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Some Good News about an Ageing Workforce

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1 Introduction

The macroeconomic costs of population ageing depend, essentially, on how ageing affects two variables: labour productivity and the employment to population ratio which Cutler et al. (1990) called the support ratio. Here, macroeconomic costs refer to the loss of average living standards which are in turn defined as consumption per person. Consider the following decomposition of consumption per person:

$$\frac{C}{N} \equiv \frac{C}{Y}\frac{Y}{L}\frac{L}{N} \tag{1}$$

where C is consumption, Y is output, N is population, L is employment, and where all variables are national aggregates. This is a simple yet powerful identity. It shows that any macroeconomic effects of ageing on consumption per person must operate through one or more of three variables: the consumption share of output (C/Y), labour productivity (Y/L), and the support ratio (L/N).

The C/Y ratio increases slightly as the population ages – in particular as workforce growth slows down – because each year there are fewer new workers to be equipped with capital goods therefore allowing a greater share of national output to be available for consumption.[1] Simulations have shown that the effect on C/Y is very small relative to the effects on L/N and Y/L (Guest and McDonald, 2002). For simplicity changes in C/Y will be ignored in the rest of this paper. C/Y can be omitted from (1) and the identity re-expressed in terms of output per capita, Y/N:

$$\frac{Y}{N} \equiv \frac{Y}{L}\frac{L}{N} \tag{2}$$

This paper is concerned with the effects of an older workforce on the two variables on the right hand side of (2): Y/L and L/N. The next section looks at policies being developed in OECD countries to boost the employment of older workers as a way of offsetting the falls in L/N that would occur at current labour force participation rates. Section 3 reviews the some of the channels, recognized in the literature, through which an ageing workforce can affect Y/L. Sections 4 and 5 provide the substantive contribution of the paper, which is to explain a new channel through which an ageing workforce can affect Y/L and provide a simulation analysis. It is “new” in the sense that it had not been considered in the literature until Prskawetz and Fent (2004) provided a discussion and simulation analysis for the Austrian economy. The purpose here is to extend the analysis in Prskawetz and Fent (2004) and to apply it to the Australian economy. The simulation results suggest that the size of the effect of an older workforce on labour productivity could be positive and of substantial magnitude.

2 Policies to boost the support ratio (L/N)

Population ageing eventually lowers the support ratio because declining fertility and declining mortality ultimately imply a lower ratio of workers to consumers. Assuming no

[1] This slight boost is somewhat offset for debtor countries, like Australia and New Zealand, because fewer new workers implies that a smaller increase in total foreign debt, which can be generated through consumption, would be permitted for a given debt per worker.

changes in labour force participation rates (LFPRs) and no change in unemployment, the support ratio for Australia falls would fall steadily to be 15 percent lower by the year 2050 than it is today, and a further 5 percent lower by the year 2100. This would imply commensurately lower living standards – that is, lower than they would have been with a constant support ratio.

In OECD countries, considerably policy attention is being directed towards increasing LFPRs of older workers and ensuring that there is demand for their labour.[2] This attention to both the supply-side and demand side is important. It would wasteful if older workers seeking jobs were unable to find them due to discrimination – however subtle it may be – against older workers. Similarly ineffective would be a situation where the benefits of older workers were being increasingly recognized by employers but older workers showed no greater willingness to participate in the labour market.

On the supply-side, policies are focusing on:

- superannuation and pension policy in order to encourage later retirement
- welfare reform to discourage older workers from moving on to forms of welfare other than the pension, such as disability benefits
- incentives for older workers to enter labour market training programs
- careers counseling for older workers

Of these, the changes to superannuation and pension arrangements have received the greatest attention and represent the best chance of increasing the LFPRs of older people. In Australia, bodies that represent superannuation funds such as ASFA and IFSA[3] have called for changes to taxation arrangements affecting superannuation, such as:

- abolishing the superannuation contributions surcharge[4] which is an additional tax imposed on the superannuation contributions of high income individuals;
- removing the limits on tax-deductible superannuation contributions - the limits are in dollar amounts and increase with age of the taxpayer;
- removing the work test on contributions;
- increasing the age up to which working members of super funds can make personal super contributions
- lowering tax rates on superannuation contributions and superannuation fund earnings;
- changing the tax rules on income streams by allowing variable income streams and therefore to encourage superannuation funds to incest away from interest-bearing assets to growth assets;

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[2] The OECD has published a series of country studies called *Ageing and Employment Policies*, covering about 20 countries, in which it explains the policy initiatives that have been taken, and are planned to be taken, to boost the employment of older workers. The series is unfinished at the time of writing (Feb, 2005). See http://www.oecd.org/findDocument/0,2350,en_2649_34747_1_119699_1_1_37435,00.html

[3] ASFA is the Association of Superannuation Funds of Australia and IFSA is the Investment and Financial Services Association Limited. See for example IFSA (2002) “Retirement Incomes and Long Term Savings: Living Well in an Ageing Society”.

[4] The tax is being reduced from 15% to 10% over 3 years and applies to income over $114981 in the 2003/4 year. A lower surcharge applies to income over $94691 in the 2003/4 year.

Aside from policy drivers, there are reasons to expect the participation rate of older workers to increase in the coming decades. Day and Dowrick (2004) provide evidence that the decline in fertility since the 1960s has been associated with a substantial increase in female LFPRs. They argue that this will continue - in particular with respect to older women as the higher education attainments of young women today will result in much higher LFPRs of older women in the future. On that latter point, Dowrick and McDonald (2002) make the following interesting comparison. Only 25 percent of the current cohort of 50 year old women completed the final years of high school education. Whereas in coming decades nearly 80 percent of 50 year old women will have completed Year 12. They argue that the resulting boost in LFPR of older women in the future will be enhanced by the fact that the LFPRs of the current 25-34 year old women are much higher than in the past. The resulting familiarization with the paid workforce and the accumulation of human capital which this implies, will lead to higher LFPRs of this cohort as they enter their 50s and beyond compared with the current cohorts of women of those ages.

Policies are being directed to the demand side of the labour market. An important determinant of labour demand is labour costs per unit of output which depend on labour productivity relative to labour costs. For given labour costs, increased investment in human capital of workers increases their productivity and therefore increases labour demand. Hence policy attention is being directed largely toward boosting the human capital of older workers. Measures to do this include (see Encel, 2003 for further details):

- career counseling , courses to upgrade skills and assistance with job applications
- financial assistance with accredited training courses undertaken by job seekers
- Personal advisers to provide more intensive individual assistance to unemployed people with special difficulties
- Training places in information technology for persons aged over 45

Other policies to boost demand for older workers include the abolition in 1999 of compulsory retirement in the Australian public service and the introduction of Anti-age discrimination legislation.

The rationale for this intensive policy focus on older workers derives from the fact that the duration of unemployment for workers over 45 years is twice the duration for younger workers, even though the rate of unemployment is lower for older workers. Long term unemployment is a more serious problem in social and economic terms than short term unemployment.

Later in the paper simulations are reported which show the potential for these policies to offset the effect of ageing on L/N and therefore on output per capita (see the discussion of Figure 7).

3 The effect of ageing on labour productivity (Y/L)

The effect of population ageing on labour productivity is a critical but elusive factor in determining the costs of population ageing for economic welfare. It is critical because labour productivity growth could potentially offset – indeed swamp - the economic burden will occur through a falling ratio of workers to consumers. However, it is not clear to what extent if any, and in what direction, productivity growth will be affected by population ageing. This remains an elusive question - unresolved in theory and empirically. For a synthesis and critique of the literature, see Chapters 4 to 7 in Birdsall, Kelley and Sinding (2001) and Chapters 1 to 8 in Mason (2001).

It is possible that population ageing could boost labour productivity through several channels: capital deepening, stronger incentives to innovate in the face of scarce labour (Habakkuk, 1962; Romer, 1990), and a higher saving rate due to both a higher working age proportion of the population and the effects of fewer children in postponing household consumption in the lifecycle (Fry and Mason, 1982; Mason 1987, 1988). There could also be dynamic gains from falling fertility rates through greater human capital creation and therefore growth, which in turn lowers fertility (Becker, Murphy and Tamura, 1990; Day and Dowrick, 2004).

On the other hand there are several factors that could work in the opposite direction: a loss of scale economies, an older and perhaps less dynamic workforce which inhibits innovation, a reduction in the supply of new researchers (Jones, 2002), and lower saving which could result from lower fertility if parents are altruistic and generations are linked in parents' welfare functions (Rebelo, 1992).

Traditional growth accounting is a useful place to start in identifying the effects of demographic change on growth. Using this framework, four sources of growth in per capita output can be derived (see the Appendix). One factor – technical progress – is a source of long run growth; and three other factors affect short run growth: investment, the capital deepening effects of labour force growth (a negative effect if labour force growth is positive) and the rate of change in the employment to population ratio.

4 A new dividend from population ageing, affecting (Y/L)

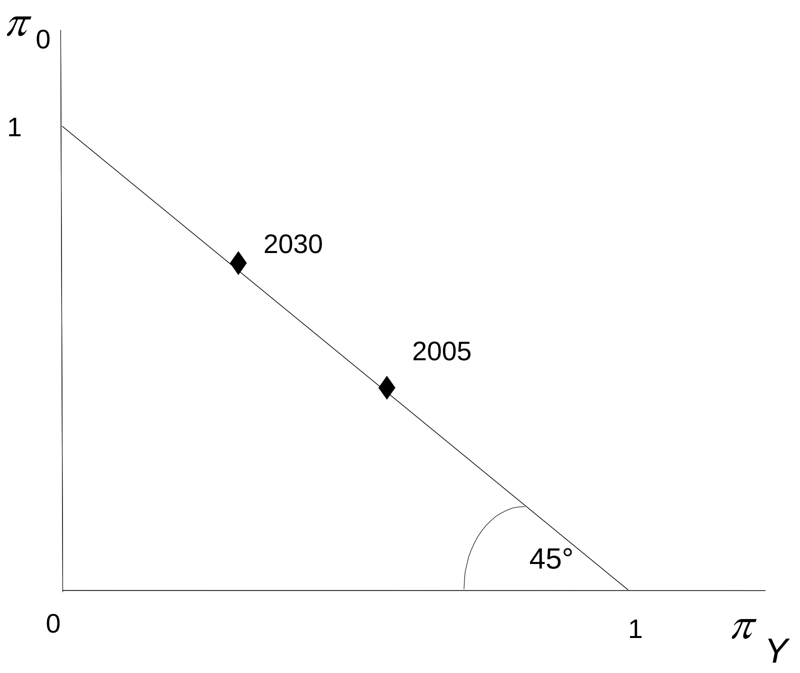
This paper focuses on a fourth source of short run growth, which occurs through changes in the age distribution of the labour force. It is well-known that age-dependent differences in the productivity of workers imply changes in aggregate labour productivity in response to changes in the age distribution of the workforce. There is an additional factor related to the age distribution, however, that is much less known. Prskawetz and Fent (2004) show that the degree of substitutability between workers of different ages is another, potentially important, factor affecting aggregate labour productivity. This is the idea that there is an optimal age mix of the workforce and that demographic change can move the actual age mix either closer or further from the optimal mix and therefore affect aggregate labour productivity. The magnitude of this "age distribution effect" depends on how substitutable workers are by age.

Macroeconomic models of population ageing have typically assumed that once workers of different ages are adjusted for their productivity differences they become identical inputs in economic terms. That is, they are infinitely substitutable. The seminal example is Cutler et al (1990); for an Australian application see Guest and McDonald (2001) and for New Zealand see Guest, Bryant and Scobie (2004). The assumption of infinite substitutability is clearly unrealistic because it doesn't account for any degree of complementarity between workers of different ages. Examples of complementary age-dependent skills include the physical strength of young male workers that complements the skills that older workers have in managing people, including mentoring younger workers, and making decisions.

Such complementarities are typically assumed away in calculating the index of aggregate employment used in modeling the macroeconomic impact of demographic change.

If workers are all equally productive, the index of labour (which is also output here) is completely independent of the age distribution of the labour force. This is illustrated in Figure 1 which that there are two types of labour: young and old. A linear isoquant is drawn giving combinations of the proportion, π_Y, of young workers, and the proportion, π_O, of old workers for a given output (maintaining the assumption of a labour only production technology). Two points are drawn on the isoquant representing the hypothetical actual age distribution of labour in 2005 and 2030. The assumed equal productivity of young and old workers is indicated by an isoquant with a 45^O angle implying that the proportions of labour can change without affecting output.

Figure 1 – The case where workers are equally productive and perfectly substitutable



Continuing with this diagrammatic representation, if the old workers are, on the other hand, more productive than the young workers, the isoquant is drawn with an angle greater than 45^O as shown in Figure 2. It is still linear, reflecting perfect substitutability between old and young workers. In this case, the older the age distribution the greater the output. Indeed the optimal age distribution is one where all workers are old, as indicated on the Figure. Hence, a shift to an older age distribution in 2030 results in higher output, represented by a point on an isoquant that is further out from the origin.

Figure 2 – The case where older workers are more productive than, but equally substitutable with, younger workers

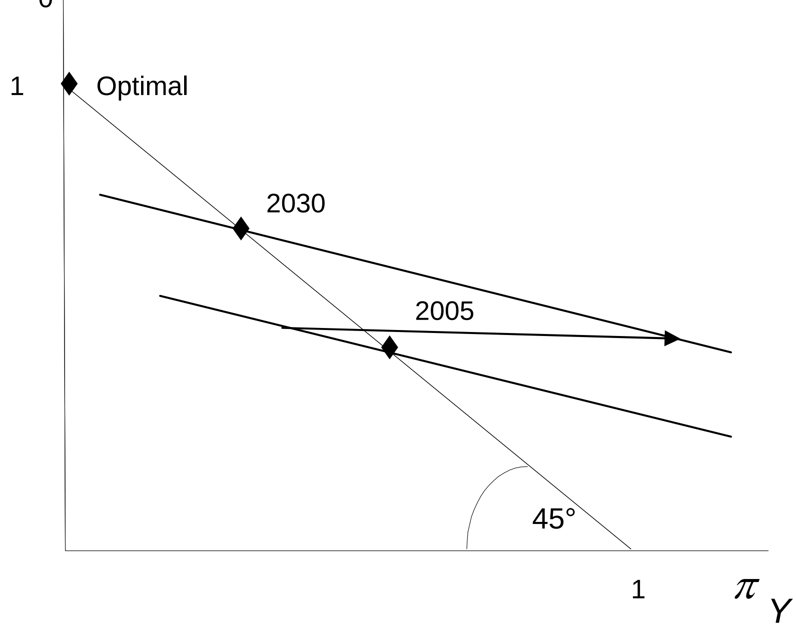
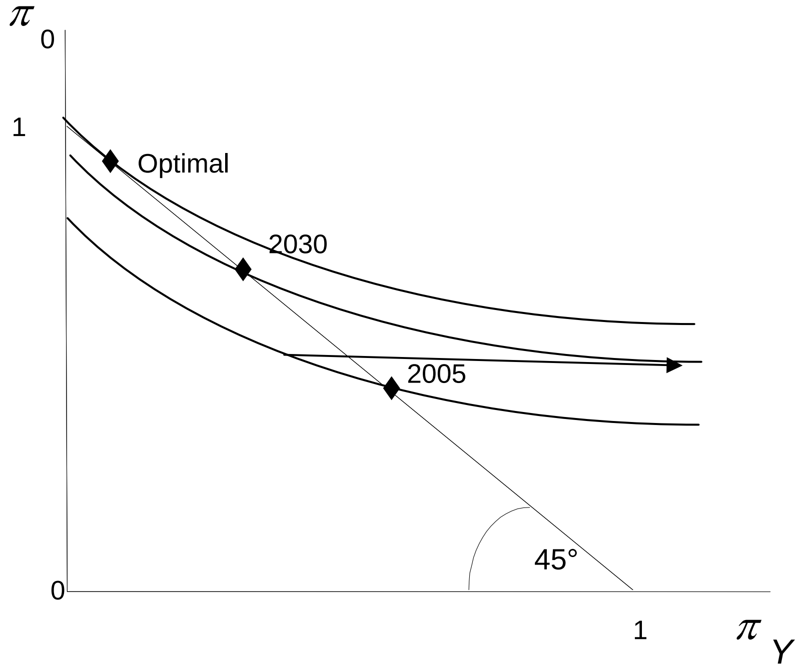


Figure 3 – The case where older workers are more productive than, and imperfectly substitutable with, younger workers



Finally, if labour inputs are less than perfectly substitutable, the isoquant is convex as illustrated in Figure 3. In that case, a shift to an older population may or may not result in higher output. It depends on where the optimal age distribution is in relation to the actual age distribution in 2005 and 2030. Figure 3 is drawn such that the optimal age distribution is older than that in 2030 which is in turn older than that in 2005. This implies higher output in 2030 than in 2005.

Note that, in Figure 3, relative wages of young workers increase as the age distribution becomes older, that is, moving from 2005 to 2030 to the optimal distribution. This is because the ratio of young wages to old wages at any given point on the 45^{O} line is equal to the slope of the isoquant at that point. That in turn is derived from the profit maximizing condition that the marginal labour costs of producing an extra unit of output are equal for labour inputs of all ages:

$$\frac{w_Y}{MPL_Y} = \frac{w_O}{MPL_O}$$ or, rearranging,

$$\frac{w_Y}{w_O} = \frac{MPL_Y}{MPL_O} \quad (3)$$

where, respectively, w_Y and w_O are the real wage of young and old workers, and MPL_Y and MPL_O are the marginal product of labour of young and old workers. The ratio of the marginal products – the right hand side of (3) – is the slope of the isoquant. At the optimal distribution, the isoquant is tangential to the 45^O line implying that the marginal products and therefore real wages are equal.

For the general case the optimal age distribution is given in Prskawetz and Fent (2004), citing Lam (1989), as[i]

$$\frac{\pi_j}{\pi_i} = \left(\frac{\alpha_j}{\alpha_i}\right)^{\sigma} \quad (4)$$

where the π's are the shares of the labour force in age groups denoted by *i* and *j*, the α's are the productivity weights and σ is the ES between labour inputs i and j. If the productivity weights are equal then $\alpha_i=\alpha_j$ and the optimal age shares are equal no matter what the value of the ES. If, on the other hand, older workers are more productive than younger workers, then their optimal labour force share is higher than for younger workers by a factor given by the ES. The lower the ES, the less variation in optimal labour shares for any given productivity weights. Intuitively, a low ES implies that different types of labour are very complementary – they cannot be easily substituted. Therefore small adjustments to the type of labour employed cause relatively large changes in marginal products, and hence relatively small adjustments away from equality of labour shares are required to equate marginal products.

The central point that emerges from the above analysis is that the effect of changes in the age distribution of the workforce on average labour productivity depends on the degree of substitutability between workers of different ages.

The simulation analysis in this paper follows broadly the approach in Prskawetz and Fent (2004) and extends it by replacing their "fuzzy CES" function with a CRESH function, special cases of which include the CES, additive and Leonteif functions.[5] The simulations here use Australian data, whereas Prskawetz and Fent (2004) used Austrian data. The results suggest that the age distribution effect could be easily as important for growth as policies that aim to increase the labour force participation rates (LFPR) of older workers. The effect could be sufficient to completely offset the effect on output per capita of projected declines in the employment to population ratio associated with population ageing. This amounts to a dividend from population ageing that has hitherto not been recognised.

See Appendix B for further technical analysis and description of the data.

5 Simulation results

The aim of the simulations is determine the effect on labour productivity (*Y/L*) and output (*Y*) of the projected change in the age distribution of the labour force under alternative assumptions about the ES between labour of different ages. The direction and size of this

[5] CRESH stands for Constant Ratio of Elasticity, Homogeneous. The CRESH function was introduced b y Hanoch (1971).

effect will depend on whether the age distribution is projected to move closer to the optimal distribution or further away from it, and on the elasticities of substitution.

Four cases are compared:

1. An infinite ES between labour of all ages, which implies the additive production function (B6 in Appendix B).

2. CES=2.

3. CES=1 (i.e. Cobb Douglas)

4. CRESH, with the hump shaped pattern of a_{ij} given in Table 1.

Figure 4 illustrates the key result and Figure 5 illustrates the reason for that result. Aggregate labour productivity is relatively constant in the case of infinite elasticity because the lower proportion of younger workers can be replaced, with infinite ease, by older workers. Relaxing this assumption by assuming a finite elasticity leads to increases in aggregate labour productivity over time. The reason is that the age distribution is projected to become closer to the optimal distribution, as illustrated in Figure 5. The resulting effect on labour productivity is therefore described here as the age distribution effect. The magnitude of the age distribution effect depends on the differences between the optimal age distribution and the projected actual distribution. The optimal distribution itself depends on the ES; the higher the ES, the more hump-shaped is the optimal distribution and therefore the closer it is to the projected actual distributions. This is illustrated in Figure 5 by plotting the optimal distribution under two constant values of the ES: ES=1 and ES=2. Therefore the gain in labour productivity over time is greater for CES=1 than for CES=2 (see Figure 4).

Figure 4 – Aggregate labour productivity

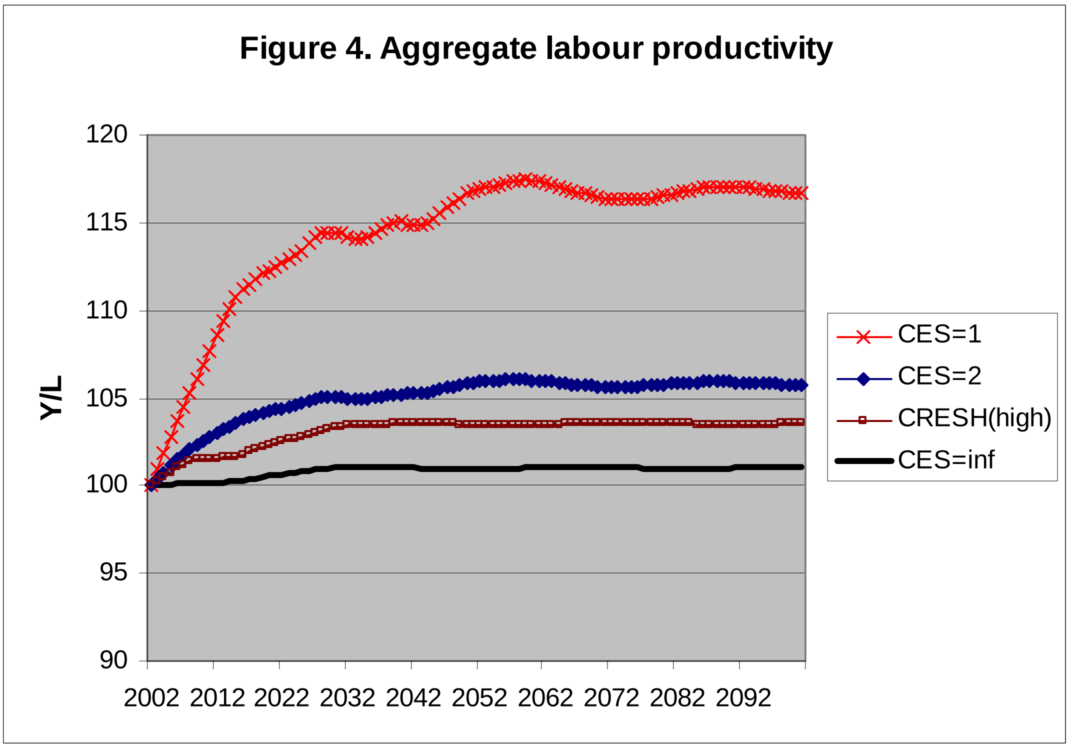
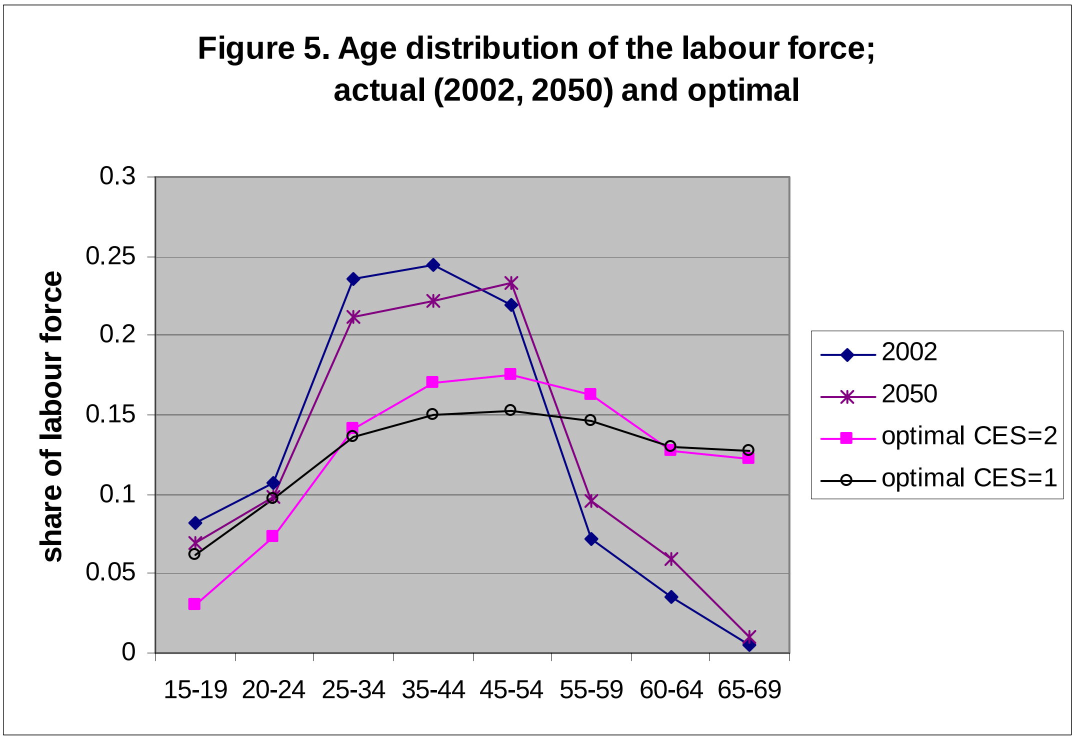


Figure 5 – Age distribution of the labour force: actual (2002, 2050) and optimal



One way to evaluate the impact of the age distribution effect is to consider its effect on output per capita. Consider the following simple decomposition: $\frac{Y}{N}=\frac{Y}{L}\frac{L}{N}$ where N is population (see Appendix A for a growth accounting analysis commencing with this identity). Hence output per person (Y/N) is the product of labour productivity (Y/L) and employment per person, or the support ratio, (L/N). Given current LFPRs, the support ratio would be expected to fall by about 15 percent over the next 50 years on the basis of the ABS Series B projections used in these simulations. The effect on Y/N is illustrated in Figure 7 by the series "CES=inf (base LFPR)" which shows that Y/N declines by 15 percent over the next 50 years. In order to offset this, there would have to be increase in Y/L and/or LFPRs. Figure 4 shows that an increase in Y/L of roughly 15 percent would be achieved through the age distribution effect in the case of CES=1. The net effect on Y/N is illustrated in Figure 7 which shows that a CES of 1 prevents Y/N from falling below its present level over the next 100 years. Higher elasticities would generate smaller increases in Y/L and therefore a smaller offset of the effect of a declining support ratio on Y/N. See, for example, the series for CES=2 in Figure 7 in which case Y/N finishes up 10 percent below its current level after about 50 years.

Figure 6 – Simulated increase in old age LFPRs

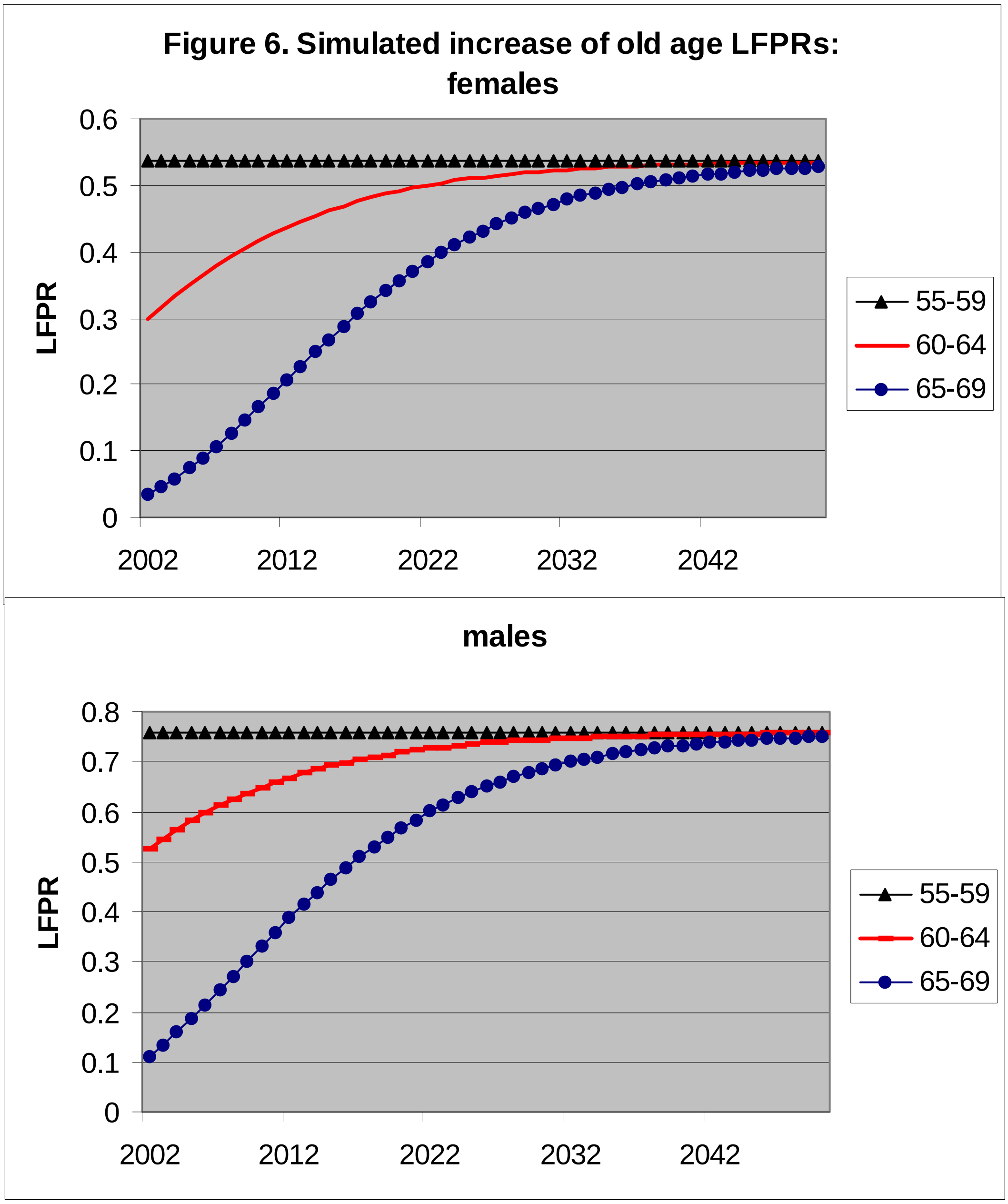
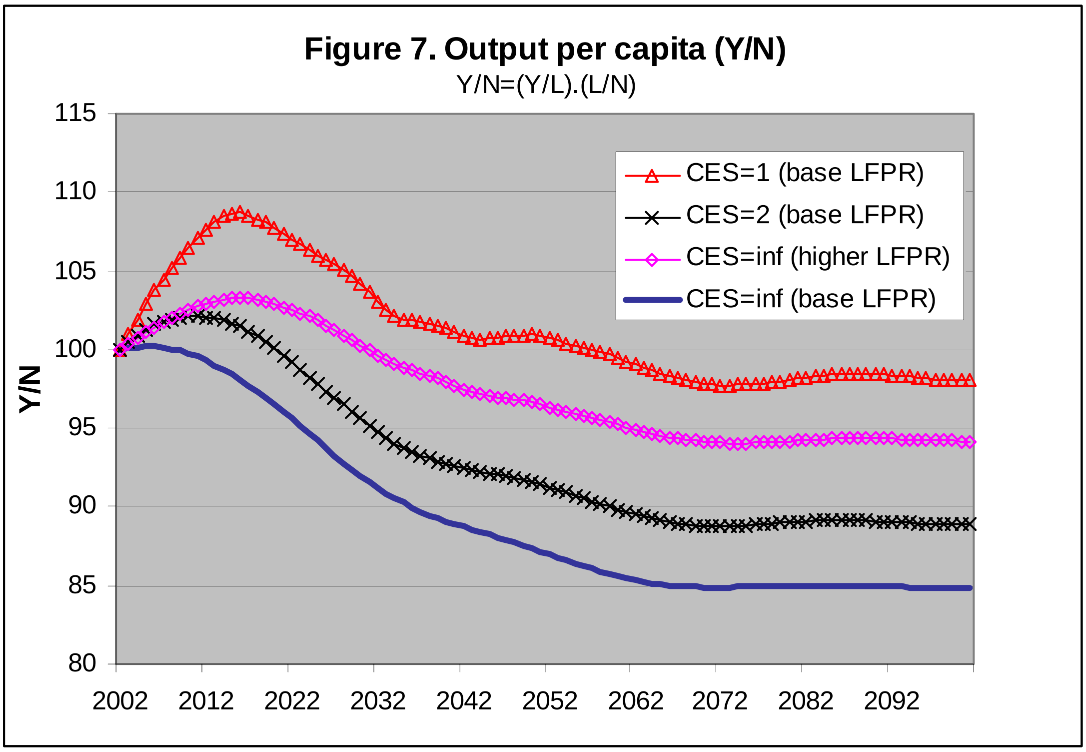


Figure 7 – Output per capita



As suggested above, an increase in LFPRs, especially of older workers, could also prevent a falling support ratio from reducing output per person. It is therefore interesting to compare the effect on output per person of the age distribution effect with that of an increase in the LFPRs of older workers. This question is addressed by running a simulation in which the LFPR of 60-64 and 65-69 year olds both increase gradually over time until they equal the LFPR of 55-59 year olds in 2002. The simulated paths of the LFPR for these age groups are illustrated in Figure 6 for both males and females. The result is that by about 2040 the LFPR of all workers over 55 is the same. This is quite an extreme scenario. For example, it implies that by 2040 three out of four 69 year old males will be in the labour force.[6] This can be compared with the scenario simulated by Gruen and Garbutt (2003) in which the LFPR of all workers increased gradually over the next 20 years to reach the 80th percentile of the distribution of current participation rates across the OECD. Their scenario implies that the LFPR of 60-64 year olds will reach 60 and 40 percent for males and females, respectively, after 20 years. This compares with 70 and 50 percent, simulated here, which is therefore extremely optimistic. The simulated increases in LFPR assume an infinite ES (i.e. equation B6 in Appendix B), since this is the typical assumption implicitly adopted in modelling demographic change including the Gruen and Garbutt (2003) simulations.

The effect on Y/N is illustrated in Figure 7 by the series "CES=inf (higher LFPR)". The gap between this series and the series with base LFPR, labelled "CES=inf (higher LFPR)"gives the "LFPR effect" on Y/N. The result is that Y/N falls by only 5 percent rather than 15 percent. Hence the LFPR effect boosts Y/N in the long run by about 10 percent. This is equivalent to the age distribution effect for an ES somewhere in between 1 and 2 (see Figure 7).

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[6] The baseline assumption that the LFPR of workers 70 and over is zero is maintained. That is, the LFPR of workers over 65 reported by the ABS is assumed to apply to only to workers aged 65-69. This implies some downward bias in the employment of older workers in the simulations because the LFPR of workers 70 years and over is positive although smaller than that of 65-69 year olds.

The important policy implication is that the existence of a positive age distribution effect would take some pressure off the need to raise LFPRs of older workers in order to prevent declining output per capita as a result of population ageing.

6 Conclusion

Higher labour productivity growth and/or higher LFPRs would be required to offset the costs of a falling worker to population ratio on output per capita. Neither of these remedies are easily achieved through government policies . For example, increasing the LFPRs of older workers requires difficult policy changes in the areas of taxation, welfare and retirement incomes.

The analysis in this paper, extending earlier work of Prskawetz and Fent (2004), identifies a dividend from the change in the age distribution of the workforce that raises labour productivity without any requiring any change in policy. The fundamental reason for this is that workers of different ages are not perfectly substitutable. The degree of substitutability - or, inversely, complementarity – affects the optimal age mix of the workforce. It appears that the age mix of the workforce is projected to move closer to the optimal mix, yielding a dividend in terms of higher labour productivity.

The simulations suggest that the magnitude of this dividend is sensitive to the elasticities of substitution between labour of different ages about which little is known. The size of the dividend could be anywhere from trivial to substantial. However, with elasticities that seem plausible the dividend could be roughly equivalent to the gains that would occur through the largest conceivable increases in the LFPRs of older workers. Put another way, the gains could be sufficient to substantially offset the effect on output per capita of the decline in the worker to population ratio that will occur with no increase in LFPRs. This should not deter policy makers from removing distortions that act as disincentives to labour force participation. This would be good policy irrespective of population ageing. However, the need to do so in order to offset the effect of ageing on output per person could be less pressing than previously thought, due to the age distribution dividend.

Sensitivity of the magnitude of the dividend to the values of the elasticity parameters, clearly suggests that there is a need for empirical work to investigate the magnitudes of these elasticities. Nevertheless, it is encouraging that the sign and magnitude of the dividend found here is consistent with that found by Prskawetz and Fent (2004) for Austria.

Appendix A

This Appendix derives the four sources of growth in per capita output referred to in the introduction. See Mason (2001, p.224) for a similar derivation.

We define the following variables and parameters: Y is output, N is population, L is employment, A is an exogenous technology parameter, i is investment net of depreciation, α is the output elasticity of capital. Let y_N=Y/N, y_L=Y/L, y_K=Y/K, k=K/L, and let "^" denote growth rate.

We start with the following decomposition of output per capita:

$$\frac{Y}{N} = \frac{Y}{L}\frac{L}{N} \text{ and therefore}$$

$$\hat{y}_N = \hat{y}_L + \hat{L} - \hat{N}\ . \qquad \text{(A1)}$$

qualitative result:

$$\frac{Y}{L} = A\left(\frac{K}{L}\right)^{\alpha} \text{ and therefore}$$

$$\hat{y}_L = \hat{A} + \alpha\hat{k}\ . \qquad \text{(A2)}$$

By definition:

$$\hat{k} = \frac{\hat{K}}{K} - \hat{L} \text{ and } \hat{K} = iY\text{. Therefore}$$

$$\hat{k} = iy_K - \hat{L}\ . \qquad \text{(A3)}$$

Substituting (A3) into (A2):

$$\hat{y}_L = \hat{A} + \alpha i y_K - \alpha\hat{L} \qquad \text{(A4)}$$

Substituting (A4) into (A1)

$$y_N = \hat{A} + \alpha i y_K - \alpha\hat{L} + \left(\hat{L} - \hat{N}\right) \qquad \text{(A5)}$$

Equation (A5) shows that growth of output per capita is a function of four variables which are captured in four right hand side terms in (A5): technical progress, investment, the effect of labour force growth on capital deepening, and the the rate of change in the employment to population ratio.

Appendix B

This Appendix provides further technical analysis of the model used for the simulations.

Following Prskawetz and Fent (2004), it is assumed that output is produced by labour only and that there is no productivity growth. Hence output is simply an index of labour inputs. The "fuzzy CES" function in Prskawetz and Fent (2004) is replaced here by a CRESH function of labour inputs distinguished by age:

$$\sum_{i=1}^{k} \alpha_i \left[\frac{L_i}{f(Y)} \right]^{\rho_i} = 1 \qquad \text{(B1)}$$

where, α_i is the productivity weight of labour of age *i*, *k* is the number of age groups, L_i is the number of workers of age *i,* Y is the index of composite labour inputs and ρ_i is a parameter that represents the flexibility, or versatility, of L_i, meaning the degree to which L_i can substitute for any other input, L_j. It is assumed here that all labour inputs are substitutes to some degree, which restricts ρ_i such that $-\infty<\rho_i<1$. The larger the absolute value of ρ_i the more easily L_i is substitutable for any other labour input with a given value of ρ_j. This implies that two labour inputs with high absolute values of ρ_i will be good substitutes and two inputs with low absolute values of ρ_i will be poor substitutes.

The elasticity of substitution[7] (ES) between L_i and L_j, which we will define as σ_{ij} , is given by (Hanock, 1971, p.699)

$$\sigma_{ij} = \frac{a_i a_j}{\sum_{m=1}^{k} s_m a_m} \qquad \text{(B2)}$$

where $a_i = \dfrac{1}{1-\rho_i}$ and s_m is the factor share of L_i.

Returns to scale are assumed here to be constant which implies *f(Y)=Y*. If $\rho_i=\rho$ then (B1) becomes CES:

$$Y = \left[\sum_{i=1}^{k} \alpha_i L_i^{\rho} \right]^{1/\rho} \qquad \text{(B3)}$$

As ρ in (B3) approaches zero we have the Cobb Douglas function in which the ES=1:

$$Y = \prod_{i=1}^{k} L_i^{\alpha_i} \qquad \text{(B4)}$$

As ρ in (B3) approaches ∞, the ES approaches zero in which case the function is of the Leonteif form:

[7] ES refers here to the Allen-Uzawa pairwise elasticity of substitution which is the n-factor analogue of the two-factor Hicks ES of substitution. The former Allen-Uzawa elasticity is equal to the Miroshima elasticity in the case where the technology is CES and therefore all the a's are equal (Blackorby and Russell, 1981).

$$Y = \min_{i=1,..,k} \frac{L}{a_i} \quad \text{(B5)}$$

where the a_i are constants.

Finally, if ρ in (B3) is equal to 1, we have the additive function for which the ES=∞:

$$Y = \sum_{i=1}^{k} \alpha_i L_i \quad \text{(B6)}$$

In (B6) the index of labour inputs is a sum of labour inputs by age, weighted by age-dependent productivity levels. The implication of (B6) is that effective units of labour of different ages, $\alpha_i L_i$, are infinitely substitutable. In other words, once workers of different ages are adjusted for their productivity differences they become identical inputs in economic terms. In (B6), the age distribution of the labour force affects output only if workers have age-dependent productivity levels, α_i. If workers are all equally productive, the index of labour (which is also output here) is completely independent of the age distribution of the labour force. Even if workers are not equally productive, output is completely independent of a change in the age distribution of effective labour units, defined as $\alpha_i L_i$. This implies that for a given labour force size, a decrease in supply of younger workers and increase in supply of older workers is matched exactly by a decrease and increase in demand for younger workers and older workers, respectively. The result is no change in the relative wages of young and older workers, and no change in output per effective labour unit.

Despite this shortcomings of (B6), it is the typical functional form that has been commonly adopted to define the labour index in macroeconomic models applied to modelling demographic change. The seminal example is Cutler et al (1990); for an Australian application see Guest and McDonald (2001) and for New Zealand see Guest, Bryant and Scobie (2004). The production functions in these models include capital. Here capital is excluded in order to focus on the effect of the ES on the quantum of labour input.

Other assumptions and data

In applying the CRESH function it is assumed that the degree of flexibility of workers varies with their age. In particular, it is assumed that middle age workers are more flexible than either young workers or older workers. The degree of flexibility is a hump shape function of age, rising to middle age then falling to old age. The intuition for this is that middle age workers, defined here as workers in the 35 to 54 age group, will be more substitutable for young workers than will older workers; and will also be more substitutable for older workers than will younger workers.

However, it would also be desirable to parameterise the CRESH function such that workers in adjacent age groups are better substitutes than are workers further away in age. Unfortunately the hump-shaped pattern violates this property. The example in the table below makes this clear. The table gives the matrix of values of the ES in (B2), where the denominator in (B2) is equal to 2 which is the value based on the data applied in the simulations. The hump shape of a_i is evident, as is the intuitively appealing result, described above, that workers closer in age to the versatile middle age groups have higher ES. However, it is also evident from the table that workers in adjacent groups are not necessarily better substitutes for workers further away in age. For example, the ES between 15-19 year olds and 20-24 year olds is lower than that between 15-19 year olds and 35-44 year olds.

***Appendix Table 1 – ES for CRESH function for hump-shaped pattern of* a_i**

i	a_i	σ_{ij} i=15-19	σ_{ij} i=20-24	σ_{ij} i=25-34	σ_{ij} i=35-44	σ_{ij} i=45-54	σ_{ij} i=55-59	σ_{ij} i=60-64
15-19	1.00							
20-24	1.50	0.75						
25-34	2.00	1.00	1.50					
35-44	3.00	1.50	2.25	3.00				
45-54	3.00	1.50	2.25	3.00	4.50			
55-59	2.00	1.00	1.50	2.00	3.00	3.00		
60-64	1.50	0.75	1.13	1.50	2.25	2.25	1.50	
65-69	1.00	0.50	0.75	1.00	1.50	1.50	1.00	0.75

An alternative to the hump shape pattern of a_i is a declining pattern as illustrated in Appendix Table 2. The advantage of this pattern is that the ES between labour of a given age, *i*, and labour of any other age, *j*, becomes less the further away *j* is from *i*. That is, $\sigma_{ij} > \sigma_{ik}$ for all *k*>*j*.[8] However, the problem with this pattern is the implicit assumption that the youngest workers are the most versatile and substitute more easily for other workers than do any other age group. This implies for example that 15-19 and 20-24 year olds substitute more easily than 20-24 and 25-34 year olds, and so on, such that the ES between adjacent groups declines monotonically as the ages of the adjacent groups increase. Hence, this is a disadvantage of the pattern in Appendix Table 2.

***Appendix Table 2 – ES for CRESH function for declining pattern of* a_i**

I	a_i	σ_{ij} i=15-19	σ_{ij} i=20-24	σ_{ij} i=25-34	σ_{ij} i=35-44	σ_{ij} i=45-54	σ_{ij} i=55-59	σ_{ij} i=60-64
15-19	3.0							
20-24	2.7	1.36						
25-34	2.3	1.16	1.74					
35-44	2.0	1.00	1.50	1.99				
45-54	1.7	0.85	1.28	1.71	2.56			
55-59	1.5	0.73	1.10	1.47	2.20	2.20		
60-64	1.3	0.63	0.94	1.26	1.88	1.88	1.26	
65-69	1.0	0.50	0.75	1.00	1.50	1.50	1.00	0.75

Other patterns of a_i would yield alternative matrices of ES. However, the two patterns illustrated in Tables 1 and 2 seem to be the most logical alternatives. Of these, Table 1 is considered here to be the preferred pattern and it is the pattern that is applied in the simulations.

[8] Ideally we would impose the less restrictive condition that $\sigma_{ij} > \sigma_{km}$ for all *i-j*>*k-m*, but it is not possible to find a unique set of values of ρ_i that satisfies this condition.

Males and females are distinguished in terms of their age-dependent productivity weights but their productivity adjusted labour units are assumed to be infinitely substitutable at any given age. Hence $(\alpha L)_i=(\alpha L)_{i,males}+(\alpha L)_{i,,females}$ and therefore α_i is a weighted average of the productivity weights for males and females:[9] $\alpha_i = \frac{(\alpha L)_{i,males} + (\alpha L)_{i,females}}{L_i}$.

Restrictions exist on the range of values of the σ_{ij} that yield a unique solution for the CRESH function (Hanoch, 1971). The binding restriction in the present application is that $\alpha_i\rho_i$ must be of the same sign for all i, assuming that all $a_i>0$ which implies that all labour inputs are substitutes to some degree. Given all $\alpha_i>0$ by definition, we must have for all i, either $0<\rho_i<1$ ($a_i>1$) or $\rho_i<0$ ($0<a_i<1$). This restricts values of the σ_{ij} that can be simulated to values that are all relatively high (as in Tables 1 and 2) or relatively low. This is a shortcoming because ideally one would like to simulate the case where some σ_{ij} are high and some are low, an example of the latter being the ES between 15-19 and 60-64 year olds.

In the simulations, the productivity weights by sex were proxied by the age distribution of full-time average weekly earnings by sex at August 2003, obtained from ABS Catalogue 6310.0. The labour force (in persons, not hours worked) and the labour force participation rate by age and sex were obtained from ABS Cat 6291.055.001. The population projections by age and sex, 2002-2100, were obtained from ABS Cat 3222.0, Series B. Of the three series of population projections published by the ABS, Series B is based on assumptions for international migration, fertility and mortality which yield population levels that are in between those of Series A and C.

[9] It is of course possible to treat females as a separate labour group with age dependent ES but this creates additional anomalies of the kind illustrated in Tables 1 and 2. The assumption of infinite elasticities between males and females of any given age is preferred in order to avoid these anomalies.

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[i] This derivation requires the assumption, adopted throughout the analysis, that workers are paid their marginal products. In that case the marginal rate of substitution is equal to the relative prices of labour i.e. $\frac{w_i}{w_j} = \frac{\alpha_i}{\alpha_j}\left(\frac{\pi_j}{\pi_i}\right)^{1/\sigma}$. Equality of marginal products implies $w_j = w_i$ and therefore $\frac{\pi_j}{\pi_i} = \left(\frac{\alpha_j}{\alpha_i}\right)^{\sigma}$.