

# Estimating New Zealand's Output Gap Using a Small Macro Model

Kam Leong Szeto

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New Zealand Treasury Working Paper 13/18

July 2013



**NZ TREASURY WORKING  
PAPER 13/18**

Estimating New Zealand's Output Gap Using a Small Macro Model

**MONTH/YEAR**

July 2013

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**ISBN (ONLINE)**

978-0-478-40362-6

**URL**

Treasury website at July 2013:  
<http://www.treasury.govt.nz/publications/research-policy/wp/2013/13-18>  
Persistent URL: <http://purl.oclc.org/nzt/p-1579>

**ACKNOWLEDGEMENTS**

I would like to thank Michael Ryan, Peter Mawson, Jamie Murray, Patrick Conway, Michael Reddell, Thora Helgadottir, Graeme Wells, Kirdan Lees and Brigid Monagel for helpful comments on an earlier version of this paper.

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# Abstract

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The Treasury has been testing the assumptions on the potential growth rate of the New Zealand economy. In this paper, we estimate a small macro model using Bayesian techniques, which allows us to assess the level of uncertainty of the estimates of the output gap. The model is based on the work of Benes et al. (2010) with some modifications reflecting New Zealand economic conditions. Although this new technique does not reduce the uncertainty in measures of potential output as indicated by large confidence bands for the estimates, it provides us a useful tool with an economic framework for measuring potential output.

**JEL CLASSIFICATION** C32, C53, E31, E32

**KEYWORDS** Potential output, Potential growth rate, Output gap, Unemployment, NAIRU, Inflation and Capacity.

## Executive Summary

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Since the onset of the Global Financial Crisis (GFC) many forecasting groups in OECD economies such as the UK Treasury and the US Congressional Budget Office have successively revised down the potential growth rates of their economies, reflecting the fact that the credit-driven growth model in the years leading up to the GFC was not sustainable. Although New Zealand is far away from the epicentre of the crisis and does not have a large financial sector, New Zealand is not immune to the impact of the GFC on potential output growth. In the 2009 *Budget Economic and Fiscal Update*, the Treasury lowered the level of potential GDP in the 2013 March year by around 5% compared to the previous forecasts released in December 2008.

Like other countries in the OECD, New Zealand is grappling with the difficulties of measuring the underlying potential growth rate of the economy. In particular, the crisis has significantly increased uncertainty surrounding the estimates of potential growth and spare capacity of the economy. The amount of spare capacity is defined as the difference between potential output and actual output, where potential output is the level of output at which stable inflation is maintained.

This paper provides estimates of the potential growth and spare capacity of the New Zealand economy using a small macro model for the period 1994-2012. This methodology makes use of the theoretical relationships between inflation, unemployment, business survey indicators and the output gap to infer the level of potential output. This method allows us to take into account a wider range of economic data when reaching a judgement on potential output than do other methods currently used by the Treasury.

The new estimates suggest more spare capacity in the post-crisis period than other indicators used by the Treasury. According to the new methodology, the estimate of the output gap remains negative at around 2% in 2012. In contrast, other measures suggest that there was no spare capacity in the economy, i.e., a zero output gap. This finding suggests that the slow recovery in the economy after the GFC is partly due to weak demand in the economy.

Another key finding is that all the methods point to the onset of the slowdown in potential growth occurring before the financial crisis. A possible reason for this is that growth over the 2000s ultimately proved unsustainable as it was associated with large build-ups of household debt that could not continue indefinitely. As a result, the economy is going through a protracted adjustment period.

Although the main advantage of the new methodology is to incorporate additional relevant economic information, this new technique does not reduce the uncertainty in measures of potential output and its output gap as indicated by large confidence bands for the estimates. Furthermore, this new methodology, like other approaches, also suffers from the “endpoint” problem.

Whatever method is chosen for use, it is important to bear in mind that users need to understand all the judgements and assumptions behind the method as there are always implicit judgements built in to different approaches.

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# Estimating New Zealand's Potential Output Using a Small Macro Model

## 1 Introduction

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The global financial crisis (GFC) has cast a long shadow on many developed economies, particularly the US and the Euro area. Since the crisis, many countries have continued to revise down their growth outlook. Furthermore, many researchers and economic commentators have debated to what extent the crisis may have affected the supply side of the economy.

In 2009, the UK Treasury assumed that the GFC would reduce the level of potential output by 5% over three years, but that the long-term growth rate of potential GDP would remain unaffected at 2¾% (UK Treasury 2009). However, in the 2011 Budget, the judgement of the Office for Budget Responsibility (OBR) is that the long-term trend rate of growth is projected to be 2.35%, falling back to 2.10% from 2014 as demographic changes reduce the growth of potential labour supply. In the economic outlook released in November 2011, the OBR further reduced the estimate of the level of potential output in 2016 by about 3.5% but revised up the long-term growth rate to 2.3% (UK Treasury 2011).

In the US, the Congressional Budget Office (CBO) has not changed its estimate of potential output significantly since the GFC. In 2009, CBO's projection for the growth of potential was estimated to be 2.3% on average during the 2009-2019 period which is only one-tenth of a percentage point slower than what was estimated in the 2008 report (CBO 2009). This view reflects the CBO's assumption that potential output in the US has been largely unaffected by the crisis. As the CBO assumes that the recent slowdown in output growth is cyclical rather than structural, the CBO's estimates of the output gap are relatively large suggesting that there is significant spare capacity in the US economy.

However, some economists question the CBO assessment of the output gap because core inflation has fallen relatively little, hinting that there may be less slack in the US economy than suggested by the CBO estimates of the output gap (Weidner and Williams 2009).

In the 2009 quarterly report (EC 2009), the European Commission (EC) revised down not only future, but also historical, potential growth rates in the euro area owing to higher unemployment and plummeting investment. As a result, the EC predicted that the crisis will entail a permanent loss in the level of potential output. At that juncture, the EC forecast that the European economy will return to its pre-crisis long-term growth rate of 2% in the medium run.

**Table 1 – Estimates of the Euro-area’s potential growth**

	Potential Growth (%)
2000-2006	1.8
2007	1.6
2008	1.3
2009	0.7
2010	0.7

Source: European Commission

Although New Zealand is far away from the epicentre of the crisis, we are not immune to the impact of the GFC on potential output growth. In the 2009 Budget Economic and Fiscal Update, the Treasury lowered the level of potential GDP in the 2013 March year by around 5% compared to the previous forecasts released in December 2008.

The New Zealand Treasury relies on a set of standard tools in estimating potential output. The methods include the Hodrick and Prescott filter (HP filter), the HP filter augmented with structural variables (MV filter), the unobservable components model (or Kalman filter) and the on-trend method. The methods have been described in detail by Downing et al (2003).

Apart from the on-trend method, all other methods are statistical tools that do not have much economic underpinning. Quah (1992) notes that focusing on the univariate time series properties of a variable means that it is not possible to identify what causes the fluctuations that occur in the variable. As a result, unless one imposes additional restrictions in an ad hoc manner, it is not possible to accurately assess the extent to which fluctuations in a variable are permanent or transitory.

The use of statistical filters may work well during a typical business cycle, where fluctuations are predominantly driven by demand shocks. However, in periods such as the GFC, the economy was hit by financial shocks, which could affect both the demand and supply sides of the economy. Reinhart and Rogoff (2009) also argue that recessions associated with a *systemic* banking crisis tend to be deep and protracted.

Therefore, the GFC has significantly increased the uncertainties surrounding the outlook for potential output and the estimates of potential growth rate are expected to be subject to large revisions. In this paper, we use a new technique for measuring and updating potential output, which incorporates a few key economic concepts such as Okun’s law and the concept of equilibrium unemployment rate (NAIRU) within the framework of a small macroeconomic model. The model was first developed by IMF to understand past developments in the US economy (Carabenciov et al. 2009). The model was then extended further to cover a wide range of economies (Benes et al. 2010). In this paper, we modify the model slightly to better reflect some characteristics of the New Zealand economy.



The advantage of this approach is that it is designed to address the criticism of the atheoretical nature of statistical filters as it contains structural equations describing an economy. Like the unobservable components model, this new approach allows us to quantify the uncertainty of the estimates. The new approach will become part of a toolkit in the New Zealand Treasury for assessing potential output and the results of this new approach will complement insights from other approaches.

The paper is organised as follows. Section 2 presents the small macroeconomic model. Section 3 outlines the data and the initial parameters of the model. Section 4 assesses the model's empirical fit by comparing how well the model matches the second moments of the data. The results of the estimates and their confidence intervals are presented in Section 5. A sensitivity analysis is performed in Section 6, where we analyse whether the estimates of the output gap are robust to an alternative measure of core inflation, different periods of estimation, a different assumption for the steady-state growth rate and different assumptions on the priors used in the estimation. Finally, Section 7 concludes.

## 2 Model

The model is composed of 4 main blocks, namely potential output, inflation, unemployment and capacity utilisation. In the equations that follow, a bar over a variable denotes the equilibrium value of the variable and  $\varepsilon$  represents the error term for each transition equation. In this paper, the model is called SMM (small macro model).

### 2.1 The model

#### 2.1.1 Potential output

$$Y_t = \bar{Y}_t + y_t \quad (1)$$

$$\bar{Y}_t = \bar{Y}_{t-1} - \theta(\bar{U}_t - \bar{U}_{t-1}) - \frac{(1-\theta)(\bar{U}_{t-1} - \bar{U}_{t-2})}{19} + \frac{G_t^{\bar{Y}}}{4} + \varepsilon_t^{\bar{Y}} \quad (2)$$

where  $Y_t = 100 \cdot \log(\text{gdp})$ ,  $\bar{Y}_t$  is potential output,  $y_t$  is the output gap and  $\theta$  is the labour share of income.

In the long run, the growth rate of potential GDP is mainly determined by the medium-term growth of potential  $G_t^{\bar{Y}}$ . Changes in the NAIRU  $\bar{U}_t$  may cause the short- and medium-term growth of potential output to differ from  $G_t^{\bar{Y}}$  and  $\theta$  is the labour share of a Cobb-Douglas production function (see Appendix 1 for the derivation).

$$G_t^{\bar{Y}} = \tau G_{ss}^{\bar{Y}} + (1 - \tau) G_{t-1}^{\bar{Y}} + \varepsilon_t^{G^{\bar{Y}}} \quad (3)$$

Equation (3) states that  $G_t^{\bar{Y}}$  is a function of its past value and the steady-state growth rate  $G_{ss}^{\bar{Y}}$ . The model incorporates a fairly flexible stochastic process – allowing for permanent changes in the underlying estimates of the equilibrium values for potential output ( $\varepsilon_t^{\bar{Y}}$ ) and temporary changes in the medium-term growth rate of potential output ( $\varepsilon_t^{G^{\bar{Y}}}$ ).

As monetary conditions are not modelled in the model, the deviation of the lagged core inflation rate ( $\pi_t$ ) from its expectations ( $\pi_t^e$ ) is used as a proxy of the movement of the short-term real interest rate. Therefore, the output gap equation is specified as follows:

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} - \frac{\rho_3}{100} (\pi_{t-2} - \pi_{t-2}^e) + \varepsilon_t^y \quad (4)$$

In order to capture richer dynamics of GDP growth in New Zealand, we add two lags of the output gap in the above equation instead of one lag in the original model.

Inflation expectations are modelled as follows:

$$\pi_t^e = (1 - \eta)\pi_{t-1}^e + \eta\pi_{ss} + \varepsilon_t^{\pi^e} \quad (5)$$

where inflation expectations are determined by their lag and the steady-state rate of inflation  $\pi_{ss}$ .

We augment the inflation expectations equation from the original model by adding the steady-state inflation  $\pi_{ss}$  variable. The rationale of adding  $\pi_{ss}$  is to reflect the current monetary policy framework of keeping the inflation rate between 1 and 3 percent over the medium run and the fact that inflation expectations were relatively well anchored over the observation period.

### 2.1.2 Inflation

$$\pi_t = (1 - \gamma)\pi_{t-1}^e + \gamma\pi_{t-1} + \beta y_t + \Omega(y_t - y_{t-1}) + \varepsilon_t^\pi \quad (6)$$

As shown in the above equation, the core inflation rate,  $\pi_t$ , is not only driven by the level of the output gap as specified in the New Zealand Treasury model (see Ryan and Szeto 2009) but also by the change in the gap. The reason for including the change in the gap in the inflation equation is to capture rigidities in the adjustment of the economy. The equation also includes the lagged inflation rate and inflation expectations.

### 2.1.3 Unemployment

The unemployment gap ( $u_t$ ) is defined as the difference between the equilibrium unemployment rate or NAIRU, ( $\bar{U}_t$ ) and the actual unemployment rate ( $U_t$ ). The dynamics of the labour market are mainly determined by the Okun's law. As shown in the following equation, the unemployment gap is a function of the output gap and the lag of the unemployment gap:

$$u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u \quad (7)$$

The equilibrium unemployment rate depends on the steady state of the unemployment rate ( $U^{ss}$ ) as well as the level of output gap, which captures a partial hysteresis effect from economy-wide demand fluctuations:

$$\bar{U}_t = \bar{U}_{t-1} + G_t^{\bar{U}} - \frac{\omega}{100} y_{t-1} - \frac{\lambda}{100} (\bar{U}_{t-1} - U^{ss}) + \varepsilon_t^{\bar{U}} \quad (8)$$

The inclusion of a persistent shock  $G_t^{\bar{U}}$  in the above equation allows for a persistent deviation of the NAIRU from the steady-state level of unemployment. The persistent shocks to the NAIRU follow a dampened autoregressive process.

### 2.1.4 Capacity utilisation

As in the preceding equation, the equilibrium capacity utilisation ( $\bar{C}_t$ ) includes transitory shocks ( $\varepsilon_t^{\bar{C}}$ ) as well as more persistent shocks ( $G_t^{\bar{C}}$ ). Similarly, the persistent shock also follows a damped autoregressive process and the capacity utilisation gap ( $c_t = C_t - \bar{C}_t$ ) is a function of the output gap and its own lag:

$$\bar{C}_t = \bar{C}_{t-1} + G_t^{\bar{C}} + \varepsilon_t^{\bar{C}} \quad (9)$$

$$c_t = \kappa_1 c_{t-1} + \kappa_2 y_t + \varepsilon_t^c \quad (10)$$

## 2.2 Steady-state assumptions

There are four key steady-state assumptions in the model. The values of these variables are shown as follows:

**Table 2 – Steady-state assumptions**

Labour share $\theta$	55%
Steady-state growth rate $G_{SS}^{\bar{Y}}$	2.5%
Steady-state unemployment rate $U^{SS}$	5.0%
Steady-state inflation $\pi_{SS}$	2.0%

In New Zealand, the labour share fell from 60% in 1988 to a low of 53% in 2002. According to the latest data, the labour share rose to 58% in the March 2012 year.<sup>1</sup> As a result, the steady-state labour share is set at 55%, as the average of the low and the latest value of the labour share.

The steady-state growth rate is set at 2.5% per year, which implies labour input and labour productivity are assumed to contribute 1% and 1.5% to annual GDP growth in steady state respectively.

The steady-state unemployment rate was assumed to be 4.5% in the 2012 Budget Economic and Fiscal Update (BEFU 2012). Since BEFU 2012, the unemployment rate rose unexpectedly to 7.3% in the September quarter 2012. In the context of the recent weakness in the labour market, the steady-state unemployment rate is set at 5% in the baseline model.

Although annual inflation averaged around 2.7% over the past 10 years, the steady-state inflation rate is set at 2.0%, which is the target midpoint of the Policy Targets Agreement 2012.

<sup>1</sup> The adjustment for the self employed made in the calculation of the labour share assumes that half of entrepreneurial income is classified as labour income.

### 3 Data and Parameterisation

The model is estimated using quarterly data from 1994q1 to 2012q3. Benes et al (2010) used the year-on-year rate of the core measure of the Consumer Price Index (CPI) in their paper. In this paper, the inflation used in this paper is the annualised quarterly percentage change of the 10% trimmed mean of the Consumer Price Index (CPI).

The reason for using quarterly percentage changes in CPI is to avoid the problem of overlapping data<sup>2</sup> with the year-on-year rate of inflation. As the quarterly percentage change in the core measure of CPI is very volatile for New Zealand, we use the 10% trimmed mean in our baseline model.

As the rate of Goods and Services Tax increased from 12.5% to 15% in the December quarter 2010, we removed the impact of the policy change on the inflation by lowering quarterly inflation by 1.8 percentage points for that quarter. The inflation expectations variable is obtained from expected annual CPI two years from now in the Reserve Bank New Zealand (RBNZ) survey of expectations. The data sources of the other variables can be found in the following table:

**Table 3 – Data and sources**

Variable	Source
$Y_t$	Real Production GDP (sa)
$\pi_t$	10% trimmed mean CPI inflation (sa)
$\pi_t^e$	Expected annual CPI 2 years from now from the RBNZ Survey of Expectations
$U_t$	Unemployment rate from the Household Labour Force Survey (sa)
$C_t$	NZIER's Quarterly Survey of Business Opinion (QSBO) - economy-wide capacity utilisation (sa)

The model is estimated using regularised maximum likelihood (Ljung, 1999), which requires users to specify priors for each parameter. The advantage of using regularised maximum likelihood is that if a model contains parameters that lack identification, the penalty function in the regularised maximum likelihood helps to shape up the likelihood so that its estimate is pushed towards the initial value. It is equivalent to a Bayesian methodology<sup>3</sup>.

As part of the estimation process, you need to specify the lower bound, the upper bound and the penalty expression of each parameter. It is important to stress that like any Bayesian estimation, the prior density for each parameter plays a significant role in determining the estimates and confidence intervals for potential output and other latent variables. For example, if one believes that potential output growth is less volatile, one

<sup>2</sup> The problem of using overlapping data could introduce a moving average term in the error term (see Hansen and Hodrik, 1980).

<sup>3</sup> See Appendix 2 for technical details.

way to achieve this is to reduce the size of the priors on the standard deviations of the shocks of the potential output process,  $\varepsilon_t^{\bar{Y}}$  and  $\varepsilon_t^{G\bar{Y}}$ .

The following table presents prior distributions and estimated posterior distributions for the model.

**Table 4 – Maximum regularised likelihood**

Parameter	Prior		Posterior	
	Mode	Dispersion	Mode	Dispersion
$\alpha$	0.900	0.158	0.898	0.022
$\beta$	0.400	0.316	0.321	0.045
$\Omega$	0.500	0.316	0.458	0.053
$\rho_1$	1.200	0.158	1.207	0.022
$\rho_2$	-0.400	0.158	-0.390	0.022
$\rho_3$	5.000	3.162	4.982	0.442
$\kappa_1$	0.100	0.632	0.061	0.076
$\kappa_2$	1.500	1.581	0.676	0.124
$\phi_1$	0.800	0.158	0.767	0.022
$\phi_2$	0.300	0.158	0.228	0.023
$\tau$	0.100	0.158	0.042	0.018
$\delta$	0.500	0.158	0.501	0.023
$\omega$	3.000	1.581	2.928	0.247
$\lambda$	2.000	3.162	2.013	0.440
$\gamma$	0.600	1.897	0.400	1.014
$\eta$	0.600	1.897	0.936	0.037
$\sigma_{\varepsilon^y}$	0.700	0.316	0.995	0.047
$\sigma_{\varepsilon^u}$	1.000	3.162	0.280	0.030
$\sigma_{\varepsilon^{\bar{u}}}$	0.080	0.158	0.155	0.021
$\sigma_{\varepsilon^{G\bar{u}}}$	0.050	0.158	0.105	0.021
$\sigma_{\varepsilon^c}$	0.400	0.316	0.710	0.036
$\sigma_{\varepsilon^{\bar{c}}}$	0.250	0.158	0.426	0.022
$\sigma_{\varepsilon^{G\bar{c}}}$	0.075	0.032	0.131	0.004
$\sigma_{\varepsilon^\pi}$	0.500	0.316	0.989	0.044
$\sigma_{\varepsilon^{\pi^e}}$	0.300	0.316	0.161	0.011
$\sigma_{\varepsilon^{G\bar{Y}}}$	0.040	0.032	0.084	0.004
$\sigma_{\varepsilon^{\bar{Y}}}$	0.067	0.032	0.122	0.005

## 4 Evaluating empirical fit

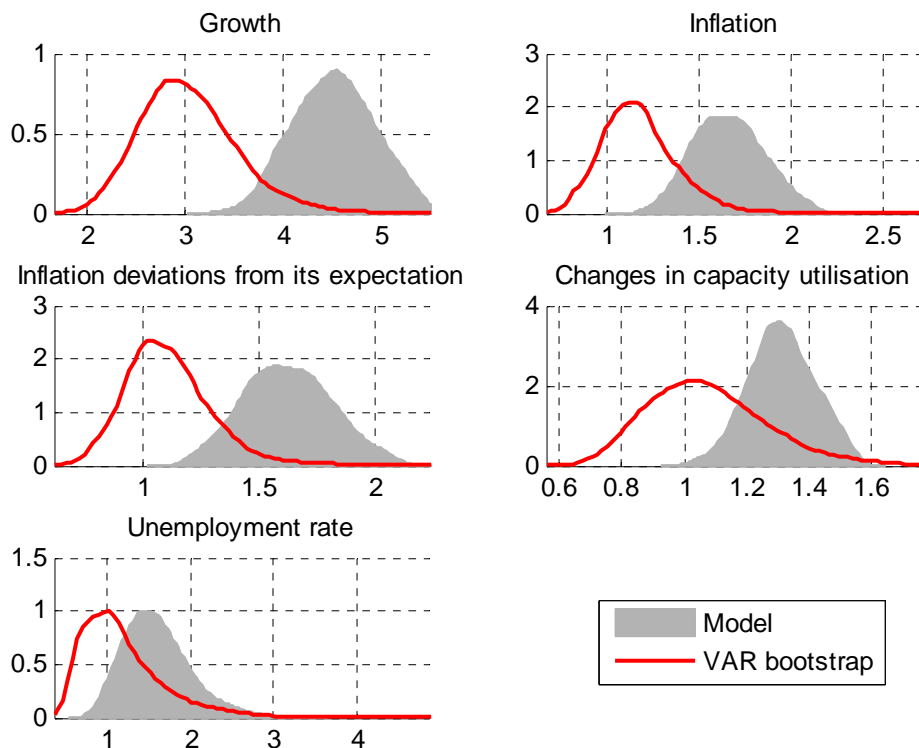
In this section, we compare the moment statistics of the model's estimates for the measurement variables to those from an unrestricted VAR of all the measurement variables. The order of the unrestricted VAR is chosen to be 4 in this experiment. The main purpose of this exercise is to evaluate how well the model fits the data. We chose only standard deviations and autocorrelation functions in the comparison.

In this exercise, we first simulate SMM for 2000 times using the Monte Carlo approach to generate 2000 data sets with each of these data sets containing the same number of observations as the historical data set.

The second step of the exercise is to generate another 2000 data set by bootstrapping the unrestricted VAR(4). We then compare summary statistics from these two large simulated data sets.

The volatility of fluctuations is usually measured using the standard deviation. In Figure 1, we check how well the model mimics the volatility of the data.

**Figure 1 – The probability density function of standard deviations**

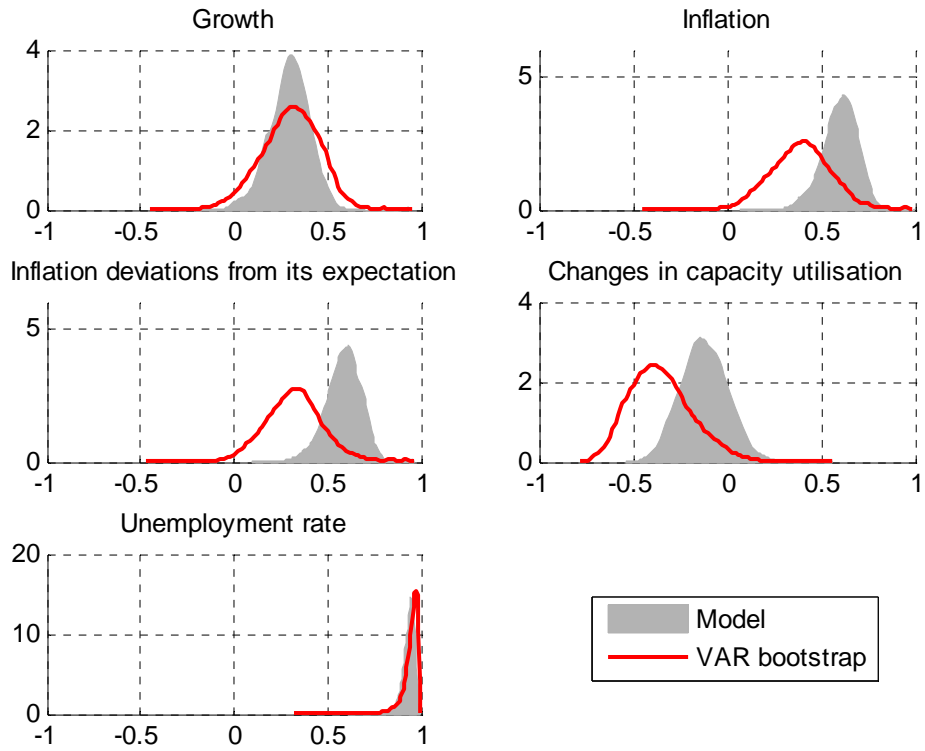


In general, the model exhibits a larger degree of volatility than those implied by the benchmark VAR. This excessive volatility suggests that we could develop the model further to improve its fit in the future.

Figure 2 shows the autocorrelation of order 1, which indicates the degree of persistence in the data. The model shows a consistent pattern of persistence as suggested by the VAR.

However, when we extend the comparison to a higher order autocorrelation (these moments are not presented in this paper), only core inflation, inflation deviations from its expectation and unemployment rate match those suggested by the data.

**Figure 2 – The probability density function of the first order autocorrelation coefficients**





## 5 Empirical analysis

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### 5.1 Estimates of the output gap and potential growth

Figure 3 presents the model's estimate of the output gap as well as the confidence band around the estimate. For comparison, we also report other output gap estimates (Multivariate filter,<sup>4</sup> MV, and Hodrick-Prescott, HP filter). The value of lambda is set at 1600 for both HP and MV filters. It appears that before the GFC, all the output gap measures moved together in broad terms and their differences were not statistically significant in general.

However, the crisis might have changed the dynamics of the economy. As a result, the output gap generated from the model over the post-crisis period is statistically different from the HP and MV filter output gap measures. However, caution is required in using the estimated confidence band to interpret the level of uncertainty, particularly at the end of the observation period. It will be shown in Section 6.4 that the estimated confidence band overstates the precision of estimates at the end of the sample.

According to the SMM, the estimate of the output gap troughs at about 2.7% in 2009Q2, which is slightly larger than the trough of 2.5% at the time of the Asian financial crisis. Furthermore, the SMM's estimate of the output gap remains negative at around 2% through the post crisis period, suggesting that there is spare capacity in the New Zealand economy.

This contrasts to the MV and HP filter gaps which were -1.7% and -2.7% respectively at the trough of the last cycle in 2009Q2 and converged back quickly to their trend. Currently, both the HP and MV gaps are positive, which suggests that there is no spare capacity in the economy.

The output gap estimates from the SMM suggest that care must be taken in using both the HP and MV filter to assess the level of potential output at the current juncture because the extent of the slack could be underestimated using the HP and MV gaps

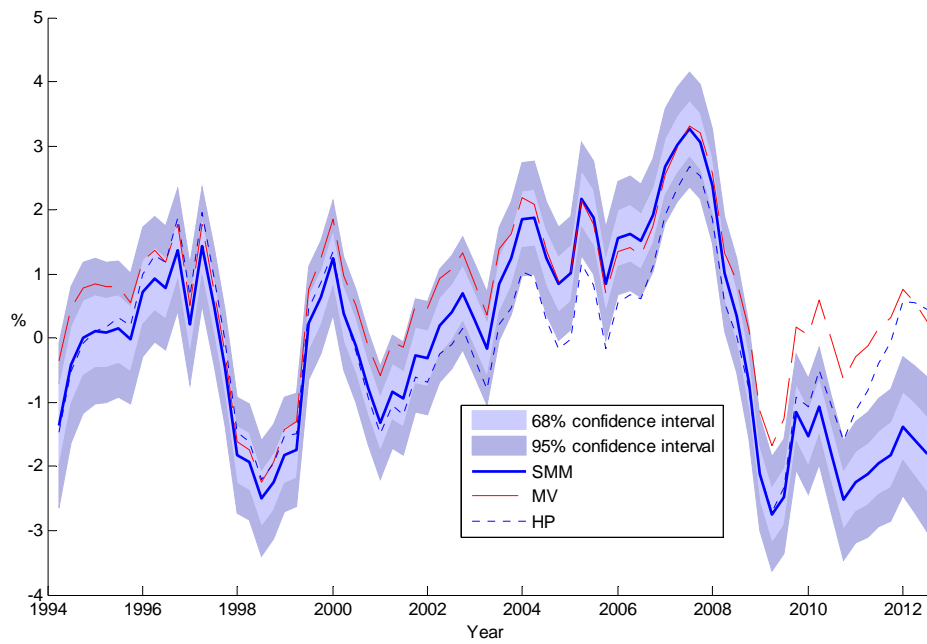
Figure 4 shows the estimated annual growth rate of potential output with confidence bands from the model, together with the growth rates stemming from the HP and MV measures of potential output. Like the output gap measures, all the measures of potential growth show a similar pattern over the pre-crisis period. However, both the HP and MV potential growth measures are below that of the SMM for most of the period 2007-2011, which leads to the divergence of the output gap between the SMM model and the HP/MV measures.

All the measures indicate that potential output growth has declined since 2003, well before the onset of GFC. According to the SMM, the rate of growth of potential output has fallen from a peak of 3.8% in 2003 to a trough of 1.1% in 2009. Currently, the growth rate of potential output is estimated at 1.8% with a 95% confidence band of 1.1% to 2.5%. The estimate is currently below the steady-state value of potential output growth of 2.5%. Labour input and labour productivity are assumed to contribute 1% and 1.5% to potential output growth in the steady state respectively. It is important to note that the current estimate is not a permanent state. However, the estimated parameter of equation (3) means that the speed of converging to the steady state is very slow.

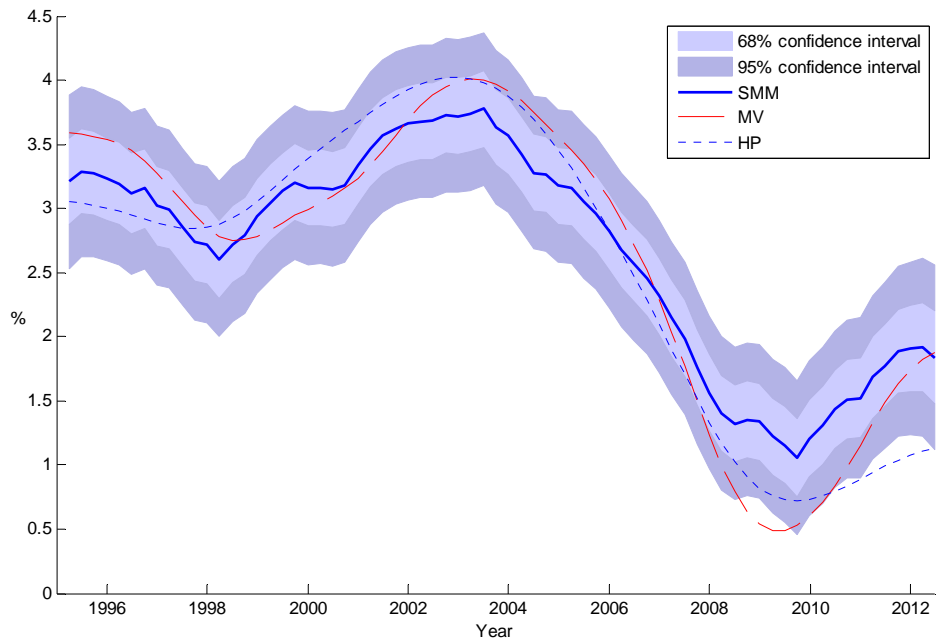
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<sup>4</sup> For technical details on multivariate filter, see Conway and Hunt (1997). We mitigate the end-point bias of the HP and MV filters by using forecasts to extend the sample by two quarters.

**Figure 3 – Measures of the output gap with its confidence interval**



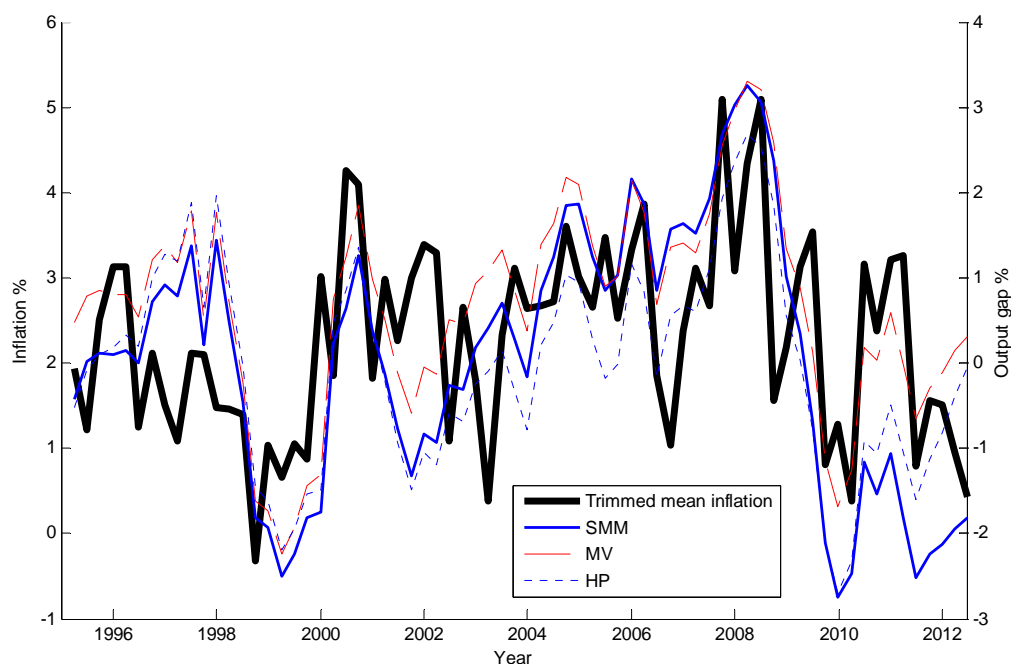
**Figure 4 – Potential growth rates (annual percentage change)**



## 5.2 Inflation and output gap

Figure 5 shows that there is a reasonably strong relationship between the inflation rate and the estimated output gap from SMM. In particular, inflation falls sharply a year after the onset of the GFC, which corresponds well with the development of the negative output gap. Overall, the degree of movement in inflation is closely in line with the evolution of the output gap.

**Figure 5 – Inflation and the output gap (lagged 3 quarters)**

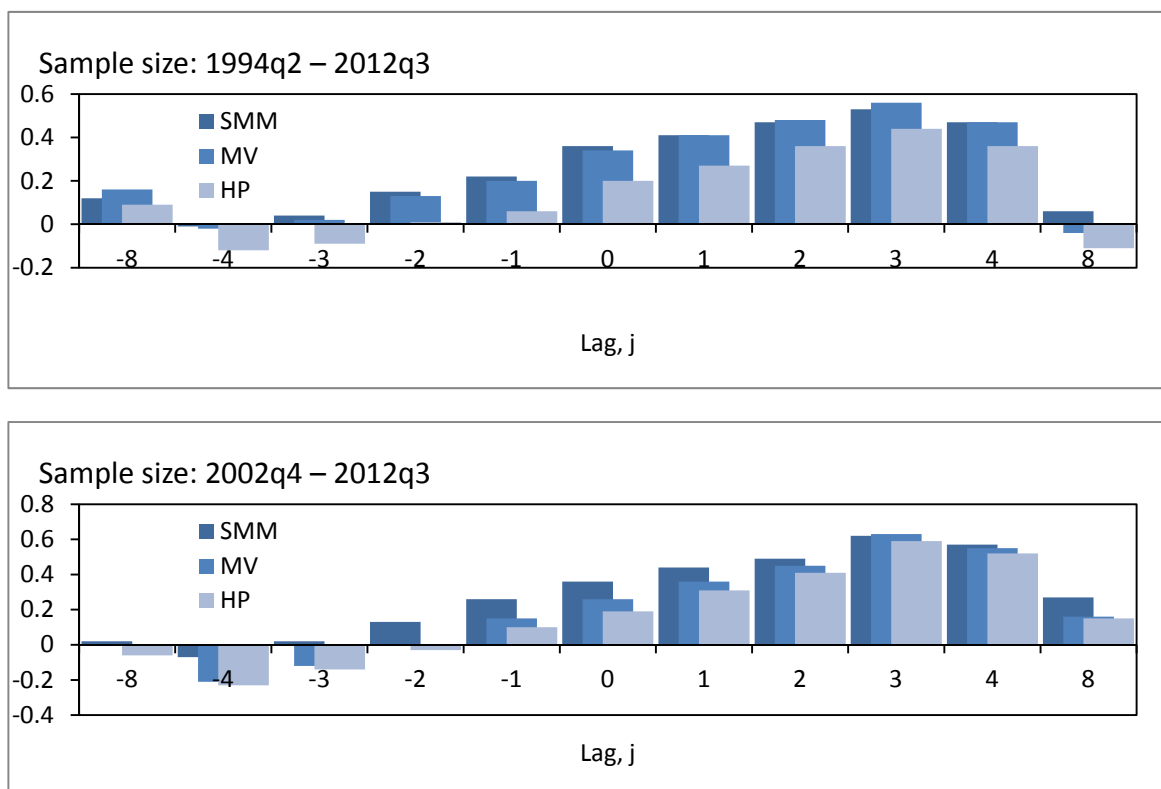


To test whether the SMM output gap outperforms the other output gap measures in explaining inflation, we use the cross-correlation coefficients between inflation and the output gap. Figure 6 presents the cross-correlation between inflation and the various output gap measures.

In general, the cross correlation coefficients indicate that both the MV filter and SMM output gaps move more closely with inflation than the HP filter output gap. The MV filter output gap records the largest value of cross-correlation of 0.56 at lag 3. If we restrict the sample size to the last 10 years of observations, the performance of the SMM output gap is still similar to that of the MV filter in tracking the movement of inflation.

With regard to timing, the largest value of the cross-correlation coefficient for the SMM output gap is 0.53 which also occurs at lag 3, indicating that inflation lags behind the SMM output cycle by 3 quarters. However, the contemporary correlation coefficient of 0.36 is rather large, reflecting that the model output gap has a positive impact on inflation in the same quarter.

**Figure 6 – Cross-correlations between inflation and output gaps**



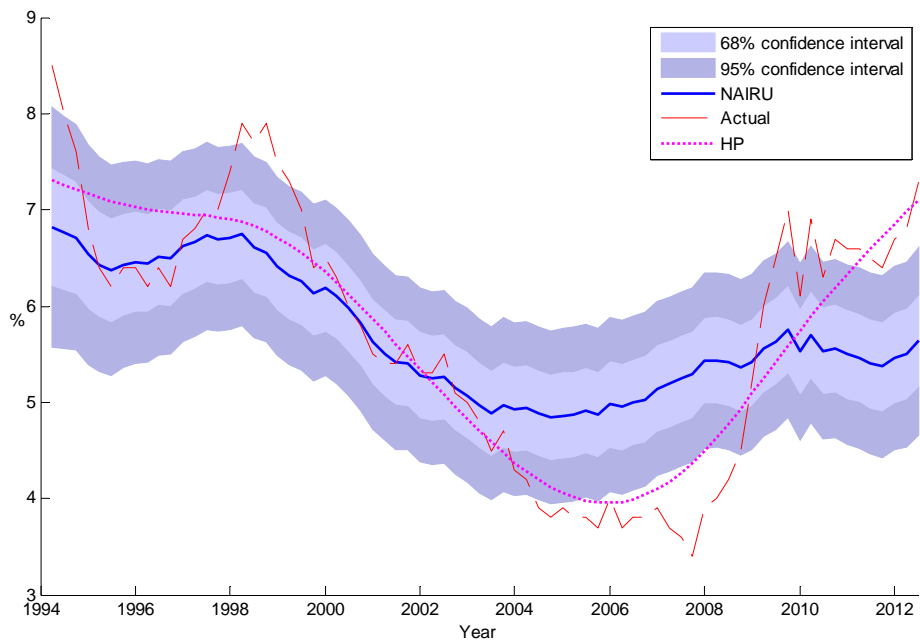
### 5.3 The unemployment gap and the output gap

Figure 7 shows the estimate of the NAIRU from the model. The NAIRU is estimated to be in the range of 6.4%-6.8% for the period of 1994-1998. Since then the NAIRU has declined steadily and reached a trough of 4.8% by the end of 2004. This finding is consistent with the previous work of Guy and Szeto (2004) who estimated the NAIRU over the period 1988q3 to 2003q4 using a Kalman filter on a reduced form approach. Based on the SMM estimates, the GFC may have caused the NAIRU to drift upwards to 5.6% at end of the sample period with a 95% confidence interval of 4.7% to 6.6%.

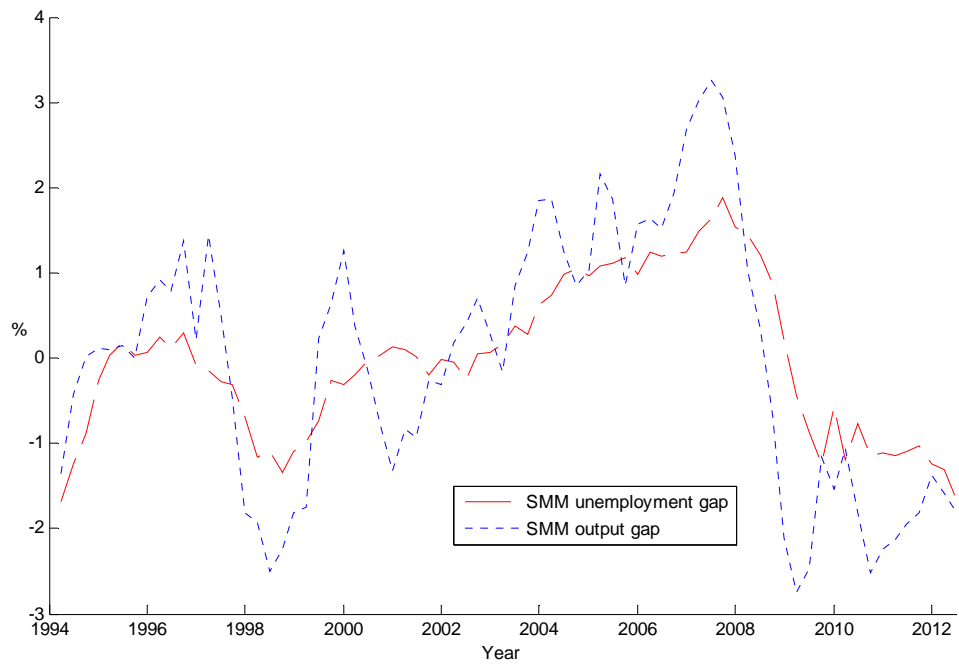
It is interesting to note that according to the HP filter, the NAIRU is estimated to be around 7% at the end of the sample period. Based on the HP filter, there is no spare capacity in the labour market, which is consistent with a positive HP filter output gap. However, the result of no slack in the labour market is definitely at odds with our perceptions and understanding of the economy – both inflation and wage outturns are still weak.

Finally, we assess the relationship between the estimated unemployment gap of the model and the production cycle as measured by the output gap by computing the cross-correlation of coefficients. The table below indicates that there is a very strong relationship between the unemployment gap and the output gap. The results are also supported by Figure 8, which compares the estimated output gap with the estimated unemployment gap.

**Figure 7 – the time-varying NAIRU with its 95% confidence interval**



**Figure 8 – Unemployment gap and SMM output gap**



**Table 5 – Cross-correlations between the unemployment gap and the SMM output gap**

	Lag j										
	-8	-4	-3	-2	-1	0	1	2	3	4	8
SMM Output gap	0.09	0.42	0.52	0.63	0.73	0.83	0.89	0.88	0.84	0.75	0.30

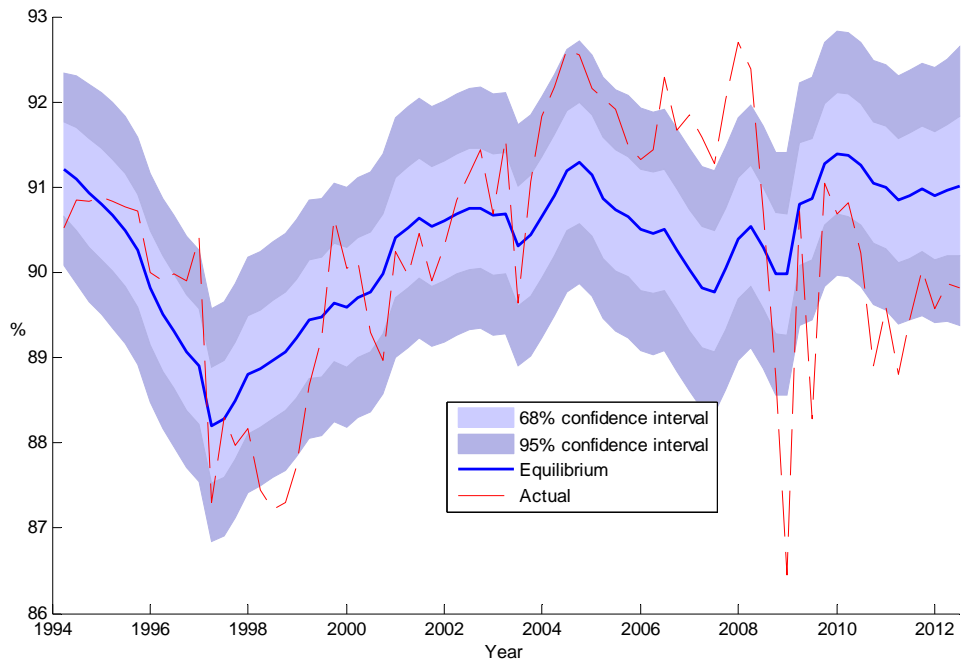
## 5.4 Capacity utilisation

Many commentators cited that the size of the output gap can be gauged by looking at surveyed measures of spare capacity from the Quarterly Survey of Business Opinion (QSBO). They argue that the size of the output gap is close to zero as the current rate of capacity utilisation (CUBO) was 89.8% in the 2012 September quarter, close to the 20-year average (90.2%). However, using the 20-year average or any other long-run average may not be appropriate as the average rate of utilisation has shifted over time, reflecting changes in sampling methodology or on-going changes in the structure of the economy. Figure 9 presents the estimated equilibrium capacity utilisation of the model as well as its confidence interval. This shows the estimated equilibrium at the end of the observation period is close to 91%, indicating some spare capacity.

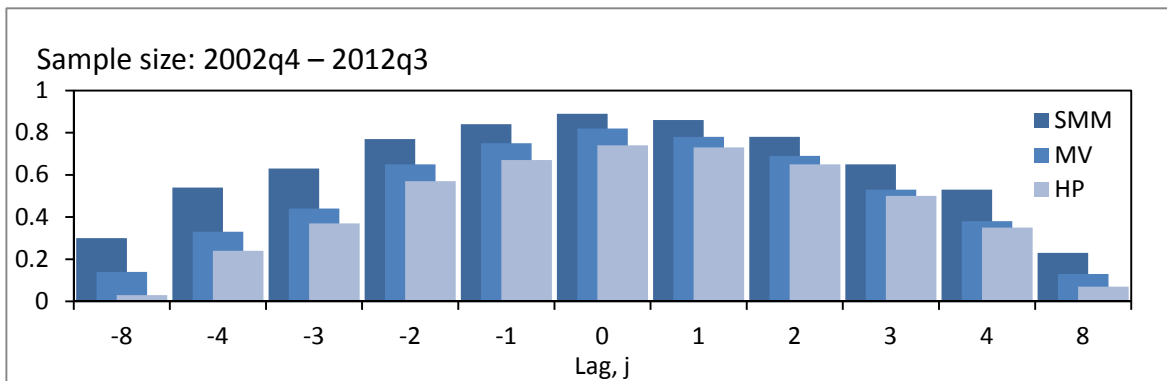
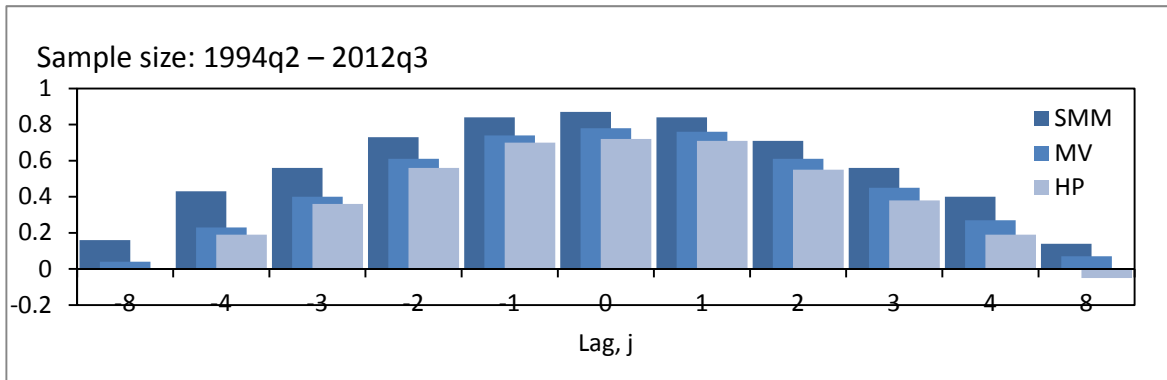
In order to test whether the model's equilibrium capacity utilisation is better than the 20-year average in explaining the level of potential output, we compute the cross correlation coefficients between various output gaps and its capacity utilisation gaps. The results, as shown in Figure 10a and Figure 10b, indicate that all the output gaps are more strongly correlated with the estimated gap from the model than the gap derived from the 20-year average, irrespective to the sample period of estimation.

From this evidence, we can conclude that compared with the estimated capacity utilisation gap derived from the 20-year average, the estimated gap derived from the model is a more reliable measure of changes in the intensity of utilisation of capital as suggested by various output gaps.

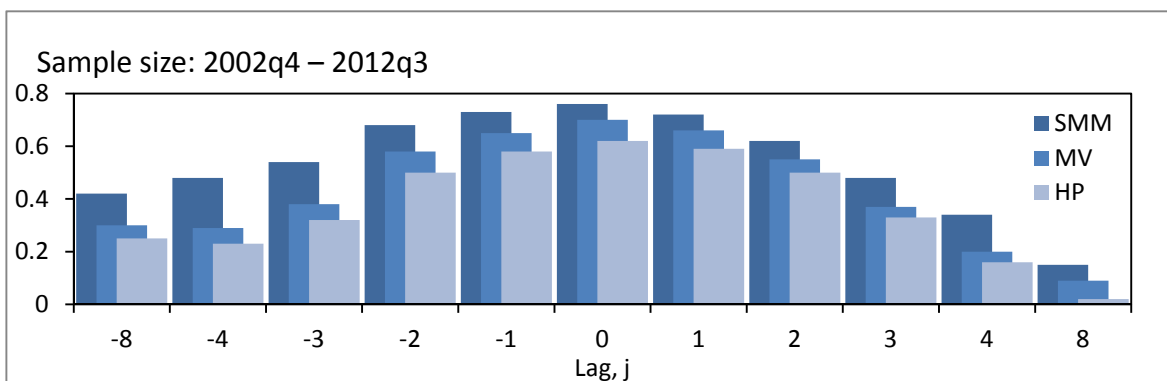
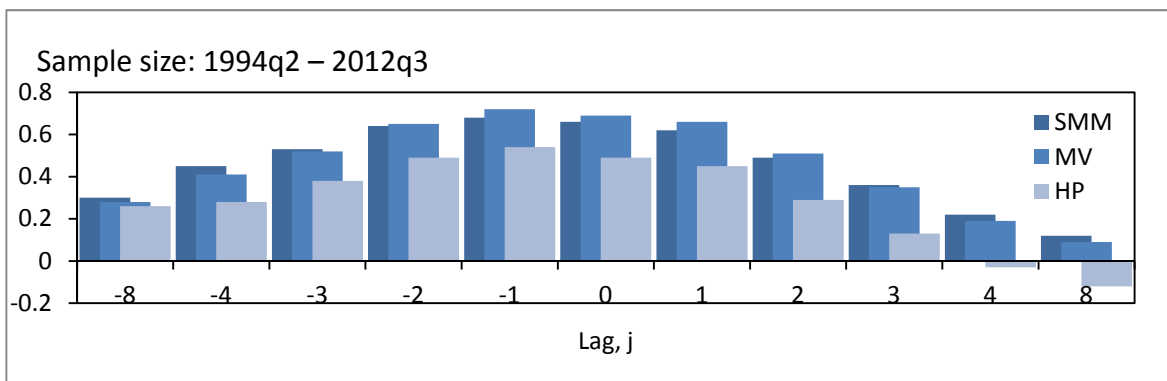
**Figure 9 – The estimated equilibrium capacity utilisation with its 95% confidence interval**



**Figure 10a – Cross-correlations between the estimated capacity utilisation gap from the model and various output gaps**



**Figure 10b – Cross-correlations between the estimated capacity utilisation gap from the 20-year average and various output gaps**





## 6 Sensitivity analysis

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In this section, we analyse whether the estimates of the output gap presented in the previous section are robust to alternative measures of inflation, different assumptions on the priors used in the estimation, different steady-state assumptions and different periods of estimation.

### 6.1 Different measures of core inflation

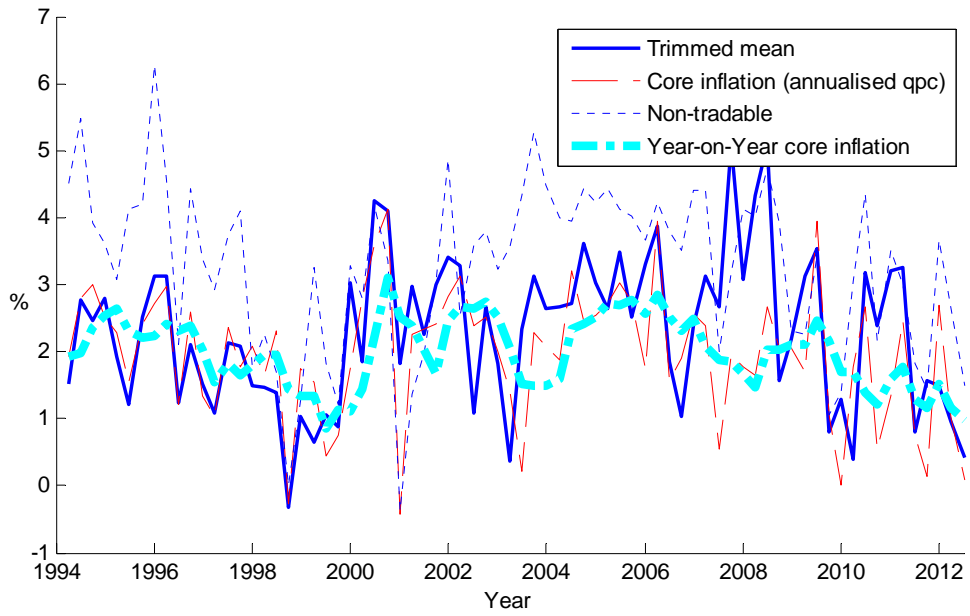
In this section, we analyse whether the model output gap estimates are sensitive to different measures of inflation. The alternative measures of inflation we use are core inflation and non-tradable inflation. Like the Benes et.al paper, the core inflation rate used in this paper is the Consumer Price Index (CPI) excluding the food group, household energy and fuel. We also test the sensitivity of the results by using two different forms of the core inflation rate, namely, the year-on-year change and the annualised quarterly percentage change.

Figure 11 illustrates the three alternative measures of inflation compared to the trimmed mean inflation used in the baseline model. The annualised quarterly percentage change of core inflation is very volatile and the volatility is significantly reduced by expressing core inflation in the form of year-on-year change. Non-tradable inflation shows the largest inflationary pressures over the entire sample period.

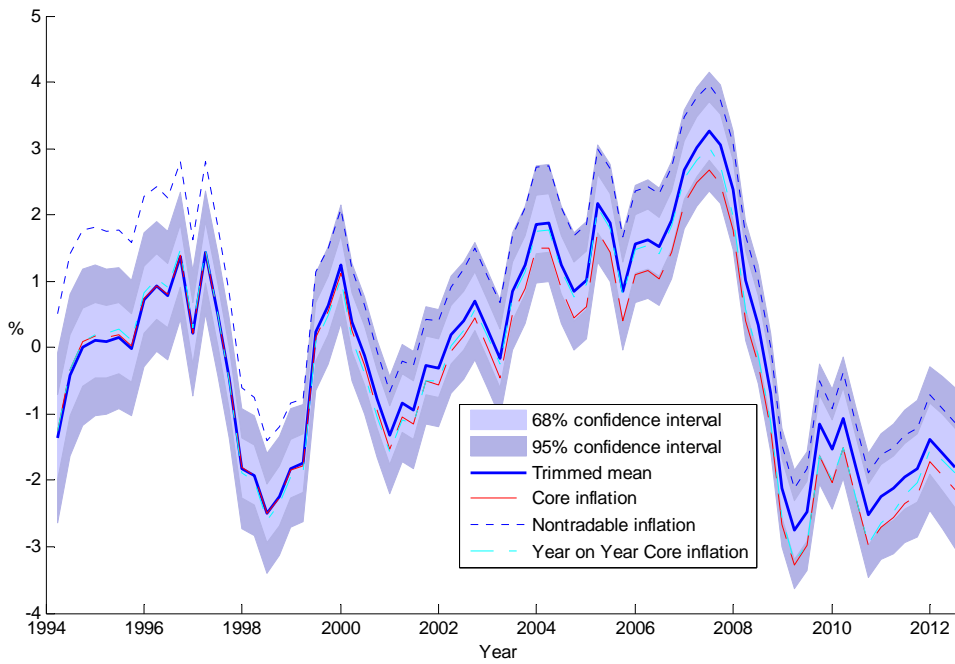
Figure 12 displays the output gap estimates using various measures of inflation, showing that the four different estimates of the output gap exhibit a similar pattern and the differences are well within the 95% confidence band. The results suggest that the estimate of the output gap is robust to different measures of inflation, reflecting the fact that the new methodology does not depend entirely on one particular piece of information but takes account of all available economic information in estimating potential output.

Although both measures of core inflation result in a similar measure of the estimates of the output gap, the inflation dynamics from the model does match well that of the annualised quarterly percentage change of core inflation. The inflation pressures generated from the model is more persistent and less volatile than that from the actual data. It is unlikely that one can develop a model which could generate such volatility in inflation.

**Figure 11 – Different measures of inflation**



**Figure 12 – Output gap estimates under different measures of inflation**



## 6.2 Alternative assumptions on the priors

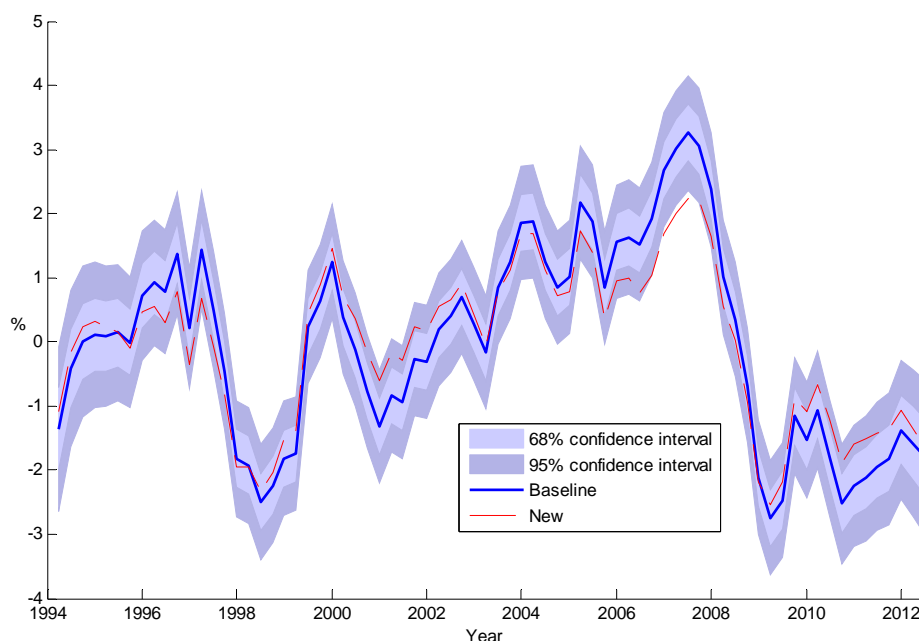
Owing to the ad hoc nature of setting the priors on the model's parameters, we assess the impact of the priors on potential output by doubling the size of the prior mean of the standard deviations of the shocks which drive the potential output process, namely,  $\sigma_{\varepsilon^{G\bar{Y}}}$  and  $\sigma_{\varepsilon^{\bar{Y}}}$ . The following table presents the prior and posterior means of the standard deviations and their variances for the new and baseline specifications.

**Table 6 – Different specifications on the priors**

	New				Baseline			
	Prior		Posterior		Prior		Posterior	
	mode	dispersion	mode	dispersion	mode	dispersion	mode	dispersion
$\sigma_{\varepsilon^{G\bar{Y}}}$	0.080	0.063	0.149	0.009	0.040	0.032	0.084	0.004
$\sigma_{\varepsilon^{\bar{Y}}}$	0.133	0.063	0.233	0.009	0.067	0.032	0.122	0.005

Figure 13 displays the output gap estimates obtained under our baseline and new specifications. The new estimates exhibit a similar pattern and are also within the 95% confidence band associated with the baseline estimate. However, the estimate of the new output gap is smaller than that of the baseline model, reflecting the fact that the new prior implies a more volatile potential growth rate. The result points to the importance of the size of the priors as they are key assumptions in determining the level of volatility of the measures of potential output. In other words, the model would generate a smoother potential output series (i.e., less volatile potential growth rate) if you reduce the size of the priors on  $\sigma_{\varepsilon^{G\bar{Y}}}$  and  $\sigma_{\varepsilon^{\bar{Y}}}$ .

**Figure 13 – Output gap estimates under different priors on  $\sigma_{\varepsilon G\bar{Y}}$  and  $\sigma_{\varepsilon \bar{Y}}$**



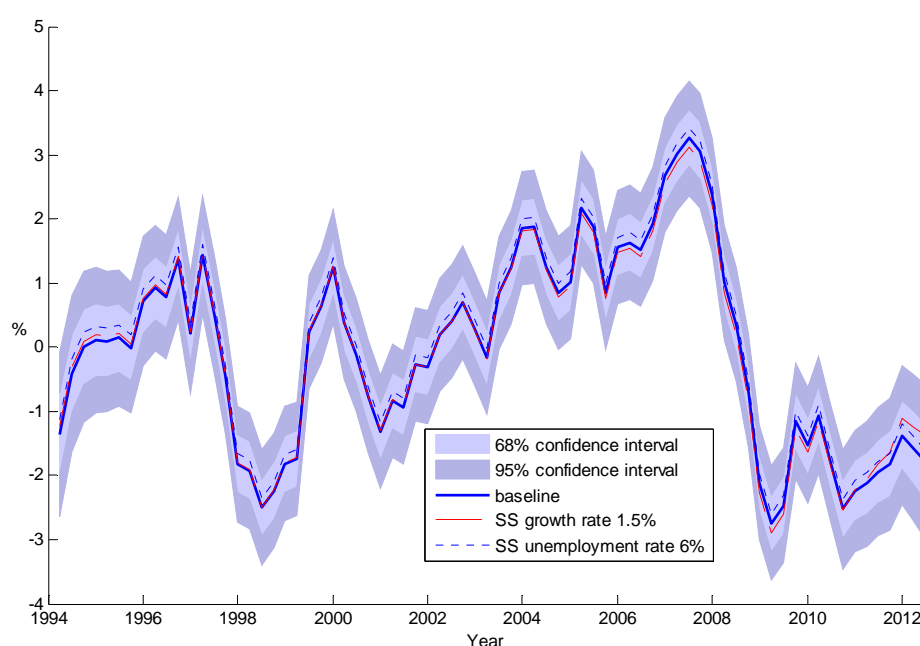
### 6.3 Steady-state assumptions

In this section, we check whether the output gap estimates are sensitive to the steady-state growth rate parameter and the steady-state unemployment rate parameter. Apart from at the end of the sample period, the output gap measures are almost identical to the baseline estimates when we lower the steady-state growth rate from 2.5% to 1.5% (see Figure 14). With an assumption of lower steady-state growth rate, the size of the negative output gap in the September quarter 2012 is estimated to be 1.4% which is slightly smaller than the baseline estimate of 1.8%.

Figure 14 also shows that increasing the steady-state unemployment rate from 5% to 6% does not result in a drastic change in the output gap estimates. Overall, the level of the output gap estimate shifts upward slightly across the whole sample period in comparison with the baseline estimates.

Overall, the output gap estimates are robust to alternative steady-state growth and unemployment rate assumptions.

**Figure 14 – Output gap estimates under different steady-state assumptions**



## 6.4 Endpoint problem

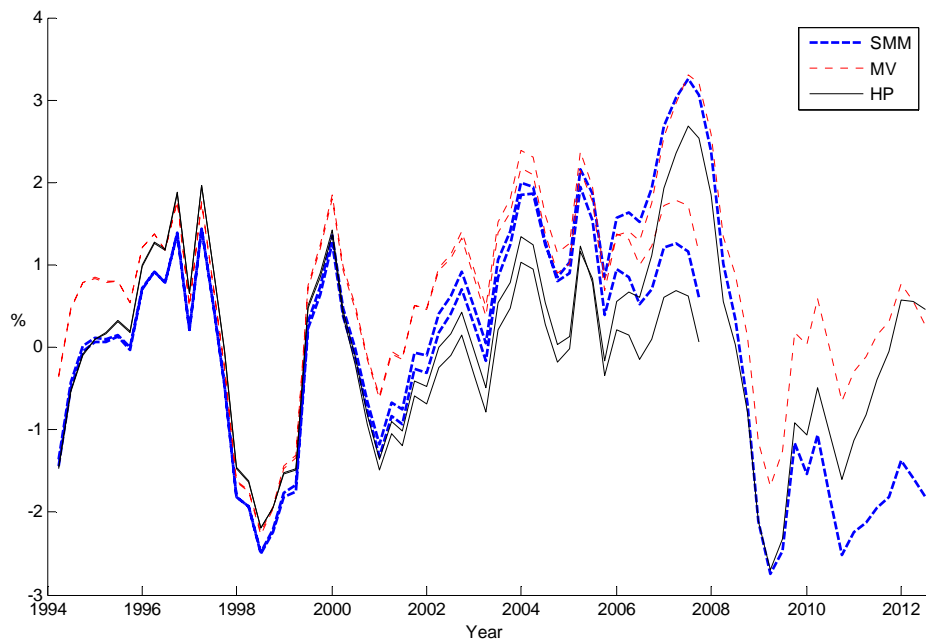
Although the output gap is a useful theoretical concept, it is very difficult to measure because of the unobservable nature of potential output. It is also well known that all the statistical filters employed in measuring potential output are subject to the “endpoint problem”, reflecting the fact the future is unknown. Ideally, the output gap estimates should not be revised drastically when new observations are added into the analysis. Figure 15a and 15b illustrates how various output gaps evolve when the sample is ended at two alternative points: 2007q4 (at the peak of a business cycle) and 2010q2 (one quarter before the September 2010 Canterbury earthquake).

When the sample is ended at 2007q4, all the output gap estimates are substantially different from the full sample estimates. The model output gap is initially estimated to be 0.6% at the December quarter of 2007q4 and the estimate is subsequently revised up to 3.1%. Similarly, the MV filter and HP filter estimates are revised up from 1.1% to 3.2% and from 0.1% to 2.5% respectively.

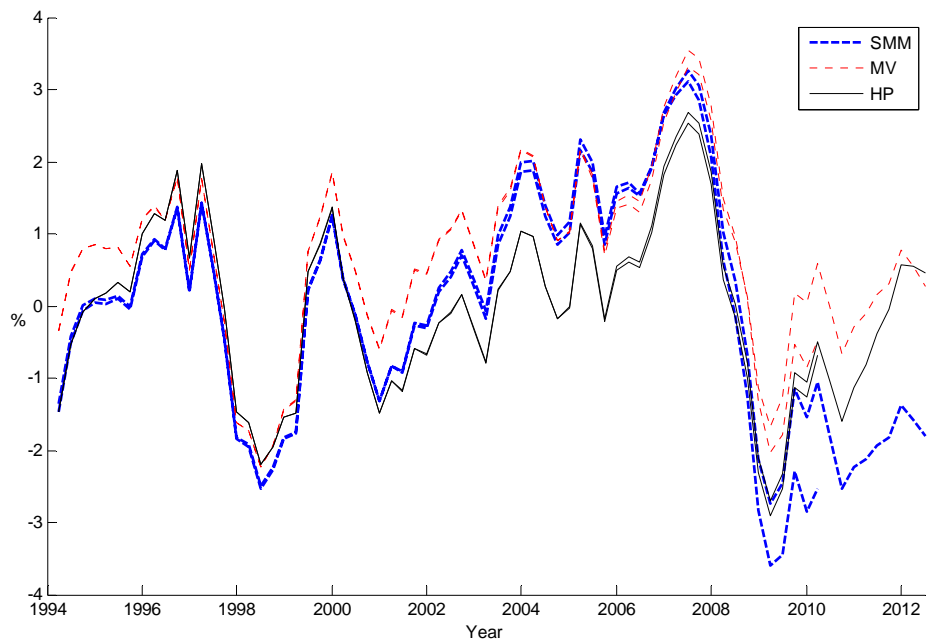
Apart from the HP filter estimates, the divergence between the initial output gap estimate and the full sample estimate at 2010q2 is also very large. According to the SMM, the output gap is initially estimated to be negative with a magnitude of 2.5%, which is revised up to -1.1%. Likewise, the MV filter estimate is revised up from -0.5% to 0.6%.

These findings suggest that although the new methodology incorporates more structural relationships, it is not able to eliminate large revisions in the output gap when the economy is subject to large shocks such as the GFC and the Canterbury earthquake. These findings are consistent with those found by Plantier and Karagedikli (2005). Thus, the addition of structural equations does not resolve the inherent difficulty of measuring the level of output gap with sufficient reliability in “real time”.

**Figure 15a – Output gap updates – sample until 2007q4**



**Figure 15b – Output gap updates – sample until 2010q2**



## 7 Conclusions

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In this paper, we estimated New Zealand's potential output within a framework of a small macro model. Compared with other statistical tools, the approach taken in this paper is considered to provide an economic framework for measuring potential output.

The key finding of this paper is that although there are a variety of methods for measuring output gaps, all the methods used in this paper provide a similar profile of output gaps over the pre-crisis period 1994-2008 when the economy fluctuated over a typical business cycle. Since the GFC, it has become increasingly difficult to distinguish between supply shocks and demand shocks. Both the HP and MV filter tend to generate a lower level of potential output. Thus, both filters interpret the crisis as having a larger negative impact on the supply side than is suggested by the SMM. Overall, the potential output estimates based on SMM result in a less cyclical profile.

Another key finding is that all the methods point to the onset of the slowdown of potential growth occurring before the financial crisis. A possible reason for this is that growth over the 2000s ultimately proved unsustainable as it was associated with large build-ups of household debt that could not continue indefinitely. As such the economy is going through a protracted adjustment period.

Although the main advantage of the method used in this paper is to incorporate additional relevant economic information, this new technique does not reduce the uncertainty in measures of potential output and its output gap as indicated by large confidence bands for the estimates. Furthermore, this new methodology, like other approaches, also suffers from the "endpoint" problem.

Whatever method is chosen for use, it is important to bear in mind that users need to understand all the judgements and assumptions behind the method as there are always implicit judgements built in to different approaches.

## Appendix 1: The derivation of the potential output equation

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In a neoclassical model, the level of capital stock (K) is determined by the following equation:

The real rate of return on capital ( $r$ ) = the marginal product of capital (MPk) (a1)

Assuming a Cobb-Douglas production function, the marginal product of capital is equal to

$$MPk = (1 - \theta)K^{-\theta}L^{\theta} \quad (\text{a2})$$

where L is the level of labour input.

Using (a1) and (a2), K can be expressed as the following function:

$$K = \left[ \frac{(1-\theta)L^{\theta}}{r} \right]^{\frac{1}{\theta}} \quad (\text{a3})$$

Therefore, the level of capital is determined by the level of labour supply. With all else being equal, one percent fall in labour supply should lead to one percent fall in the level of capital in the long run. As a result, a permanent 1% fall in labour supply should lower the level of potential output by 1%.

Hence,  $(1 - \theta)(\bar{U}_{t-1} - \bar{U}_{t-20})/19$  captures the effect of gradual changes in the capital stock as a result of a permanent change in NAIRU.



## Appendix 2: Regularised maximum likelihood

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Let  $\theta$  be a vector of parameters, let  $Y$  be the data and  $L(\theta; Y)$  be the data likelihood function. Then the parameters of the model are estimated by maximising the likelihood function subject to a penalty function:

$$\max_{\theta} \log L(\theta; Y) - p \sum_i \frac{(\theta_i - \bar{\theta}_i)^2}{\sigma_{\bar{\theta}_i}^2}$$

where  $\theta_i \in \{\theta_i^L, \theta_i^T\}$ .

The penalty function is a function of the distance between  $\theta_i$  and the prior  $\bar{\theta}_i$ , and the initial variance ( $\sigma_{\bar{\theta}_i}^2$ ) of the parameter. We can put more weight on the prior by increasing the value of  $p$ .

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