed as being equivalent to a proportional change in \bar{x} of $\Delta I_x/(1-I_x)$.

In addition, these inequality measures are defined only for $x_i \geq 0$. Although any attempt to include negative values in calculating Atkinson measures immediately runs into difficulties, a value of G can mechanically be obtained, leading some investigators to overlook the fact that Ginis should not include negative incomes.¹⁵ For example, the distribution $[-2, 1, 1, 3]$ has an arithmetic mean of $\bar{x} = 3/4$ but a value of $\bar{x}_R = 0$. Substitution into (8) gives $G = 5/4 = 1.25$.

¹⁵ Among New Zealand studies, Hyslop and Yahanpath (2005, p. 7) compute Ginis including negative values.

4 Construction of Distributions

The data used here were obtained from the Household Economic Survey (HES) for the years 2006/07, 2009/10 and 2010/11. Each year's survey runs from July to June of the next year and contains detailed information about incomes and household characteristics for approximately 8000 individuals, grouped by households. This is sufficient to enable the calculation of market income, welfare benefits and direct and indirect taxes. Each individual in the survey is assigned a weight which makes it possible to aggregate from the sample to population values.¹⁶

For the purposes of applying the tax and transfer system, 'Economic Family Units' (henceforth referred to as 'families') were constructed. A family is defined as a person, a partner (if relevant), and any children under the age of 19 who is not in full time employment (henceforth referred to as 'dependent children'). This construction is required to model a significant proportion of the New Zealand tax and transfer system, such as Working for Families and the core benefits.

From these data, a measure of market income was obtained for each individual. It is possible for some individuals to have negative market incomes, almost exclusively through negative self-employed income or large capital market losses. As discussed in the previous section, it is invalid to include negative values in the calculation of the Gini and impossible to include in the Atkinson measure, and neither of the reasons for negative income provide a strong indication where they should lie in the income distribution. Hence they were removed from the sample.¹⁷

The New Zealand tax and transfer system was then applied to the data by using Treasury's non-behavioural microsimulation model, Taxwell. This incorporates the majority of the rules of the tax and transfer system, including: Direct tax and ACC levies; the core and supplementary benefits; Independent Earner and Working for Families tax credits; New Zealand Superannuation; the Accommodation Supplement. Of relevance here are earlier policies, such as the Low Income Earner Rebate, the Child Taxpayer Rebate, and Transitional Tax Allowance. Many of these policies need information at the family or household level to calculate entitlements, but all amounts are attributed back to the eligible individual. An assumption is made that the receiver of Working for Families tax credits is the adult rather than the child, and in the case of partnered adults it is attributed to the partner with the least amount of market income, assumed to be the primary caregiver of any dependent children. Thus, processing by Taxwell gives the components of disposable income at the individual level. While Taxwell works with the New Zealand system, many of the same types of policies and issues arise when applying other countries' systems and it is straightforward to apply these.

¹⁶ The weights used for this study are taken from Taxwell, rather than Statistics New Zealand's HES weights.

¹⁷ Some investigators set negative values to zero, rather than removing them from the sample.

In the New Zealand tax and transfer system there are two benefits – the student allowance and the accommodation supplement – where a cash payment is tied to a particular choice the person makes (regarding decisions about study and location and type of housing). Both benefits are included in the calculation of disposable income. A scheme similar in purpose to the Accommodation Supplement, the Income Related Rental Subsidy, is not a cash payment and was instead treated as an ‘in-kind’ payment.

This process produces disposable income at an individual level. Those with zero or negative disposable incomes are excluded. This may arise in cases of retired families living entirely from savings, or sole students (who are counted as a family of one) incurring debt. For the present comparisons it is important to maintain a consistent sample across the alternative distributions. If a person or family was marked for deletion their entire household was deleted. Thus, people and families in these households that did not have invalid data were also deleted, even for distributions that concern themselves with the individual. This ensured that all 13 distributions use an identical dataset. This does not extend to when individuals with zero market incomes were removed for only distributions 9 and 10, as these are valid members of the distributions.

As in Aziz *et al.* (2013), the explicit sharing rule in equation (2) was applied only with families, rather than across households. The rationale behind this is that sharing is much more likely to occur within a family. Sharing of financial resources is not likely to occur to the same extent between families of adults, and is rare across shared living arrangements where the individuals are not related. Similarly, the allocation of health and education expenditure follows Aziz *et al.* (2013). Education expenditure is based on total government spending on particular types of education. For example, primary and secondary education is decomposed into schooling year or age groups, and those in each category are allocated the appropriate expenditure. Health expenditure is attributed using demographic per capita expenditure profiles provided by the Ministry of Health.

Unlike earlier studies, the Income Related Rental Subsidy is not included in disposable income, but is included in the calculation of final income. In contrast, Student Allowance payments are included in the calculation of disposable income. Thus, only cash payments are in disposable income. A second difference is in the calculation of indirect tax which, in view of NZ’s broad base, is here treated simply as a constant proportion of disposable income, using the tax-inclusive GST rate. Furthermore, the data were not scaled to fiscal aggregates, since relativities of income, taxes and transfers are important to preserve, and it is of less importance to match macroeconomic variables with the national accounts.

5 Inequality Comparisons 2007, 2010 and 2011

This section compares the alternative distributions and inequality measures. First, the period 2007 to 2010 is chosen as covering the years of the global financial crises and a period in which there were few tax and transfer changes. In view of the tax changes made in 2010, additional comparisons are made including 2011. All results in subsections 5.1 and 5.2 are obtained using, where relevant, adult equivalent scales with $\theta = 0.5$ and $\alpha = 0.8$. Sensitivity analyses are reported in subsection 5.3.

5.1 Comparisons from 2007 to 2010

A range of inequality measures for 2007 and 2010 are presented in Table 2. A dash (–) in the table in the column relating to Atkinson measures for $\varepsilon = 1.2$ indicates that, in view of the presence of zero values in the distribution, $A_{1,2}$ is unity, as discussed above. The table reveals quite substantial differences in the absolute values of inequality, depending on the measure used (the degree of inequality aversion in the case of the Atkinson measures) and the combination of welfare metric and unit of analysis.

In comparing results for different values of relative inequality aversion, it is useful to consider the hypothetical ‘leaky bucket’ experiment suggested by Atkinson when proposing his measure. Consider taking a small amount from person 2, with income of x_2 , and transferring part of this to person 1, with $x_1 < x_2$, so that the ‘tax-transfer’ is equalising. The transfer is thought to involve the use of a leaky bucket, so that some income is lost in the process. A judge’s aversion to inequality is reflected in the tolerance of leaks. Totally differentiating W in (3) with respect to x_1 and x_2 gives:

$$dW = x_1^{-\varepsilon} dx_1 + x_2^{-\varepsilon} dx_2 \quad (11)$$

Transfers which leave W unchanged are thus given by:

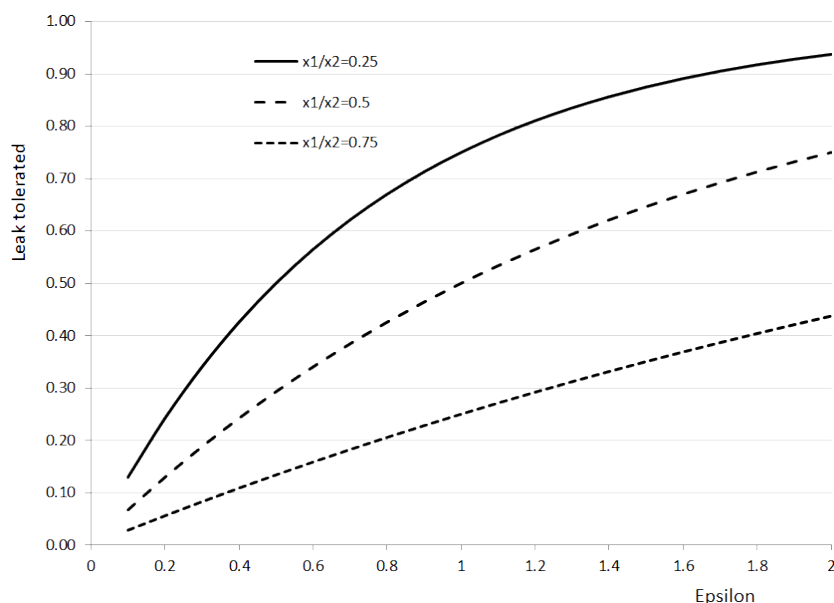
$$\left. \frac{dx_1}{dx_2} \right|_W = - \left(\frac{x_1}{x_2} \right)^\varepsilon \quad (12)$$

Convert changes to discrete form, and consider taking 1 unit from the richer person, so that $\Delta x_2 = -1$. The minimum amount that must be given to person 1 is thus:

$$\Delta x_1 = \left(\frac{x_1}{x_2} \right)^\varepsilon \quad (13)$$

and the judge would tolerate a leaking bucket up to a maximum leak of $1 - \left(\frac{x_1}{x_2} \right)^\varepsilon$. The tolerance thus depends on the initial relative incomes of the two individuals and the value of ε . Figure 3 illustrates the leakage from a ‘tax’ of 1 unit that would be tolerated by a judge with varying ε values, for three different ratios of x_1 to x_2 .

Figure 3: The Leaky Bucket Experiment



The implications of the direct tax and transfer system, in reducing inequality when moving from a gross to a net income metric, are shown in Table 3. These percentage reductions are substantial, but again they vary considerably depending on the comparisons used: for example in 2007 the Gini measure for the comparison between distributions 9 and 10 shows a reduction of 20.9 per cent, whereas the Atkinson measure, for $A_\epsilon = 0.8$, falls by 75.3 per cent when comparing distributions 3 and 4. Any comments about the redistributive effects of taxes and transfers must therefore be clear about the precise nature of the comparisons being made. The changes for 2010 are generally slightly higher than for 2007, although otherwise similar.

It is also of interest to examine the percentage changes in inequality between the two years. These are shown in Table 4. With just two exceptions – for the distribution of individual market income after sharing (number 11) and the Atkinson measure for $\epsilon = 0.5$ and $\epsilon = 0.8$ – it could be said that inequality fell from 2007 to 2010. Despite this large degree of agreement among measures and metrics, the extent of the reduction varies substantially. Disposable incomes have generally shown the largest inequality reductions compared with market incomes.

Table 2: Inequality Measures: 2007 and 2010

No.	Welfare metric	Atkinson for ε of:				Gini
		0.2	0.5	0.8	1.2	
<i>Year 2007</i>						
1	HH market income (H)	0.110	0.310	0.628	–	0.533
2	HH disposable income (H)	0.049	0.122	0.197	0.308	0.382
3	HH market income per AE (H)	0.106	0.300	0.615	–	0.523
4	HH disposable income per AE (H)	0.039	0.095	0.152	0.237	0.337
5	Market income per AE (N)	0.088	0.247	0.514	–	0.481
6	Disposable income per AE (N)	0.034	0.084	0.132	0.201	0.317
7	Market income per AE (N_E)	0.092	0.258	0.535	–	0.489
8	Disposable income per AE (N_E)	0.036	0.088	0.139	0.213	0.324
9	Individual market income (N_W)	0.094	0.242	0.407	0.670	0.502
10	Individual disposable income (N_W)	0.057	0.145	0.243	0.427	0.397
11	Individual market income (N)	0.126	0.334	0.633	–	0.583
12	Individual disposable income (N)	0.065	0.158	0.246	0.375	0.438
13	Individual final income (N)	0.044	0.107	0.165	0.236	0.364
<i>Year 2010</i>						
1	HH market income (H)	0.102	0.289	0.591	–	0.513
2	HH disposable income (H)	0.042	0.105	0.169	0.260	0.356
3	HH market income per AE (H)	0.099	0.281	0.580	–	0.507
4	HH disposable income per AE (H)	0.034	0.082	0.130	0.196	0.316
5	Market income per AE (N)	0.087	0.244	0.505	–	0.476
6	Disposable income per AE (N)	0.030	0.072	0.113	0.168	0.297
7	Market income per AE (N_E)	0.088	0.247	0.511	–	0.477
8	Disposable income per AE (N_E)	0.031	0.075	0.118	0.176	0.302
9	Individual market income (N_W)	0.090	0.235	0.398	0.645	0.497
10	Individual disposable income (N_W)	0.050	0.127	0.211	0.357	0.376
11	Individual market income (N)	0.125	0.335	0.644	–	0.580
12	Individual disposable income (N)	0.060	0.145	0.226	0.331	0.423
13	Individual final income (N)	0.041	0.099	0.154	0.222	0.353

Table 3: Percentage Inequality Reduction from Market to Disposable Income

Change	Atkinson for ϵ of:				Gini
	0.2	0.5	0.8	1.2	
<i>Reductions for 2007</i>					
1 to 2	-55.3	-60.5	-68.6	–	-28.3
3 to 4	-63.4	-68.2	-75.3	–	-35.5
5 to 6	-61.2	-66.2	-74.3	–	-34.1
7 to 8	-61.0	-65.9	-74.0	–	-33.7
9 to 10	-39.1	-40.0	-40.4	-36.3	-20.9
11 to 12	-48.3	-52.7	-61.1	–	-24.8
<i>Reductions for 2010</i>					
1 to 2	-58.6	-63.6	-71.4	–	-30.6
3 to 4	-66.2	-70.8	-77.6	–	-37.6
5 to 6	-65.8	-70.4	-77.6	–	-37.7
7 to 8	-64.9	-69.6	-76.9	–	-36.7
9 to 10	-44.8	-46.1	-46.9	-44.7	-24.4
11 to 12	-52.2	-56.7	-64.8	–	-27.0

Table 4: Percentage Change in Inequality from 2007 to 2010

No.	Welfare metric	Atkinson for ϵ of:				Gini
		0.2	0.5	0.8	1.2	
1	HH market income (H)	-7.6	-6.7	-5.9	–	-3.7
2	HH disposable income (H)	-14.5	-14.1	-14.0	-15.6	-6.8
3	HH market income per AE (H)	-6.6	-6.1	-5.7	–	-3.1
4	HH disposable income per AE (H)	-13.8	-13.8	-14.5	-17.3	-6.3
5	Market income per AE (N)	-2.0	-1.4	-1.8	–	-1.1
6	Disposable income per AE (N)	-13.6	-13.8	-14.3	-16.3	-6.4
7	Market income per AE (N_E)	-4.8	-4.2	-4.3	–	-2.4
8	Disposable income per AE (N_E)	-14.4	-14.5	-15.1	-17.3	-6.9
9	Individual market income (N_W)	-3.9	-2.7	-2.2	-3.7	-1.0
10	Individual disposable income (N_W)	-12.9	-12.6	-12.9	-16.4	-5.3
11	Individual market income (N)	-1.3	0.5	1.8	–	-0.5
12	Individual disposable income (N)	-8.7	-7.9	-7.9	-11.6	-3.4
13	Individual final income (N)	-8.2	-7.2	-6.5	-5.8	-3.1

5.2 Comparisons from 2007 to 2011

As discussed earlier, it is useful to consider changes in inequality over the longer period 2007 to 2011, in view of the tax changes announced in 2010.

The main policy changes made in 2010 concerned a partial shift in the tax mix from personal income taxation towards indirect taxation, with associated adjustments to a number of benefit levels.¹⁸ In particular, the percentage marginal income tax rates, which in 2006/07 were [19.5, 30, 39], were changed to [10.5, 17.5, 30, 33]. Corresponding thresholds, above which the respective rates applied, were [0, 38000, 60000] in 2006/07 and became [0, 14000, 48000, 70000] in 2010. The GST rate was increased from 12.5% to 15%.¹⁹ A range of benefit abatement thresholds, such as Domestic Purposes Benefit (DPB), Invalid's Benefit (IB), Widow's Benefit (WB) were changed from \$80 and \$180 per week to \$100 and \$200 per week, with abatement rates of 30% and 70% continuing to apply. The New Zealand Superannuation Non Qualifying Spouse benefit (NZS NQS) threshold was changed from \$80 to \$100 per week, with the abatement rate of 70% remaining unchanged. In addition, in 2006/07 there was a Low Income Rebate (a 4.5% tax rebate until \$9500 per year where a 1.5% abatement begins). In 2010 an Independent Earner Tax Credit applied, involving a \$520 tax credit for income over \$24000, abated at 13% after \$44000 per year. The Accident Compensation Corporation (ACC) Levy was 1.3% in 2006/07, and 2.04% in 2010.²⁰

Table 5 shows the percentage change over the period for each of the metrics and inequality measures (although distribution number 13 could not be considered for 2011). Unlike comparisons between 2007 and 2010, it can be seen that the direction of change is more ambiguous. The Gini measures show small percentage increases for all distributions except for the disposable income distributions in numbers 10 and 12.

There is more ambiguity among the Atkinson measures. Measured inequality in any period is higher, the higher is the degree of relative inequality aversion. But of relevance here is the *change* in the Atkinson measure between two time periods. It is not necessarily the case that judges will agree about the direction of changes in inequality. If there are equalising changes in the lower ranges of the distribution, more importance will be attached to these by a judge with high inequality aversion, who attaches less importance to high-income changes. Alternatively a judge with low aversion is relatively more concerned with the changes taking place in higher-income groups. From the evaluation function, W , associated with the Atkinson

¹⁸ The HES is conducted from July to June, and Taxwell uses this for modelling the April to March tax year. However, when modelling the so called '2011 year' we use the 2010/2011 HES but apply the policies that came into force at October 2010.

¹⁹ For further discussion of the tax mix change, see Creedy and Mellish (2011).

²⁰ Changes to Portfolio Investment Entities (PIEs) could not be incorporated into the analysis. There was a temporary additional payment to some benefit categories to compensate for price rises due to the GST increase. This was paid from October until April 2011 when benefits would next be indexed by the CPI. As a compromise, benefit payments were modelled according to the Taxwell tax year, thus including only half the temporary payment.

measure, given in equation (3):

$$\frac{\partial W}{\partial x_i} = x_i^{-\varepsilon} \quad (14)$$

Hence, the increase in W associated with an increase in x_i is not only lower for higher incomes, but is lower for higher values of ε , for a given income. This has the potential to lead to the counter-intuitive result, depending on the precise nature of the distributional changes, whereby a higher aversion implies a decrease in inequality over time, where a low aversion implies an increase.

Interestingly, in the present context, a judge with a higher degree of aversion to inequality takes the view that there has been a reduction in inequality from 2007 to 2011 for all disposable income distributions (except for individual market incomes in distribution 9, which shows as increase of about 1 per cent). However, a lower ε implies a reduction in the cases of distributions of disposable incomes 2, 10 and 12, but an increase for distributions 4, 6 and 8. Both the direction and extent of the measured change in inequality depend on the particular combination of the welfare metric, the unit of analysis and the inequality measure being considered.

It is also of interest to consider whether, for each social welfare function, the value of W increases over the period. That is, in those cases where inequality is seen to increase, is this compensated by an increase in real incomes (in the view of the independent judge's evaluation function)? This question is answered by comparing values of $W = \bar{x}(1 - I_x)$, discussed above, where \bar{x} is suitably adjusted for inflation over the period. Table 6 reports percentage changes in the (abbreviated) social welfare function from 2007 to 2011. Generally the changes are positive. Hence, the increase in real incomes over the period is judged to more than compensate for the increase in inequality, where relevant, although again the percentage changes differ. The exceptions are for distribution number 5 (for Gini and $A_{0.8}$) and distribution 11 (for $A_{0.8}$). These cases relate to market rather than disposable incomes, and the distributions contain zero values. Furthermore, the higher values for $\varepsilon = 0.8$ imply that there is greater sensitivity to changes at the lower end of the distribution.

5.3 Adult Equivalence Scales

The results presented above are all obtained for a single set of parameters in $m = (n_a + \theta n_c)^\alpha$, the expression for adult equivalent scales given in (1). Obviously these scales do not affect all the distributions discussed here, but where they are relevant the sensitivity of comparisons to the value of α , which reflects the extent of economies of scale, was examined.

Consider, for example, the distribution of disposable income per adult equivalent, using the individual as unit, and the Atkinson inequality measure. Using $\varepsilon = 0.5$ inequality in 2011 exceeds that in 2007 for all values of α , but when $\varepsilon = 1.2$ the order is reversed and the distribution in 2007 is more unequal than in 2011. The case where $\varepsilon = 0.8$ is illustrated in Figure 4. The shape of the profiles indicates that simple assumptions about the way in which changes in the economies of

Table 5: Percentage Change in Inequality from 2007 to 2011

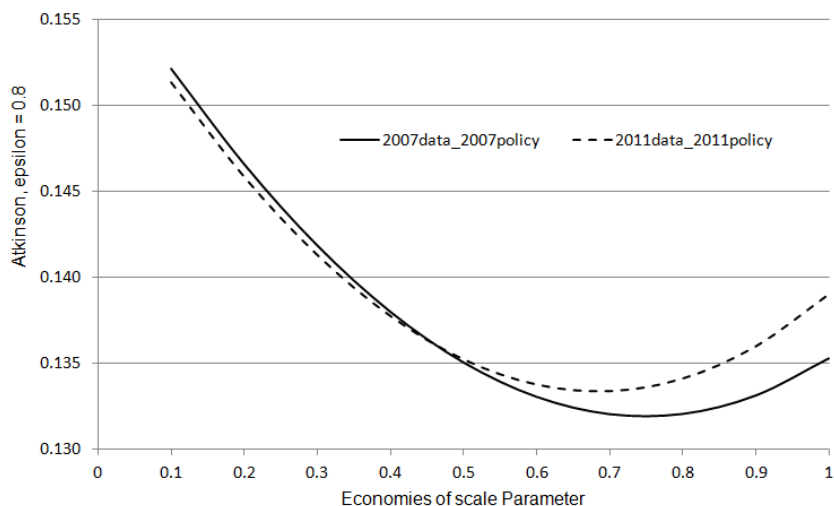
No.	Welfare metric	Atkinson for ε of:				Gini
		0.2	0.5	0.8	1.2	
1	HH market income (H)	-0.01	-0.69	-1.17	–	0.57
2	HH disposable income (H)	-0.47	-1.58	-3.12	-6.90	0.07
3	HH market income per AE (H)	0.87	-0.15	-0.97	–	0.82
4	HH disposable income per AE (H)	1.88	0.22	-1.90	-6.55	0.65
5	Market income per AE (N)	4.46	3.44	2.40	–	2.40
6	Disposable income per AE (N)	4.78	3.30	1.58	-1.66	2.01
7	Market income per AE (N_E)	2.35	1.39	0.42	–	1.49
8	Disposable income per AE (N_E)	2.85	1.48	-0.19	-3.60	1.18
9	Individual market income (N_W)	0.05	0.32	0.83	0.89	0.04
10	Individual disposable income (N_W)	-4.32	-5.71	-7.31	-11.03	-1.81
11	Individual market income (N)	1.39	2.01	2.60	–	0.54
12	Individual disposable income (N)	-1.01	-1.40	-2.25	-7.04	-0.40

Table 6: Percentage Changes in Abbreviated Social Welfare from 2007 to 2011

No.	Welfare metric	Atkinson for ε of:				Gini
		0.2	0.5	0.8	1.2	
1	HH market income (H)	2.0	2.3	4.0	–	1.4
2	HH disposable income (H)	10.0	10.2	10.8	13.3	9.9
3	HH market income per AE (H)	2.9	3.1	4.6	–	2.1
4	HH disposable income per AE (H)	10.5	10.6	11.0	12.9	10.2
5	Market income per AE (N)	1.3	0.5	-0.9	–	-0.6
6	Disposable income per AE (N)	9.5	9.3	9.4	10.1	8.6
7	Market income per AE (N_E)	1.4	1.2	1.2	–	0.2
8	Disposable income per AE (N_E)	9.5	9.4	9.6	10.6	9.0
9	Individual market income (N_W)	3.1	3.0	2.5	1.2	3.0
10	Individual disposable income (N_W)	11.4	12.2	13.7	20.2	12.4
11	Individual market income (N)	1.3	0.5	-3.0	–	0.8
12	Individual disposable income (N)	9.5	9.7	10.2	14.0	9.8

scale parameter affect inequality may be misleading. There is a range over which an increase in α is associated with reduction in inequality, and a range over which an increase in α produces an increase in inequality.²¹ In addition, the two profiles intersect, so that for values of α above around 0.45, inequality in 2011 is judged to be higher than in 2007, but for α less than 0.45, the inequality ranking is reversed.

Figure 4: Inequality of Disposable Income per Adult Equivalent with Individual as Unit: $\varepsilon = 0.8$



A similar kind of sensitivity arises for the distribution of disposable income per adult equivalent person, when the income unit is the equivalent adult. Figures 5 and 6 show the corresponding profiles for inequality aversion of $\varepsilon = 0.2$ and $\varepsilon = 0.8$ respectively. For the higher value of $\varepsilon = 1.2$, inequality is judged to be greater in 2007 than in 2011 for all values of α .

An example involving the distribution of market income per adult equivalent person, with the equivalent adult as the unit of analysis, is illustrated in Figure 7. In this case, the distributions contain some zero values, so Atkinson inequality measures are reported only for $\varepsilon < 1$. In this case, with $\varepsilon = 0.8$, inequality falls consistently as α increases (that is, as the extent of scale economies falls), but again the two profiles intersect. For α less than about 0.65, inequality in 2007 is judged to be greater than in 2011.

²¹ For discussion of the precise conditions in terms of the relevant joint distributions and the correlation between equivalent income and the number of individuals in the household, see Creedy and Sleeman (2005, pp. 58–60).

Figure 5: Inequality of Disposable Income per Adult Equivalent with Equivalent Adult as Unit: $\varepsilon = 0.2$

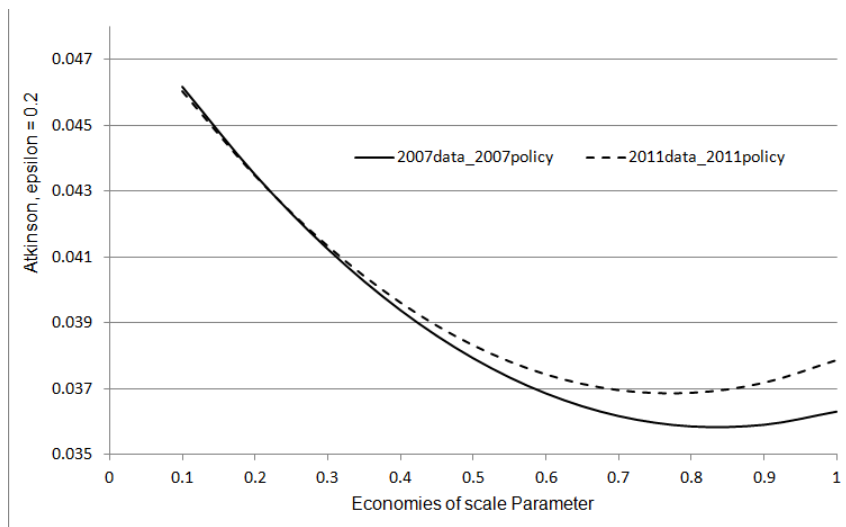


Figure 6: Inequality of Disposable Income per Adult Equivalent with Equivalent Adult as Unit: $\varepsilon = 0.8$

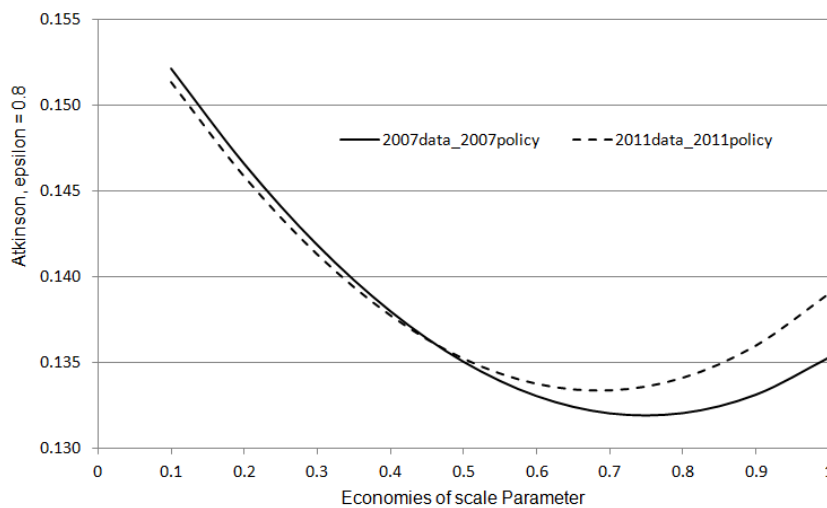
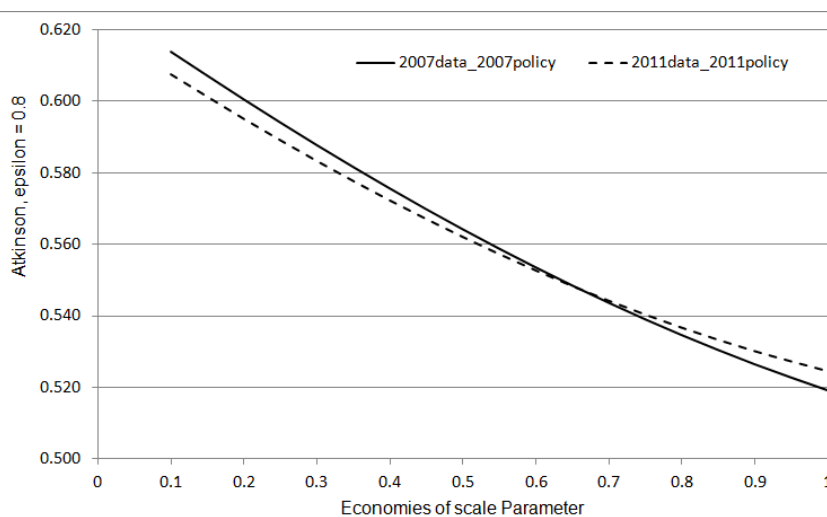


Figure 7: Inequality of Market Income per Adult Equivalent with Equivalent Adult as Unit: $\varepsilon = 0.8$



6 Inequality Decompositions

The previous sections of this paper have discussed alternative income distribution comparisons, either for a single time period (in moving from market to disposable income), or for two periods. However, changes in measured inequality over time depend on the structure of the population as well as the tax and transfer system. For example, there are systematic variations in incomes over the life cycle. A change in the age distribution of the population could therefore give rise to an observed increase in overall inequality even if the tax system is changed in ways which are designed to make it more redistributive.

The fact that the redistributive effect of any tax system cannot be evaluated independently of the population (the pre-tax income distribution) raises the question of how comparisons can be made over time, where typically both the population and the tax structure are different. In fiscal incidence studies the question is thus: has the income tax and transfer system become more or less redistributive? The difficulty is therefore to isolate the marginal effect of the tax policy change from that of the population change. The various components are explained in subsection 6.1, and subsection 6.2 applies the method to examine inequality changes between 2007 and 2011.

Such decompositions must nevertheless be treated with caution. First, the following analysis does not consider endogenous labour supply responses to tax changes.²² In addition, there may be other responses to tax changes which are nevertheless included in population structure changes. These include changes in fertility, household formation, migration and so on.

6.1 Tax and Population Components

Given two cross-sectional household surveys, let T_i denote the tax structure for $i = 0, 1$ (an initial period and subsequent period respectively). Similarly let P_i denote the population in period i . For convenience, consider the Gini inequality measure, although the following approach may be used for other summary measures. There are therefore four possible Gini inequality measures of both gross market income and disposable income; denote these by $G_m(P_i, T_j)$ and $G_d(P_i, T_j)$ for $i, j = 0, 1$. These four measures can be obtained using each of the combinations of income concept and unit of analysis discussed above. It is of course required that each survey contains enough information about the characteristics of households so that the disposable incomes of each population can be computed for each of the tax structures.

²² On decompositions which allow also for labour supply responses, see Bargain (2012a, 2012b), and Creedy and Herault (2011). The latter paper also considers the use of money metric utility as the welfare metric in decompositions.

The comparisons in previous sections above are of Gini measures, in the case of disposable income in each period, using $G_d(P_0, T_0)$ and $G_d(P_1, T_1)$. But the finding that, for example, $G_d(P_0, T_0) > G_d(P_1, T_1)$, following a policy change, does not support the inference that the policy reform has reduced inequality. The reduction may have arisen from the population structure changes. The separate effects of tax and population changes can be obtained as follows. Consider the following decomposition:

$$G_d(P_1, T_1) - G_d(P_0, T_0) = [G_d(P_1, T_1) - G_d(P_0, T_1)] + [G_d(P_0, T_1) - G_d(P_0, T_0)] \quad (15)$$

The first term in square brackets on the right hand side of (15) is the population effect given tax structure 1, and the second term in square brackets is the tax policy effect given initial population 0. However, there is another possible decomposition of the change in inequality, since:

$$G_d(P_1, T_1) - G_d(P_0, T_0) = [G_d(P_1, T_0) - G_d(P_0, T_0)] + [G_d(P_1, T_1) - G_d(P_1, T_0)] \quad (16)$$

The first term in square brackets on the right hand side of (16) is the population effect given tax structure 0, while the second term is the tax policy effect given population structure 1. Faced with two values for each of the marginal effects, an approach is to obtain the unweighted arithmetic mean: this average is recommended by Shorrocks (2011), who links it to the Shapley Value, familiar from game theory.

6.2 New Zealand Comparisons 2007 to 2011

Table 7 reports the effects of applying the above decomposition to changes between 2007 and 2011. Values shown are absolute changes, and are the arithmetic means of the relevant components. It is clear from the final two columns of the table that, for all disposable income distributions and inequality measures, the effect of the tax and transfer changes between 2007 and 2011 was to increase inequality of disposable incomes slightly. However, the population structure changes had the effect, in all cases, of reducing measured inequality. Whether the overall effect was to reduce inequality therefore depends on whether the population component outweighed the tax change effect. For the Atkinson measures this is seen to be more likely for the higher inequality aversion cases, where measures are more sensitive to changes at the lower end of the income distribution.

Table 7: Decomposition of Absolute Changes in Inequality of Disposable Income: 2007 to 2011

No.	Welfare metric	Component:		
		Total	Population	Tax
<i>Decomposition based on Gini</i>				
2	HH disposable income (H)	0.0003	-0.0061	0.0064
4	HH disposable income per AE (H)	0.0022	-0.0061	0.0083
6	Disposable income per AE (N)	0.0064	-0.0024	0.0088
8	Disposable income per AE (N_E)	0.0038	-0.0044	0.0082
10	Individual disposable income (N_W)	-0.0072	-0.0126	0.0054
12	Individual disposable income (N)	-0.0017	-0.0085	0.0067
<i>Decomposition based on Atkinson with $\varepsilon = 0.2$</i>				
2	HH disposable income (H)	-0.0002	-0.0019	0.0016
4	HH disposable income per AE (H)	0.0007	-0.0010	0.0018
6	Disposable income per AE (N)	0.0016	-0.0001	0.0018
8	Disposable income per AE (N_E)	0.0010	-0.0007	0.0017
10	Individual disposable income (N_W)	-0.0025	-0.0039	0.0014
12	Individual disposable income (N)	-0.0007	-0.0027	0.0020
<i>Decomposition based on Atkinson with $\varepsilon = 0.8$</i>				
2	HH disposable income (H)	-0.0061	-0.0122	0.0061
4	HH disposable income per AE (H)	-0.0029	-0.0095	0.0066
6	Disposable income per AE (N)	0.0021	-0.0046	0.0067
8	Disposable income per AE (N_E)	-0.0003	-0.0066	0.0064
10	Individual disposable income (N_W)	-0.0177	-0.0228	0.0050
12	Individual disposable income (N)	-0.0055	-0.0129	0.0073
<i>Decomposition based on Atkinson with $\varepsilon = 1.2$</i>				
2	HH disposable income (H)	-0.0212	-0.0298	0.0086
4	HH disposable income per AE (H)	-0.0155	-0.0249	0.0094
6	Disposable income per AE (N)	-0.0033	-0.0128	0.0095
8	Disposable income per AE (N_E)	-0.0077	-0.0168	0.0091
10	Individual disposable income (N_W)	-0.0471	-0.0535	0.0064
12	Individual disposable income (N)	-0.0264	-0.0361	0.0097

7 Conclusions

This paper has emphasised the importance of making a range of value judgements explicit when attempting to measure inequality, and changes in inequality, for any particular population group. Special attention was given to comparisons of alternative distributions and the implications of using different distributions and summary measures. It is suggested that comparisons are too often based on a limited range of measures which do not provide sufficient information for readers, whose value judgements may vary widely, to make their own judgements. Using an annual accounting period, alternative welfare metrics and units of analysis were investigated. Emphasis was given to the role of value judgements in making inequality comparisons.

The use of alternative value judgements was illustrated by examining recent changes in income inequality in New Zealand. It has long been recognised that the particular summary measure of inequality used depends crucially on value judgements which are unlikely to be shared by all readers. The implications of alternative measures were illustrated here using Gini and Atkinson inequality measures, where the latter also allow for a range of degrees of aversion to inequality.

Some of the comparisons involved the use of adult equivalent scales. The use of a welfare metric defined in terms of income per adult equivalent (for example, market or disposable income) can be combined with the use of different income units, such as the household, the individual or the equivalent adult. In addition, the sensitivity to assumptions about economies of scale within households was examined. Furthermore, changes in inequality were decomposed into those arising from population and tax structure changes.

When considering the period 2007 to 2010 all measures agree that inequality fell, although the extent of the reduction varies. For the period 2007 to 2011 (after the tax reforms of 2010) the answer to the question of whether inequality in New Zealand has risen or fallen was found to depend crucially on the combination of welfare metric, income unit, adult equivalent scale and inequality measure used.

When decomposing changes in inequality into tax and population components, it was found that for all disposable income distributions and inequality measures, the effect of the tax and transfer changes between 2007 and 2011 was to increase inequality of disposable incomes slightly. However, the population structure changes had the effect, in all cases, of reducing measured inequality. The overall effect on inequality (depending on whether the population component outweighed the tax change effect) was found to depend on the inequality measure used.

It should perhaps not be surprising that such a complex phenomenon as inequality within a heterogeneous population does not allow simple unambiguous comparisons. It is all too easy for researchers, often implicitly, to impose their own value judgements or not to be aware of some of the complexities involved (such as the

fact that the use of the individual as unit, when using a measure of income per adult equivalent, can imply a preference for more inequality if household size is strongly positively related to income and there are substantial economies of scale in consumption within households). For many years the theoretical literature on inequality measurement has attempted to untangle the many complexities and implications of alternative value judgements, yet too often empirical studies have ignored the subtleties. An aim of the present paper has thus been to persuade empirical researchers and policy analysts to investigate and report results for a wider range of distributions and comparisons.

Appendix: Weighted Gini Inequality Measures

A widely used expression for the Gini measure is given in equation (6). To allow for weights, suppose that each x_i has an integer weight, w_i . Let $N = \sum_{i=1}^n w_i$, and $\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i w_i$. Define $D_{i,j}$ as follows. For $i = 1$, and $j = 1, \dots, w_1$:

$$D_{1,j} = N + 1 - j \quad (17)$$

and for $i = 2, \dots, n$, and $j = 1, \dots, w_i$:

$$D_{i,j} = N + 1 - \sum_{k=1}^{i-1} w_k - j \quad (18)$$

The summation, S , can be written as:

$$G = 1 + \frac{1}{N} - \frac{2}{N^2 \bar{x}} \sum_{i=1}^n x_i \left(\sum_{j=1}^{w_i} D_{i,j} \right) \quad (19)$$

Alternatively, it can be shown that:

$$G = 1 + \frac{\sum_{i=1}^n w_i^2 x_i}{N \sum_{i=1}^n w_i x_i} - \frac{2 \sum_{i=1}^n \left(N + w_i - \sum_{j=1}^i w_j \right) w_i x_i}{N \sum_{i=1}^n w_i x_i} \quad (20)$$

which can be directly applied to the case where weights are non-integer.

Alternatively, the reverse-order-rank weighted mean is given by:

$$x_R = \frac{\sum_{i=1}^n \sum_{j=1}^{w_i} \left(N + 1 - \left(\sum_{k=1}^{i-1} w_k \right) - j \right) x_i}{\sum_{i=1}^n \sum_{j=1}^{w_i} \left(N + 1 - \left(\sum_{k=1}^{i-1} w_k \right) - j \right)} \quad (21)$$

where it is understood that $w_0 = 0$ so that for $i = 1$, $\sum_{k=1}^{i-1} w_k = 0$. Hence it is clear that re-scaling the weights – which is equivalent to replication – has no effect on the Gini measure.

Alternatively it is possible to use an expression for the Gini measure which does not make use of ordering. For unweighted data, the standard expression involving all pairwise comparisons is:

$$G_s = \frac{\sum_{i=1}^N \sum_{j=1}^N |x_i - x_j|}{2N^2 \bar{x}} \quad (22)$$

As before, suppose the weight attached to i is w_i , with $N = \sum_{i=1}^n w_i$. Then:

$$\begin{aligned}
 G_g &= \frac{\sum_{i=1}^n w_i \sum_{j=1}^n w_j |x_i - x_j|}{2 \left(\sum_{i=1}^n w_i \right)^2 \bar{x}} \\
 &= \frac{\sum_{i=1}^n w_i \sum_{j=1}^n w_j |x_i - x_j|}{2 \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i} \tag{23}
 \end{aligned}$$

This can be rewritten as:

$$G_g = \frac{\sum_{i=1}^n w_i \sum_{j=1}^{i-1} w_j |x_i - x_j|}{2 \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i} + \frac{\sum_{i=1}^n w_i \sum_{j=i+1}^n w_j |x_i - x_j|}{2 \sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i} \tag{24}$$

The numerator of the first term is the sum of every pairwise absolute difference in x_i . It differs from the numerator of equation (23) because it does not repeat any previous comparison. These duplicate comparisons can be found in the last term of equation (24), which is equal to the first term. Thus:

$$G_g = \frac{\sum_{i=1}^n \sum_{j=i+1}^n w_i w_j |x_i - x_j|}{\sum_{i=1}^n w_i \sum_{i=1}^n w_i x_i} \tag{25}$$

References

- Aziz, O.A., Ball, C., Creedy, J. and Eedrah, J. (2013) The distributional impact of population ageing. *New Zealand Treasury Working Paper*, 13/13.
- Banks, J. and Johnson, P. (1994) Equivalence scale relativities revisited. *Economic Journal*, 104, pp. 883-890.
- Bargain, O. (2012a) The distributional effects of tax-benefit policies under New Labour: a decomposition approach. *Oxford Bulletin of Economics and Statistics*, 74, pp. 856-874.
- Bargain, O. (2012b) Decomposition analysis of distributive policies using behavioural simulations. *International Tax and Public Finance*, 19, pp. 708-731.
- Creedy, J. (2013) Alternative distributions for inequality and poverty comparisons. *New Zealand Treasury Working Paper*, 13-11.
- Creedy, J. and Scutella, R. (2004) The role of the unit of analysis in tax policy reform evaluations of inequality and social welfare. *Australian Journal of Labour Economics*, 7, pp. 89-108.
- Creedy, J. and Sleeman, C. (2005) Adult equivalence scales: inequality and poverty. *New Zealand Economic Papers*, 39, pp. 51-83.
- Creedy, J. and Hérault, N. (2011) Decomposing inequality and social welfare changes: the use of alternative welfare metrics. *The University of Melbourne Institute of Applied Economic and Social Research*, WP No. 8/11.
- Creedy, J. and Mellish, A. (2011) Changes in the tax mix from income taxation to GST: revenue and redistribution. *New Zealand Economic Papers*, 45, pp. 299-309.
- Cutler, D.M. and Katz, L. (1992) Rising inequality? changes in the distribution of income and consumption in the 1980s. *American Economic Review*, 82, pp. 546-551.
- Decoster, A. and Ooghe, E. (2003) Weighting with individuals, equivalent individuals or not weighting at all: does it matter empirically? In *Inequality, Welfare and Poverty: Theory and Measurement*. (Edited by J. and J.G. Rodríguez), pp.173-190. New York: Emerald Group Publishing.
- Glewwe, P. (1991) Household equivalence scales and the measurement of inequality: transfers from the poor to the rich could decrease inequality. *Journal of Public Economics*, 44, pp. 214-216.
- Hyslop, D. and Yahanpath, S. (2005) Income growth and earnings variations in New Zealand 1998-2004. *New Zealand Treasury Working Paper*, 05/11.
- Jenkins, S.P. and Cowell, F.A. (1994) Parametric equivalence scales and scale relativities, *Economic Journal*, 104, pp. 891-900.

Lambert, P. (2001) *The Distribution and Redistribution of Income: A Mathematical Analysis*. Manchester: Manchester University Press.

Ministry of Economic Development, Treasury and Statistics New Zealand (2011) *New Zealand Economic Development Indicators 2011*. Available from: http://www.stats.govt.nz/browse_for_stats/economic_indicators/productivity/development-indicators/2011-indicators.aspx.

Shorrocks, A.F. (2004) Inequality and welfare evaluation of heterogeneous income distributions. *Journal of Economic Inequality*, 2, pp. 193-218.

Shorrocks, A.F. (2011) Decomposition procedures for distributional analysis: a unified framework based on the Shapley value. *Journal of Economic Inequality*, 11, pp. 99-126.

Wang, C., Caminada, K. and Goudswaard, K. (2013) Income redistribution in 20 countries over time. *International Journal of Social Welfare*, Published online 23 September 2013.