The Excess Burden of Taxation and Why it (Approximately) Quadruples When the Tax Rate Doubles

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Abstract

The `excess burden' of taxation represents an efficiency loss which must be compared with any perceived gains arising either from income redistribution or the non-transfer expenditure carried out by the government. An important property is that, under certain assumptions, it increases disproportionately with the tax rate. This result provides the basis of a general presumption in favour of a broad-based and low tax rate system: any exemptions which reduce the tax base inevitably raise the tax rate required to obtain an unchanged amount of total tax revenue. The aims of this paper are to provide a non-technical explanation of the concepts of welfare change and excess burden used in the public finance literature, and to demonstrate the result that an approximation to this burden depends on the square of the tax rate.

JEL CLASSIFICATION H21
H30
D60

KEYWORDS Taxation; excess burden; equivalent variation; compensating variation

Contents

List of Figures

1 Introduction

The fact that taxation imposes welfare costs which, when expressed in money terms, exceed the amount of revenue collected, is one of the fundamental results of public finance analysis, and has been recognised for many years. This excess burden of taxation represents an efficiency loss which must be compared with any perceived gains arising either from income redistribution or the non-transfer expenditure carried out by the government. An important property of this excess burden from taxation is that it increases disproportionately with the tax rate: indeed this burden is approximately proportional to the square of the tax rate. This result provides the basis of a general presumption in favour of a broad-based and low tax rate system: any exemptions which reduce the tax base inevitably raise the tax rate required to obtain an equivalent amount of total tax revenue.

The aims of this paper are to provide an explanation of the concepts of welfare change and excess burden used in the public finance literature, and to demonstrate the result that an approximation to this burden depends on the square of the tax rate. An attempt has been made to rely largely on diagrams, although some more technical details have been added in footnotes.[1]

Section 2 introduces the main concepts of a money measure of welfare change and the excess burden. It concentrates on the welfare loss, the equivalent variation, imposed on a single individual arising from a tax on a single good.[2] Section 3 derives the approximation for this burden using a diagrammatic argument. The next two sections contain elaborations of the basic analysis. Section 4 briefly describes a different, but closely related, measure of welfare loss, the compensating variation, and its associated excess burden measure. Section 5 looks at the special context of an income tax, which is complicated by the fact that it involves simultaneously an income and a price change, and the analysis requires a slightly different concept of income. This

[1] For more technical and wide-ranging treatments, with extensive references to the literature, see for example Auerbach (1985), Auerbach and Hines (2002), Blundell *et al.* (eds) (1994) and Creedy (1998).

[2] In particular, general equilibrium effects are ingored. Income taxation, which affects both the net wage rate and the price of leisure, is also not treated explicitly here.

section, even though it uses a very simple income tax structure, is necessarily more technical and complex than the earlier sections. Brief conclusions and discussion of the role of approximations in policy analysis are in section 6.

2 A Money Measure of Welfare Change

This section explains the concept of welfare change used to examine tax burdens.[3] It defines the equivalent variation and the excess burden. The context used to examine these concepts is that of a tax on a single good.

2.1 A Tax on One Good

Consider a single individual who is maximising utility subject to a fixed budget (which, in this static analysis, is equivalent to 'income'). There are two goods, X and Y. Good Y may be considered as a composite of all other goods, with the price set equal to unity. Hence units of Y are equivalent to money units. In the initial situation, the budget line, with a slope equal to the relative price of X to that of Y, is shown as AB in Figure 1 and the optimal position is E_1 representing a tangency position on indifference curve U_0. Suppose a selective tax is then imposed on good X, which causes the price of X to increase so that the budget line pivots to AC. The new optimal position along this budget line is E_2, representing a tangency along indifference curve U_1.

There is a reduction in the individual's welfare as a result of the tax, as indicated by the move from U_0 to the lower indifference curve U_1. However, the difference $U_0 - U_1$ does not provide a useful measure of welfare change because utility is regarded as an ordinal concept: the utility levels themselves are entirely arbitrary, and the utility function provides simply a preference ordering of alternative bundles with standard properties (such as transitivity and decreasing marginal rates of substitution).

[3]These concepts apply to welfare changes arising from any set of price changes, but here the price change is assumed to arise purely from a tax change, so that the excess burden concept is relevant.

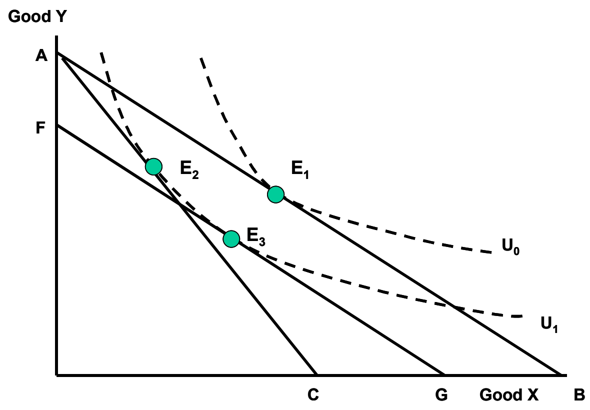

Figure 1: A Tax on Good X

2.2 The Equivalent Variation

Welfare change concepts are instead based on the change in the cost of reaching a particular indifference curve as relative prices change as a result of the tax. This cost is not affected by the arbitrary absolute value of utility attributed to the indifference curves. Figure 1 shows that utility level U_1 can be attained at the pre-tax prices if the individual faces budget line FG which is parallel to AB. The optimal position along this hypothetical constraint is shown as E_3. Hence, there is a variation in the individual's budget that is equivalent to the tax imposed on X, in the sense that the individual would be indifferent between that budget variation (at the old prices) and the tax. This 'equivalent variation', denoted EV, is measured in terms of a quantity of good Y by AF, the vertical distance between the two budget lines AB and FG.

The equivalent variation can be measured if sufficient information is available about the individual's indifference map implied by the form of the utility function. It is possible in principle to convert the utility function into another function that describes the minimum total expenditure needed to reach a specified indifference curve at a given set of prices. Such a function is called the 'expenditure function', $E(p, U)$, and has as its arguments, not the amounts of the goods consumed, as with $U(x,y)$, but prices and utility.[4] Given this function, the equivalent variation is expressed as:

$$EV = E(p_1, U_1) - E(p_0, U_1) \tag{1}$$

where p_0 and p_1 are the two vectors of pre-tax and post-tax prices respectively. The first term, $E(p_1, U_1)$, is simply the individual's budget, m_1. In this case, the budget is assumed to be fixed, so that $m_1 = m_0 = E(p_0, U_0)$.[5]

2.3 The Excess Burden

To explore the welfare change further, it is useful to concentrate on the area in Figure 1 around the two points E_2 and E_3, which lie on indifference curve U_1. This expanded area is shown in Figure 2. In this figure, since point E_2 represents the position actually reached by the individual after the imposition of the tax on X, the vertical distance AB shows the amount of tax paid, again expressed in terms of good Y. But it is clear from the diagram that this is less than the equivalent variation in income that also places the individual on U_1 at pre-tax prices. This is highlighted in Figure 3. The difference between the equivalent variation and the tax paid, T, therefore represents the excess burden, EB_{EV}, of the tax. This is the distance BC in Figure 3.

To rephrase the argument, the individual has an amount, T, taken in the form of the tax, but this causes a loss of welfare that is the same as if the budget were reduced by the equivalent variation, EV, at the old prices. Another way of describing the EV is that it represents the maximum amount the individual would be prepared to pay to avoid the tax and associated price change. This exceeds the tax by the amount EB_{EV}.

[4] For an early analysis of the role of the expenditure function in public finance, see Diamond and McFadden (1974),

[5] The expenditure function is obtained from the direct utility function $U(x,y)$ by first deriving the indirect utility function $V(p,m)$; this is produced by substituting the expressions for optimal Marshallian demands x and y into U. The function $E(p,U)$ is then produced by inverting V to express m in terms of p and V. However, these operations can be carried out for only a narrow range of utility function. In the Cobb-Douglas case, $U = x^{\alpha} y^{1-\alpha}$ and $x = \alpha m / p_x$ with $y = (1-\alpha) m / p_y$. Hence $V(p,m) = m\left(\frac{\alpha}{p_x}\right)^{\alpha}\left(\frac{1-\alpha}{p_y}\right)^{1-\alpha}$, so that $E(p,U) = kUp_x^{\alpha} p_y^{1-\alpha}$ with $k = 1/\left\{\alpha^{\alpha}(1-\alpha)^{1-\alpha}\right\}$.

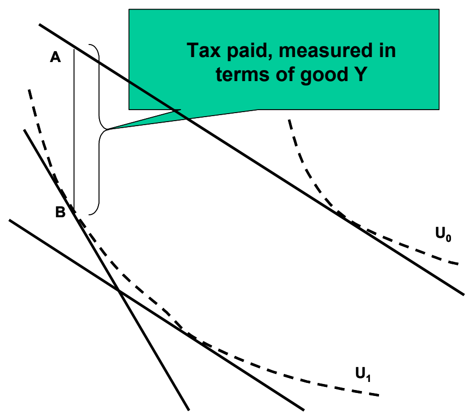

Figure 2: The Tax Paid

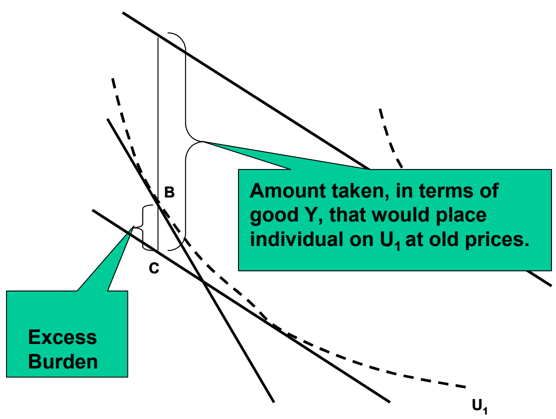

Figure 3: Welfare Change and Excess Burden

The tax revenue can actually be used to finance government projects or for redistribution to other individuals. However, the excess burden $EV - T$ is not available for anyone to spend: it represents a pure efficiency loss arising from the tax. The size of this excess burden depends partly on the degree of convexity of the indifference curve, which reflects the degree of substitutability between the two goods.

Any attempt to derive a general approximation for the excess burden along the lines of equation (1) involves a certain amount of algebra, making assumptions about the form of $E(p, U)$. However, the aim here is to provide a non-technical treatment, so another avenue must be found, as discussed in the next section.

3 Approximating The Excess Burden

The previous section emphasised the crucial role played in welfare measurement by the expenditure required to reach a specified indifference curve at a given set of prices, $E(p, U)$. It is this feature which also provides the key enabling the previous 'distance' money measure of welfare change in the above diagram to be converted into an area, expressed in terms of price and quantity changes.

3.1 The Excess Burden as an Area

As shown in the previous section, the excess burden is a vertical distance traced out by moving along the post-tax indifference curve from E_2 to E_3 as the relative price changes from the post-tax to the pre-tax ratio. The movement from E_2 to E_3 - along the fixed indifference curve U_1 - actually traces out part of a demand curve, in this case a Hicksian demand curve for good X: demand changes as the price varies, while utility is constant.[6] This contrasts with the Marshallian demand curve for X, which relates to the movement between E_1 and E_2. These two demand curves are shown in the

[6] Another Hicksian demand curve could be traced, relating to movements along U_0; this is examined below.

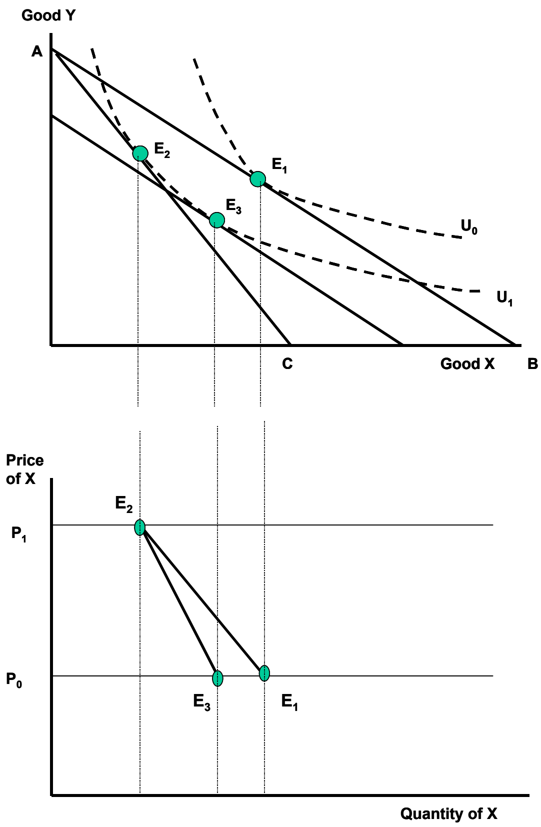

Figure 4: Demand Curves For Good X

lower part of Figure 4, for variations between the pre-tax and the post-tax prices for good X, P_0 and P_1.

For a small change in the price, the narrow area to the left of the Hicksian demand curve between the prices represents an expenditure level (since it is a money price per unit multiplied by a quantity). In just the same way that the Hicksian demand is produced by *gradually* moving along U_1 as the relative price varies, it is necessary to add up all the areas for each small price change. The total area - obtained by adding all the narrow horizontal strips - gives the expenditure required for obtaining the equivalent variation measure of welfare change. Hence the equivalent variation is the area shown in Figure 5, which is precisely equivalent to the length in Figure 3.[7]

[7] Using the concept of the expenditure function, it is evident that the Hicksian demand function can in general be obtained from the expenditure function using $x^H(p, U_1) =$

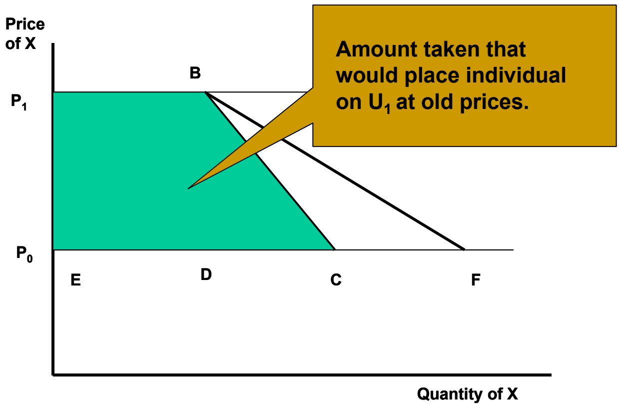

Figure 5: Area Measure of Welfare Change

The distance between the two price lines, $P_1 - P_0$, represents the tax per unit imposed on the good, so that multiplying this by the quantity of the good purchased after the imposition of the tax gives the actual tax revenue. Hence tax revenue is represented by the area indicated in Figure 6. Finally the difference between these two areas is the excess burden, shown by the area BDC in Figure 6.

3.2 The Approximation

It is the conversion of the excess burden into an area involving a demand curve that makes an approximation so simple to derive. All that is necessary is to assume that the Hicksian demand curve between the relevant prices is a straight line. This makes BCD in Figure 6 a simple triangle, whose area is known to be half the base multiplied by the height. The height, BD, is equal to the price change (the tax per unit) and the base, DC, is equal to the change in quantity demanded along the relevant Hicksian demand curve.

$\partial E(p, U) / \partial p$. Here the Hicksian demand traces the quantity change as $E(p, U)$ is varied, by changing p and keeping U fixed - equivalent to moving along the indifference curve. Furthermore, the EV area measure is the integral of the Hicksian demands between the two prices. This gives the difference between the two values of the expenditure function needed to evaluate the equivalent variation. Hence, the equivalent variation is given by: $EV = E(p_1, U_1) - E(p_0, U_1) = \int_{P_0}^{P_1} x^H(p, U_1)\, dP$. For an extensive treatment of the 'duality' properties used here, see Cornes (1992).

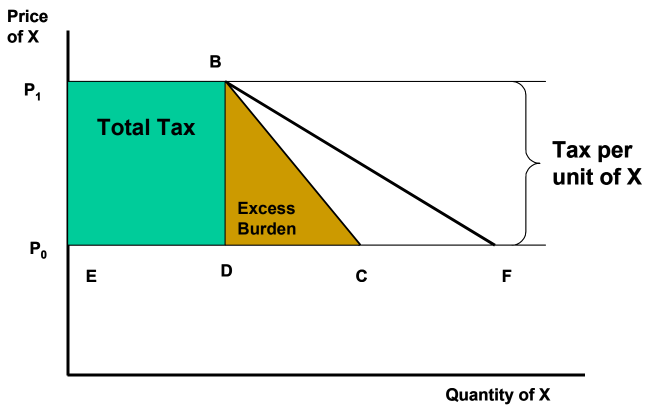

Figure 6: The Excess Burden as an Area

A little algebra cannot be avoided at this point. Let τ denote the proportional tax rate imposed on good X, defined as a tax-inclusive rate: this means that the tax paid on each unit of the good is τP_1, a proportion of the post-tax price. Hence the price change can be expressed simply as $\Delta P = \tau P_1$, and if the absolute quantity change is ΔX, the excess burden is simply:

$$EB_{EV} = \frac{1}{2} (\Delta X) (P_1 \tau) \tag{2}$$

This expression is just one step away from the approximation required. Define the point elasticity of demand along the Hicksian demand curve for constant U_1 as $\eta_1 = \frac{P}{X}\frac{dX}{dP}$. For the discrete price change $\Delta P = P_1 \tau$, the absolute change in quantity, starting from P_1 and $X_{1,}$ is given by $\Delta X = |\eta_1| X_1 \tau$.[8] The initial price and quantity are P_1 and Q_1 because movement along indifference curve U_1 is from point E_2 to E_3 (that is, along the Hicksian demand curve to the right). Substituting into (2) immediately gives:

$$EB_{EV} = \frac{|\eta_1|}{2} (X_1 P_1) \tau^2 \tag{3}$$

Hence the excess burden is approximated by one half of the Hicksian elasticity, multiplied by the initial expenditure, multiplied by the square of the

[8] The elasticity is of course likely to vary along the demand curve, unless it is a rectangular hyperbola. The use of the point elasticity to obtain a discrete change in quantity in this way itself involves an approximation, even where the elasticity is constant.

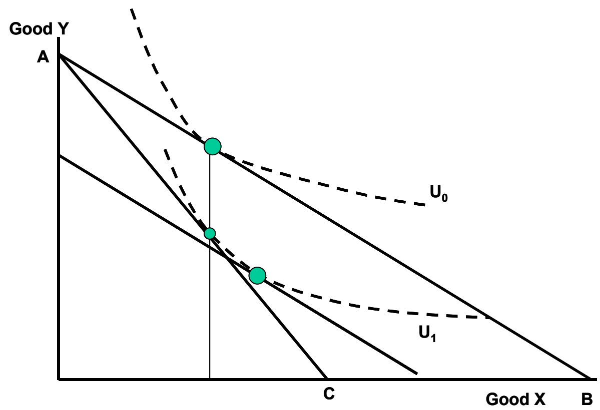

Figure 7: A Zero Marshallian Price Elasticity

proportional tax-inclusive rate of tax. This is the important result showing, for example, that a doubling of the tax rate quadruples the excess burden: if the tax rate increases from τ' to $2\tau'$, the relevant term in equation (3) increases from τ'^2 to $4\tau'^2$.

3.3 Some Further Comments

Some further comments on this important result are necessary. Although this basic property is widely known, it is often forgotten that the elasticity required is the Hicksian elasticity, not the standard Marshallian demand elasticity. A completely inelastic Marshallian demand does *not* imply that the excess burden is zero.[9] This is illustrated in Figure 7, where the post-tax tangency position on the constraint AC is immediately below the pre-tax point on AB. Thus the demand for X (though not for good Y) is unchanged, implying a vertical demand curve. Even in this extreme situation the equivalent variation exceeds the tax paid.

Furthermore, the result relates to the burden of the tax, compared with the no-tax situation. It is often useful to measure the 'marginal excess bur-

[9] In the case of the Cobb-Douglas utility function, which implies constant expenditure shares, the Marshallian demands all have an own-price elasticity of -1. However, the Hicksian demand functions are (after differentiating the expenditure function given in an earlier footnote) $x^H = k\alpha U\left(p_y/p_x\right)^{1-\alpha}$, so that the price elasticity is $\eta = -\left(1-\alpha\right)p_x^{-(1-\alpha)}$.

den', MEB, which relates to changes in the tax rate. This is obtained simply in terms of changes in the equivalent variation and the amount of tax, so that $MEB = \Delta EB - \Delta T$. This gives rise to two related concepts. First, the marginal welfare cost is given by $MWC = MEB/\Delta T = \Delta EB/\Delta T - 1$ and the marginal cost of funds, $MCF = \Delta EB/\Delta T$. Combining the two measures gives $MCF = MWC + 1$. When considering a policy of increasing a tax rate in order to finance a public project, it is the MCF that is relevant.[10]

Another important issue is that the above discussion all relates to a tax on a single good. Nevertheless, the concepts of the equivalent variation and excess burden can be applied, with little modification, to taxes on several goods, although simple approximations based on diagrams are not available.[11] With more than one good and if tax rates are initially unequal, it is possible that the equivalent variation from an increase in one tax rate can be negative, implying in turn a negative marginal excess burden and an improvement in efficiency.

No mention has yet been made of the famous Marshallian concept of consumer's surplus. This is based on the area to the left of the Marshallian demand curve, between the relevant prices, so that the welfare change is extended, as can be seen from Figure 6. An approximation based on the Marshallian demand curve simply substitutes the Marshallian elasticity for the Hicksian elasticity in equation (3). For a broad non-technical discussion of such welfare triangles in a historical context, see Hines (1999).

All the above analysis relates to a single individual. However, in some contexts (particularly when using a tax microsimulation model) it is possible to obtain a measure of welfare change for each individual or household.

[10] The efficiency condition for a public good is that the sum of the marginal rates of substitution in consumption between the public good and others (over all individuals) must equal the marginal rate of transformation in production. (This contrasts with the condition for private goods, where the common marginal rate of substitution in consumption must equal the marginal rate of transformation). Allowing for the efficiency costs of taxation means that the marginal rate of transformation must be multiplied by the marginal cost of funds. On these issues see, for example, Slemrod and Yitzhaki (2001).

[11] For several goods, the expression in an earlier footnote can be extended (for goods $i = 1, ..., n$) simply by adding a summation sign, so that $EV = E(p_1, U_1) - E(p_0, U_1) = \sum_{i=1}^{n} \int_{P_{0,i}}^{P_{1,i}} x_i^H (p, U_1)\, dP_i$. This raises no problems for equivalent variations, but introduces the 'path dependency' problem for Marshallian consumer's surpus measures.

Such money measures can then be used along with a specified social welfare function in order to provide an overall evaluation of the policy change.[12]

4 The Compensating Variation

The previous discussion defined a money measure of the welfare change arising from a price change (in this case induced by a tax) in terms of the equivalent variation, the amount of money that (at the old prices) is equivalent to the price change in that it places the individual on the new indifference curve, U_1 . However, it is also possible to consider the amount of money that, if given to the individual, would allow (at the new prices) the initial indifference curve, U_0, to be reached. This is called the compensating variation.[13]

The comparisons are illustrated in Figure 8. The hypothetical budget line FG is parallel to the post-tax budget line AC but places the individual at E_3, which is on indifference curve U_0. The relevant area of the diagram is again expanded, in Figure 9. After compensation of BC, the vertical difference between the two budget lines, at the new prices, the individual would consume at point B. This means that the tax paid is the distance DC, so the excess burden, based on the compensating variation, is BD. An important feature of this measure of excess burden is that it is not based on the amount of tax actually paid: it has to allow for the tax that would be paid if the individual were in fact compensated. Another way of looking at the compensating variation is to recognise that it is the negative of the equivalent variation arising from a price change in the opposite direction.

As with the equivalent variation, the concepts can be illustrated in a diagram containing demand curves, as in Figure 10. In this case, the Hick-

[12] The welfare function may allow for inequality aversion. One particularly useful approach involves comparing the distributions of m (initial total expenditure) and $m - EV$. For further discussion of these aspects, see King (1983).

[13] It is possible to define a money measure of utility as the total expenditure (budget) that, at some reference set of prices, places the individual on the same indifference curve as the actual expenditure and prices. Welfare changes can be expressed in terms of this money measure, and it is clear that there is a different value depending on the reference prices used. The equivalent and compensating variations are effectively such welfare measures, using pre-tax and post-tax prices respectively as the reference prices.

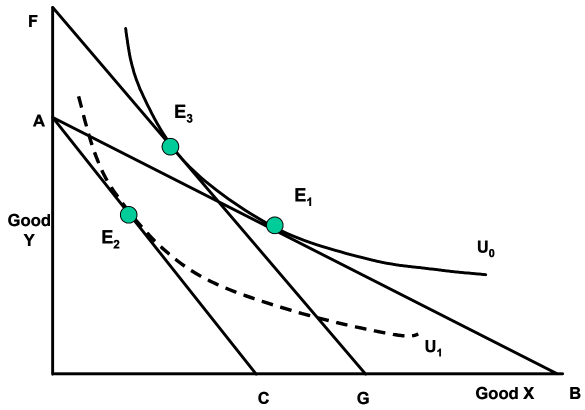

Figure 8: The Compensating Variation

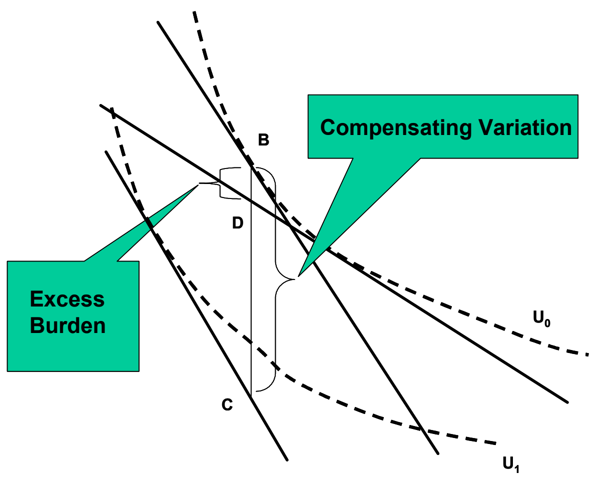

Figure 9: The Excess Burden and Compensating Variation

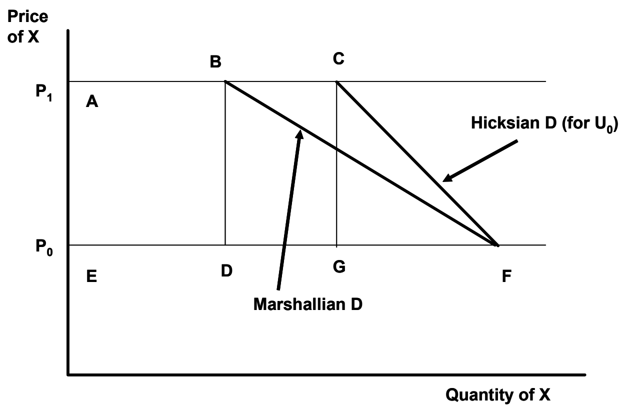

Figure 10: Hicksian Demands and The Excess Burden

sian demand curve relates to movements along U_0. The compensating variation is the area ACFE, and the relevant revenue to be deducted is the area ACGE (which exceeds the actual revenue of ABDE). Hence the excess burden, EB_{CV}, is the area CFG.

Again, on the assumption that the Hicksian demand curve is linear, an approximation to the excess burden can be obtained. In this case it is useful to express the tax per unit as a proportion of the tax-exclusive price, P_0. Hence, let t be the proportional tax-exclusive rate imposed on good X, so that the tax per unit is $P_0 t$. If this is fully shifted to consumers, the absolute price change can therefore be expressed as $\Delta P = P_0 t$. If the absolute quantity change is ΔX, the excess burden is simply:

$$EB_{CV} = \frac{1}{2}\left(\Delta X\right)\left(P_0 t\right) \tag{4}$$

If the Hicksian point elasticity of demand along this curve, for constant U_0, is $\eta_0 = \frac{P}{X}\frac{dX}{dP}$, then for the discrete price change $\Delta P = P_0 t$, the absolute change in quantity, starting from P_0 and $X_{0,}$ is given by $\Delta X = |\eta_0| \, X_0 t$.[14] In this case, the starting point is P_0 and X_0 because movement is along U_0 from E_1 (that is, moving along the Hicksian demand curve to the left). Substituting

[14] The use of the point elasticity to obtain a discrete change in quantity in this way itself involves an approximation (even where the elasticity is constant).

into (2) immediately gives:

$$EB_{CV} = \frac{|\eta_0|}{2} (X_0 P_0) t^2 \tag{5}$$

Hence the excess burden is approximated by one half of the relevant Hicksian elasticity, multiplied by the initial expenditure, multiplied by the square of the proportional tax-exclusive rate of tax. The two tax rates are connected by the relationship, $\tau = t/(1+t)$, since a tax-inclusive rate of $t/(1+t)$ raises the same revenue per unit as the tax-exclusive rate of t.

5 An Income Tax

This section considers the welfare cost of an income tax, in the context of a simple model in which the individual maximises a utility function that includes non-work, or leisure, time as well as consumption (equivalent to net income).[15] Although this case involves no new principles, the application of the welfare measures in the context of taxation and labour supply involves a number of complexities. It is therefore worth considering the details. But before turning to the income tax, it is useful to extend the earlier analysis of a tax on a single good to a policy which simultaneously imposes a tax on good X and changes the budget of the individual.

5.1 Changes in Price and Budget

Suppose that a tax is imposed on good X, as before, and at the same time the individual's budget is increased. This is shown in Figure 11, where the budget line shifts from FG to CH. The treatment of welfare changes in terms of the equivalent variation is the same as before. The optimal position shifts from E_1 to E_2. The policy change is equivalent, in terms of its effect on welfare, to a budget change measured (in terms of good Y) by the vertical distance between budget lines AB and FG.

[15] Here the term income is used rather than referring to a wage tax. The context is a static one in which only a single period is considered and income from employment is, other than government a transfer payment, the only source of income.

In terms of expenditure functions, the point F is associated with the initial budget of $E(p_0, U_0)$, while point A is associated with $E(p_0, U_1)$. Hence the equivalent variation is in this case given by:

$$EV = E(p_0, U_0) - E(p_0, U_1) \tag{6}$$

Although the basic concept of the equivalent variation is unchanged, this expression differs from (1), which has $m_1 = E(p_1, U_1)$ as the first term: indeed, in the earlier context the budget was fixed so that $E(p_0, U_0) = E(p_1, U_1)$ by assumption.

It is useful to divide equation (6) into two separate components, associated with the two aspects of the policy change. The change in the budget is equal to $E(p_0, U_0) - E(p_1, U_1) = m_0 - m_1$, while the welfare change arising purely from the price change is, from equation (1), equal to $E(p_1, U_1) - E(p_0, U_1) = m_1 - E(p_0, U_1)$. By adding and subtracting $E(p_1, U_1)$ from equation (6), it becomes:

$$EV = \{E(p_1, U_1) - E(p_0, U_1)\} + \{E(p_0, U_0) - E(p_1, U_1)\} \tag{7}$$

or:

$$EV = \{m_1 - E(p_0, U_1)\} + \{m_0 - m_1\} \tag{8}$$

Using this form, the separate price and budget effects of the policy change are evident. This result provides the basis for the treatment of an income tax in the following subsection.

5.2 A Linear Income Tax

All that is needed to apply the result of the previous subsection to an income tax is a redefinition of some of the terms to allow for the different context. With an income tax, the budget available for spending, or net income, is endogenous, since labour supply depends on the tax system. Let T be the total time available and let h denote the time devoted to working at a gross wage of w,which is fixed.[16] Net income is y, and utility can therefore be

[16]This is a partial equilibrium framework. But in a general equilibrium context, wage rates (factor pricers) would be expected to adjust.

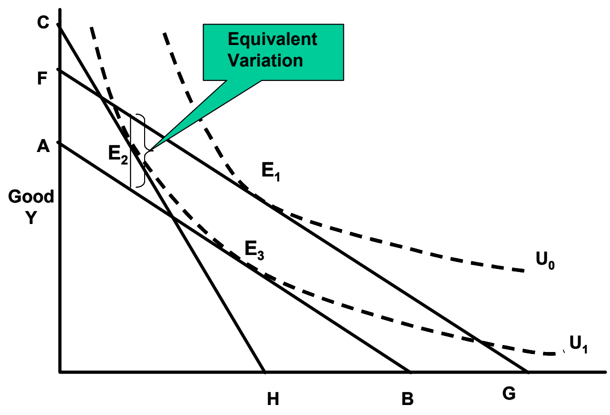

Figure 11: Changes in Price and Budget

written as the function $U(y, T-h)$, with $T-h$ measuring leisure time. Utility is maximised subject to a budget constraint that depends on the tax system. Consider a simple proportional, or linear, tax applied to income from employment, so that the net wage is $w(1-t)$. If net non-wage income is μ, the budget constraint can be written as:

$$y = w(1-t)h + \mu \tag{9}$$

By adding $Tw(1-t)$ to each side and rearranging, this can be rewritten as:

$$w(1-t)(T-h) + y = Tw(1-t) + \mu \tag{10}$$

This form of budget constraint brings out clearly the difference between the present context and the earlier case of a commodity tax with a fixed budget. Here, $w(1-t)$ represents the price per unit of leisure, so the left hand side of (10) shows the total amount spent on leisure and other consumption (net income). The right hand side shows the total resources available to the individual, consisting of the amount that could be obtained if all the endowment, T, of time were spent working. Expressed in this way, the problem is just the same as the earlier one: the individual is viewed as converting the endowment into money and using this, along with non-wage income, μ, to buy goods and leisure. Instead of having a fixed budget, the individual can be said to have a 'full income', M, equal to:

$$M = Tw(1-t) + \mu \tag{11}$$

Similarly, the expenditure function must now be defined in terms of the full income needed to reach a specified indifference curve, U, with a given net wage rate, $w' = w(1-t)$, and is thus written as $M(w', U)$.[17]

The income tax, imposed at the rate t, thus has two simultaneous effects: it reduces full income and reduces the price of leisure. The first effect reduces welfare while the second effect increases welfare. The expression in (8) can be directly translated into the income tax framework as:

$$EV = \{M_1 - M(w_0', U_1)\} + \{M_0 - M_1\} \tag{12}$$

The excess burden is, as before, obtained by subtracting the tax paid from the equivalent variation. To obtain an approximation, it is again useful to turn to diagrams.

5.3 A Diagrammatic Version

Labour supply choices under a linear income tax are illustrated in Figure 12, which shows an initial optimal position, E_1, along indifference curve U_0. The non-wage income is given by the height of BT, which represents the consumption available if the individual does not work. The point A indicates full income $M(w_0', U_0) = M_0$, the amount that can be consumed if all of T were devoted to work. If the initial position is a 'no-tax' situation, then $w_0' = w$. Point C is associated with a full income of $M(w_1', U_1) = M_1$ and F with a full income of $M(w_0', U_1)$. The equivalent variation is thus the distance AF, or $M_0 - M(w_0', U_1)$, which is easily converted into the expression in (12) by the addition and subtraction of M_1.

Figure 13 converts the information in Figure 12 into a diagram showing the Hicksian demand curve for leisure between the two net wage rates of w_0' and w_1': this is the line E_3E_2. The Marshallian demand curve is E_1E_2. Since the price of leisure has in this case fallen, the area DE_3E_2C, the area to the left of the Hicksian demand curve, is the gain from the price reduction.

[17] It is possible to define the expenditure function in terms of 'virtual' income, equal to the value of net income where the budget line (or the extension of a relevant segment of the budget line, in the case of piecewise-linear constraints) intercepts a vertical line at T hours of leisure. The form adopted is a matter only of convenience, since the results are identical.

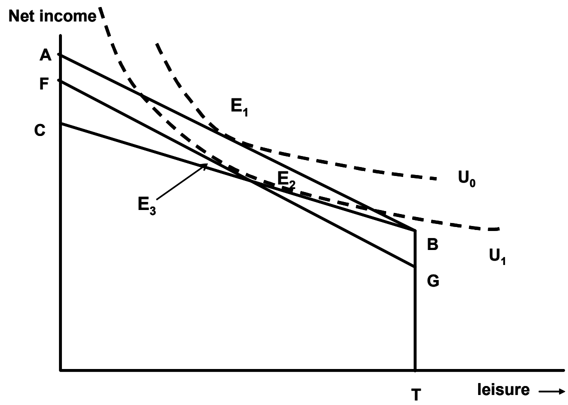

Figure 12: A Linear Income Tax

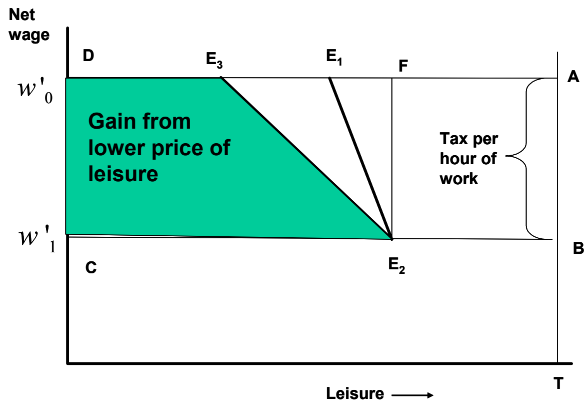

Figure 13: The Excess Burden of an Income Tax

However, there is simultaneously a change in full income. The reduction in full income is measured by the change in net income when working T hours, so in Figure 13 it is the area ABCD. The equivalent variation is thus the area ABCD minus the area DE_3E_2C. This gives the area to the right of the demand curve for leisure, that is, ABE_2E_3. The tax paid, the tax per unit multiplied by the time worked, is equal to the area of the rectangle ABE_2F. Hence the excess burden is the area of the triangle E_2E_3F.

As before, the excess burden is approximated by the area of a triangle, equal to half the product of the base and the height. The base is the change in labour supply (associated with the Hicksian demand curve for leisure),

Δh, and the height is the tax per unit of time, equal to tw. Hence:

$$EB_{EV} = \frac{1}{2}tw\left(\Delta h\right) \tag{13}$$

Following the previous analyses, the change in labour supply can be expressed in terms of the elasticity of the Hicksian supply curve (from the leisure demand curve), $\eta = \frac{w'}{h}\frac{dh}{dw'}$. Since the burden is based on the equivalent variation, movement is from the new position, with $h = h_1$ and $w'_1 = w\left(1-t\right)$, so using the approximation $\eta = \frac{w(1-t)}{h_1}\frac{\Delta h}{wt}$:

$$\Delta h = \left|\eta\right| h_1 \left(\frac{t}{1-t}\right) \tag{14}$$

and substituting (14) into (13) gives:

$$\begin{aligned} EB_{EV} &= \frac{|\eta|}{2} w h_1 \left(\frac{t^2}{1-t}\right) \\ &= \frac{|\eta|}{2} w'_1 h_1 \left(\frac{t}{1-t}\right)^2 \end{aligned} \tag{15}$$

This approximation, as expected, takes the same form as equation (3) although at first sight it looks different. This is because the tax rate, τ, in equation (3) is a tax-inclusive rate, whereas t in (15) is a tax-exclusive rate and in addition the relevant price has fallen: the equivalent tax-inclusive rate in this case is $\tau = -t/\left(1-t\right)$.

In practice, tax systems are much more complex than the simple linear tax considered here, as there are typically several tax thresholds and rates, giving rise to a piecewise-linear budget constraint. Also, means-testing of transfer payments often leads to non-convexities in the budget set. A complete analysis therefore needs to pay careful attention to corner solutions. An approximation like (15) therefore needs to be treated with considerable caution.

6 Conclusions

This paper has shown how an approximation to the excess burden of a tax can be obtained diagrammatically, involving indifference curves and associated

Hicksian demand curves. An assumption that the demand curve is linear over the relevant range produces the result that the excess burden is proportional to the square of the tax rate.

This result has considerable pedagogic value, as it provides a general warning that policies which erode the tax base, and therefore increase the tax rate, need very special justification. Furthermore, tax-financed expenditure and redistribution policies must provide benefits that outweigh the efficiency costs of raising the required revenue.

However, the question arises of whether reliance can be placed on this result in practical policy analyses. One obvious point is that practical analyses often involve more than one tax change and these are seldom 'small changes', so that any approximation is unlikely to be reliable. More importantly, such approximations are not actually *necessary*. Indeed, information about the relevant Marshallian demand curves (even where several tax changes are involved) is *sufficient* to obtain the exact welfare measures. The essential ingredients - the values of the relevant expenditure functions - can be obtained from the Marshallian demand functions using a process of integration. While this may not be possible analytically, numerical integration methods can be used.[18]

Thus, the use of approximations to produce 'back of the envelope' calculations is not recommended, although an appreciation of the basic nature of the excess burden and the arguments leading to the approximation can be valuable in informing policy analyses.

[18] An earlier footnote mentioned that, $\frac{dE(p,U)}{dp_i} = x^H(p,U)$, and since Hicksian and Marshallian demands are related by $x(p,m) = x(p,E(p,U)) = x^H(p,U)$, the differential equation to be integrated is $\frac{dE(p,U)}{dp_i} = x^M(p,E(p,U))$. Integration, along with an appropriate initial condition for the constant of integration, gives the expenditure function, $E(p,U)$. A numerical procudure was suggested by Vartia (1983); for further discussion see Creedy (1998, chapter 4).

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