Income Tax Revenue Elasticities with Endogenous Labour Supply

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Abstract

It is important for the design of tax policy to be able to measure reliably the income elasticity of tax revenue. This gives the extent to which tax revenues change as a result of a change in earnings. Analytical expressions for income tax revenue elasticities treat earnings as exogenous, so that they do not accommodate the endogenous response of labour supply to the income tax system. This paper shows how these expression can be adapted to allow for endogenous labour supply. It identifies how far, and in what circumstances, labour supply effects are quantitatively important for revenue responsiveness estimates, both for individual taxpayers and in aggregate. It is shown that even a relatively simple tax-benefit structure can produce labour supply responses which considerably alter tax revenue elasticity calculations. It is shown that , even with modest leisure preferences, tax-wage elasticities are significantly higher that tax-income elasticities.

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KEYWORDS Income Taxation; Revenue; Elasticity; Labour Supply

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1 Introduction

It is important for the design of tax policy to be able to measure reliably the income elasticity of tax revenue. For example, given some revenue target, the need for discretionary changes in tax parameters (such as tax rates, income thresholds and allowances) is conditional on the expected automatic revenue growth generated by the system's built-in flexibility. In the context of aggregate revenue over all individuals, the extent to which the aggregate effective average tax rate changes when total income changes depends on a number of factors including the precise distribution of individual income changes associated with the aggregate change.

The literature on the built-in flexibility of taxation has concentrated on the analysis of tax revenue elasticities with respect to income growth at the individual and aggregate level. Approaches have been based on regression methods, simulation models, or calculations using analytical expressions.[1] These methods are statistical in that there is no modelling of economic behaviour; they treat income changes as exogenous. If it is required to consider the effects on revenue growth of wage changes, for example arising from productivity growth, the elasticities with respect to income provide only part of the story.

The elasticity of revenue with respect to the wage rate depends on both the revenue elasticity with respect to income and the elasticity of income with respect to the wage. Furthermore, the latter is a function of the elasticity of hours worked with respect to the wage rate. When the tax-wage relationship is of interest, it is therefore necessary to consider how a change in wages affects labour-leisure choices, given the existing tax schedule.[2] Changes in wage rates and gross incomes may be quite different, especially in the neighbourhood of income thresholds in piecewise linear tax structures.

This paper explores the income tax revenue responsiveness arising from wage changes. It shows how analytical expressions can be extended to incorporate endogenous labour supply responses. Simulation methods are used to examine revenue elasticities in the face of highly nonlinear tax structures, where multiple local optima may exist and large discrete jumps in labour supply can arise from non-convex ranges of budget sets. An objective is to identify how far, and in what circumstances, labour supply effects can alter revenue responsiveness estimates that are typically computed for individual taxpayers and in aggregate.

Section 2 provides the relevant analytics of labour supply in the presence of a piecewise linear (multi-step) income tax and transfer structure. Section 3 shows how the revenue elasticities with respect to income and wages are related to each

[1] For a review of analytical expressions and references to alternative methods, see Creedy and Gemmell (2002).

[2] To the extent that there is any automatic indexation of tax thresholds, these are usually in relation to income, rather than wage changes.

other and to labour supply elasticities at the individual level. Section 4 presents some numerical examples while Section 5 turns to aggregate revenue elasticities. Brief conclusions are in section 6.

2 Labour Supply

The simplest approach to the analysis of labour supply involves the static maximisation of a (single period) direct utility function, $U(c, h)$, where h and c represent hours worked and consumption (or net income, where the price index is normalised to unity), subject to a budget constraint. Unlike the standard commodity demand model in which prices are constant irrespective of the amount of each good consumed, the individual faces a variety of net wage rates. Although the gross wage is assumed to remain constant (independent of the number of hours worked), varying effective marginal tax rates associated with the piecewise linear nature of the budget constraint mean that the net wage varies with h. The actual net wage depends on the chosen position on the budget constraint and is therefore, like the number of hours worked, endogenous.

With a piecewise linear budget constraint any interior (or tangency) solution and corner solution can be regarded as being generated by a simple linear constraint of the form:

$$c = w_n h + \mu \tag{1}$$

In the case of such tangency solutions, w_n and μ represent the appropriate net wage rate and 'virtual' income respectively. Virtual income is the intercept (where $h = 0$) corresponding to the relevant segment of the budget constraint and associated net wage; it is therefore distinct from actual non-wage income. In the case of a corner solution, the appropriate virtual income is defined as the value generated by a linear constraint having a net wage, the virtual wage, equal to the slope of the indifference curve at the kink. The concept of the virtual wage is the same as that of the virtual price used in the theory of rationing.

labour supply modelling requires the full tax and benefit system, which determines individuals' budget sets, to be specified. Any piecewise linear tax and transfer system can be described by a number of effective marginal tax rates and gross earnings values at which the marginal rates change. Subsection 2.1 discusses individual budget constraints, while subsection 2.2 briefly states the main results for the special case of Cobb-Douglas utility functions. This form is chosen merely for convenience of presentation in order to illustrate the basic properties of the models.[3]

[3]The computer program used to provide simulation results below also allows for CES utility functions. Copies of this program are available from the authors.

2.1 Budget Constraints

A piecewise linear tax and transfer system, without discontinuities, can be defined in terms of K gross income thresholds, a_k, and effective marginal tax rates applying above those thresholds, t_k, for $k = 1, ..., K$. The initial threshold, a_1, is equal to zero. This subsection explains how the thresholds and rates can be converted to a budget constraint for an individual facing a given gross wage rate. Attention is restricted to the case where earnings from employment and transfer payments are the only sources of income.

Given a gross wage rate, w, the thresholds and rates must be transformed into a set of virtual incomes, m_k, and net wages, $w(1 - t_k)$, which describe respectively the intercept and the slope of each of the K linear segments of the budget constraint.

The virtual income at the start of the first segment of the budget constraint, m_1, must be known. This corresponds to net income when the individual does not work. If there are no transfer payments, then m_1 is set to zero. Along the kth linear segment, $w_n = w(1 - t_k)$ and net income, $z = c$, corresponding to h hours of work is given, from (1), as:

$$z = m_k + w(1 - t_k)h \tag{2}$$

Let h_k^* denote the hours of work for which gross earnings are equal to the specified gross earnings thresholds. Clearly, $h_1^* = 0$, and for $k > 1$, given that two adjacent segments k and $k - 1$ must intersect at point h_k^* (assuming that there are no discontinuities), it must be true that:

$$m_k + w(1 - t_k)h_k^* = m_{k-1} + w(1 - t_{k-1})h_k^* \tag{3}$$

Hence, for $k = 2, ..., K$:

$$h_k^* = \frac{m_k - m_{k-1}}{w(t_k - t_{k-1})} \tag{4}$$

Next, consider the net incomes at the earnings thresholds. For $k = 1$, $a_1 = 0$, and $z = m_1$. At a_2 net income is $m_1 + a_2(1 - t_1)$ and for $k = 3, ..., K$:

$$z = m_1 + \sum_{j=2}^{k-1} a_j(t_j - t_{j-1}) + a_k(1 - t_{k-1}) \tag{5}$$

Equating these with the corresponding values of $m_{k-1} + w(1 - t_{k-1})h_k^*$, and substituting for h_k^* from (4), it can be seen that, for $k = 2, ..., K$:

$$m_k = m_{k-1} + a_k(t_k - t_{k-1}) \tag{6}$$

The piecewise linear budget constraint for any value of w, corresponding to a given set of a_k and t_k (along with m_1) is thus fully defined. Furthermore, the net tax paid can be expressed in terms of the difference between gross earnings and net income at the corresponding level of hours worked, h, and is $y - \{m_k + w(1 - t_k)h\}$.

2.2 Cobb-Douglas Utility Functions

Suppose an individual is on the kth segment with virtual income of m_k and marginal tax rate of t_k. If preferences are Cobb-Douglas, with consumption (net income) of c and time spent in work of h, from a total of H available hours:

$$U(c,h) = c^{\alpha}(H-h)^{1-\alpha} \tag{7}$$

In this context, it is most convenient to write the budget constraint in terms of 'full income', M, defined as the maximum amount the individual could obtain (and consume) by working all available hours. Hence:

$$M = w(1-t_k)H + m_k \tag{8}$$

The individual is thus considered to convert the endowment of time, H, into full income, which is used to purchase goods (having a price normalised to unity) and leisure at the effective price of $w(1-t_k)$.

The standard Cobb-Douglas result gives, for a gross wage rate of w, a demand for leisure of:

$$H - h = (1-\alpha)\frac{M}{w(1-t_k)} \tag{9}$$

Hence:

$$h = \alpha H - \frac{m_k(1-\alpha)}{w(1-t_k)} \tag{10}$$

Furthermore, gross earnings, $y = wh$, in the relevant range are:

$$y = \alpha w H - \frac{m_k(1-\alpha)}{(1-t_k)} \tag{11}$$

In this context the value of h is jointly determined along with the segment, giving the values of m_k and t_k, or the relevant corner of the budget constraint.

If marginal effective tax rates always increase, there is only one local optimum. However, if the budget set has non-convex ranges where the marginal tax rate declines, determination of the optimal solution is complicated by the possibility of more than one local optimum and the existence of a threshold wage rate at which the individual makes a large discrete jump from one section of the budget constraint to another. This possibility arises where transfer systems are means-tested, so that in practice an efficient algorithm is needed for determining all local optima; for further discussion of this case, see Creedy (2001) and Creedy and Duncan (2002).

3 Individual Elasticities

To see how endogenous labour supply affects tax revenue elasticities, it is helpful to begin with some basic relationships among various elasticities. Then the special case of an income tax only is investigated which allows comparison with standard built-in flexibility results.

3.1 Basic Relationships

The tax paid by an individual i, who has a gross wage rate, w_i, and is on the kth linear segment of the budget constraint, with resulting income $y_i = w_i h_i$, is expressed as:

$$T(y_i) = T(y_i(w_i)) \tag{12}$$

The elasticity of $T(y_i)$ with respect to the wage rate, referred to as the 'tax-wage elasticity', can be expressed as:

$$\eta_{T,w} = \eta_{T,y}\eta_{y,w} \tag{13}$$

where $\eta_{y,w}$ is the elasticity of earnings with respect to the wage. The standard tax revenue elasticity, $\eta_{T,y}$, referred to here as the 'tax-income elasticity', is given by:

$$\eta_{T,y} = \frac{dT(y_i)/dy_i}{T(y_i)/y_i} = \frac{mtr_i}{atr_i} \tag{14}$$

where mtr_i is the marginal tax rate and atr_i is the average tax rate faced by i. Furthermore, using $y_i = w_i h_i$, the elasticity $\eta_{y,w}$ can be expressed as:

$$\eta_{y,w} = 1 + \eta_{h,w} \tag{15}$$

giving:

$$\eta_{T,w} = \eta_{T,y}(1 + \eta_{h,w}) \tag{16}$$

Although $\eta_{T,y}$ is endogenous, it is useful to express (16) in this way because $\eta_{T,y}$ is typically estimated from Inland Revenue data on incomes and tax revenues. Thus, together with estimates of $\eta_{h,w}$, the tax-wage elasticity may be obtained.

3.2 An Income Tax Only

Consider the simple case where there is only an income tax and suppose there are no transfer payments, so that $m_1 = 0$. Consider the multi-step tax function defined by:

$$\begin{array}{rcll} T(y_i) & = & 0 & 0 < y_i \leq a_1 \\ & = & t_1(y_i - a_1) & a_1 < y_i \leq a_2 \\ & = & t_1(a_2 - a_1) \quad + \quad t_2(y_i - a_2) & a_2 < y_i \leq a_3 \end{array} \tag{17}$$

and so on. If y_i falls into the kth tax bracket, so that $a_k < y_i \leq a_{k+1}$, and $a_0 = t_0 = 0$, $T(y_i)$ can be written for $k \geq 1$ as:

$$T(y_i) = t_k (y_i - a_k) + \sum_{j=0}^{k-1} t_j (a_{j+1} - a_j) \tag{18}$$

The expression for $T(y_i)$ in (18) can be rewritten as:

$$T(y_i) = t_k y_i - \sum_{j=1}^{k} a_j (t_j - t_{j-1}) \tag{19}$$

Hence:

$$T(y_i) = t_k (y_i - a'_k) \tag{20}$$

where:

$$\begin{aligned} a'_k &= a_k - \sum_{j=0}^{k-1} \left(\frac{t_j}{t_k}\right) (a_{j+1} - a_j) \\ &= \sum_{j=1}^{k} a_j \left(\frac{t_j - t_{j-1}}{t_k}\right) \end{aligned} \tag{21}$$

The implication of (20) and (21) is that the tax function facing an individual taxpayer within the kth segment is equivalent to a tax function with a single marginal tax rate, t_k, applied to income measured in excess of a single threshold, a'_k. The term, a'_k, is the effective threshold for individuals in the kth class, and is a weighted sum of the a_js, with weights, $\frac{t_j - t_{j-1}}{t_k}$, determined by the structure of marginal rate progression. Therefore, a'_k differs across individuals, unlike a_j, depending on the marginal income tax bracket, y_{ik}, into which they fall. Hence the virtual incomes are given by:

$$m_k = t_k a'_k \tag{22}$$

From (11) it can be shown, using the abbreviations: $a = a'_k$, $t = t_k$, that:

$$\eta_{h,w} = \left\{ \frac{\alpha(1-t)wH}{t(1-\alpha)a} - 1 \right\}^{-1} \tag{23}$$

Here $mtr = t$, and $atr = t - (at/y)$, so that:

$$\eta_{T,y} = \frac{y}{y-a} \tag{24}$$

Table 1: Elasticities

α	$\eta_{h,w}$	$\eta_{a,w}$	$\eta_{T,y}$	$\eta_{T,w}$	$\frac{\eta_{h,w}}{\eta_{T,y}}$	$\frac{\eta_{T,w}}{\eta_{T,y}}$
0.5	0.046	0.261	1.331	1.391	0.034	1.046
0.7	0.027	0.257	1.331	1.367	0.021	1.027

where (11) may be used to substitute for y.

Equation (24) is the simplest income tax revenue elasticity, derived by Creedy and Gemmell (2002), for the exogenous labour supply case where the a_js are fixed. However, in many income tax systems, a number of deductions are income-related so that $da/dy > 0$. Allowing for this possibility, Creedy and Gemmell (2002) show that (24) becomes:

$$\eta_{T,y} = \frac{y - a\eta_{a,y}}{y - a} \tag{25}$$

where $\eta_{a,y}$ is the elasticity of allowances with respect to income. Using (16) allows this to be expressed as:

$$\eta_{T,y} = \frac{y(1 + \eta_{h,w}) - a\eta_{a,w}}{(y - a)(1 + \eta_{h,w})} \tag{26}$$

and from (13):

$$\eta_{T,w} = \frac{y(1 + \eta_{h,w}) - a\eta_{a,w}}{(y - a)} \tag{27}$$

These expressions show clearly how the tax-income and tax-wage elasticities can both be decomposed into components associated with the labour supply elasticity, $\eta_{h,w}$, and the elasticity of income tax allowances with respect to wages, $\eta_{a,w}$.

To illustrate possible values, consider an individual earning gross income of $y = £17,000$ per year, paying a marginal tax rate of $t = 0.23$, with $a' = 5200$ and $\eta_{a,y} = 0.25$.[4] Substitution into equations (16), (26) and (27) yields the values shown in Table 3, for values of $\alpha = 0.5$ and $\alpha = 0.7$. The resulting labour supply effects on the tax revenue elasticity are small, causing $\eta_{T,y}$ to underestimate $\eta_{T,w}$ by about 3-5 per cent, or 6 percentage points.

The small labour supply effects in column 2 are broadly in line with the empirical estimates available for the UK which generally find small positive elasticities for men.[5] However, potentially larger effects on revenue elasticities are to be

[4] These values approximate those corresponding to average income and the tax structure in the UK in the early 1990s; See Creedy and Gemmell (2002).

[5] See, for example, Blundell and Walker (1982), Blundell, Duncan and Maghir (1998). Blundell and MaCurdy (1999) provide a review of empirical estimates.

Table 2: Tax Structure

Income Tax		Tax/Transfer	
Threshold	MTR	Threshold	EMTR
0	0	0	0.50
80	0.10	130	0.10
180	0.23	180	0.23
600	0.40	600	0.40

expected for individuals closer to relevant tax thresholds and for aggregate revenue elasticities where a sufficiently large proportion of the wage distribution is affected by these thresholds. The following section explores individual effects further using a more detailed model containing a means-tested transfer payment in addition to income taxation.

4 Some Numerical Examples

In practice, tax and transfer systems display considerable complexity and must deal with problems of population heterogeneity. However, to illustrate the wide range of possible labour supply responses and resulting tax revenue elasticities it is sufficient to analyse a simple tax and transfer system.

4.1 The Tax and Transfer System

The simulations reported here combine an income tax involving four marginal tax rates and thresholds, shown in weekly terms in the left-hand section of Table 2; these values, suitably annualised, approximate the UK income tax structure around the year 2000. These numerical examples are nevertheless purely illustrative. Indeed, the benefit structure is assumed to take a highly simplified form, in which non-workers receive a weekly benefit, b, giving the initial virtual income, $m_1 = b$. There is an effective marginal tax rate, s, imposed on relatively low earned incomes, such that net income, z, is given by $z = b + (1 - s)\, y$. Eligible individuals have their net incomes brought up to this level. Hence for those with $y_2 > y > y_1 = 0$, who work but face a marginal income tax rate of zero, the benefit they receive is equal to $b - sy$.

Those with earnings in the range $y > y_2$, and who therefore pay income tax and receive a transfer, have after-tax incomes given by $z = y_2 + (1 - t_2)\,(y - y_2) = t_2 y_2 + (1 - t_2)\, y$. Hence the social transfer received by such individuals is equal to $b - y(s - t_2) - t_2 y_s$. This becomes zero at the income level $y' = (b - t_2 y_2)\,/\,(s - t_2)$. Hence y' becomes a new effective threshold, whereby only

those with $y < y'$ receive any benefits, and those with $y_3 > y > y'$ pay income tax at the marginal rate t_2 and receive no benefits.

Hence if $s = 0.5$ and $b = 60$, the effective tax structure has an earnings threshold of $y' = 130$ per week, above which the marginal tax rate is 0.10 and below which it is 0.5. This ensures that the overall tax structure has no discontinuities. The effective structure is thus given on the right-hand side of Table 2, with an initial virtual income of $m_1 = 60$.[6]

4.2 Labour Supply

The effective tax structure shown on the right hand side of Table 2 has a range where the implied budget set for each individual is non-convex, giving rise to a large discrete 'jump' in labour supply at a particular 'switching' wage. This switching wage is lower than the value that is associated with the earnings threshold of y'; indeed, the non-convexity implies that earnings in a range around y' would not be observed. The labour supply curve, for a given value of α, can be generated numerically using the results presented in subsection 2.2 for a Cobb-Douglas utility function. An efficient algorithm is used to determine the optimal labour supply for each individual.[7]

The potential importance of labour supply effects for revenue elasticities can be seen from the labour supply curve in Figure 3, which applies to an individual with $\alpha = 0.5$. This reveals a number of kinks and segments associated with the various tax and benefit thresholds and the switching wage where a jump is made from the first to the second segment. The individual does not begin working until the wage of $w = 1.5$ is reached. Below this level the full benefit of 60 is obtained. As soon as the individual starts to work, the effective marginal tax rate of 0.5 applies.

For a wage rate above 3.52 the individual begins to pay income tax and simultaneously receives a benefit. Labour supply is therefore relatively elastic over this range. The wage of approximately 3.96 is found to be the switching wage that is associated with a substantial jump in hours worked (from 24.77 to 38.88 hours). This involves a jump to the next segment of the budget constraint, so that the individual no longer receives any benefit and there is a consequent drop in the effective marginal tax rate (from 0.5 to 0.1).

Above the switching wage, labour supply is relatively inelastic, except for backward-bending segments associated with kinks in the budget constraint (where marginal rates fall) at the two highest income tax thresholds of $y = 180$ and 600. At these kink points, $\eta_{h,w} = -1$ so that $\eta_{y,w} = 0$ over the range of wage rates for

[6] The recently introduced Working Families Tax Credit in the UK, where $s = 0.55$, leads to benefits being received by some households paying the standard tax rate.

[7] For a description of the algorithm used, which allows for the possibility of multiple local optima where non-convexities exist, see Creedy and Duncan (2002).

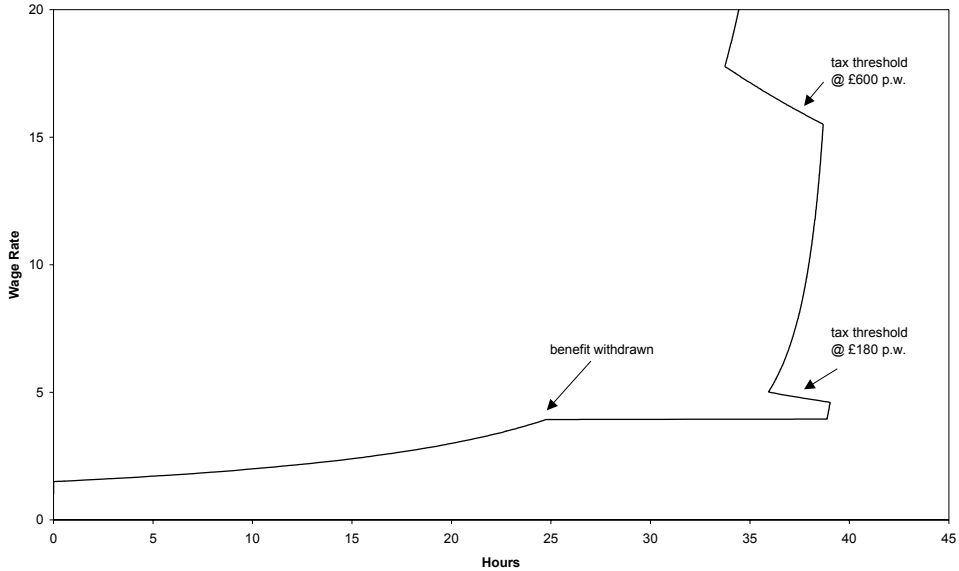

Figure 1: Individual Labour Supply Curve

which the kinks are optimal. Gross earnings remain constant at the relevant income threshold as the wage rate increases, with each of these backward-bending labour supply curve segments forming part of rectangular hyperbolas.

4.3 Revenue Elasticities

Figures 2 and 3 show the associated revenue elasticity profiles.[8] In Figure 2, $\eta_{T,w}$ is compared with the equivalent profile for the case where labour supply is exogenous.[9] Figure 3 compares $\eta_{T,w}$ with $\eta_{T,y}$ for the endogenous labour supply case. Figure 2 reveals that ignoring endogenous labour supply responses leads to $\eta_{T,y}$ underestimating $\eta_{T,w}$ except at corner solutions where $\eta_{T,w} = 0$ in the endogenous case, but the exogenous case leads to positive values. In the exogenous labour supply case, $\eta_{T,y}$ exceeds $\eta_{T,w}$, particularly at low wage levels, because the lower gross earnings associated with responses to the tax-benefit system are ignored.

With such differences between the two profiles, especially at wages below around $£5$ per hour, it is unclear how aggregate elasticities based on a wage

[8]The elasticities were computed by increasing w by a small amount, 0.2%, at each step, and recalculating optimal labour supply and taxation.

[9]The zero labour supply case is obtained by setting the exogenous level of hours worked at the maximum level chosen in the endogenous case (that is, 39 hours per week).

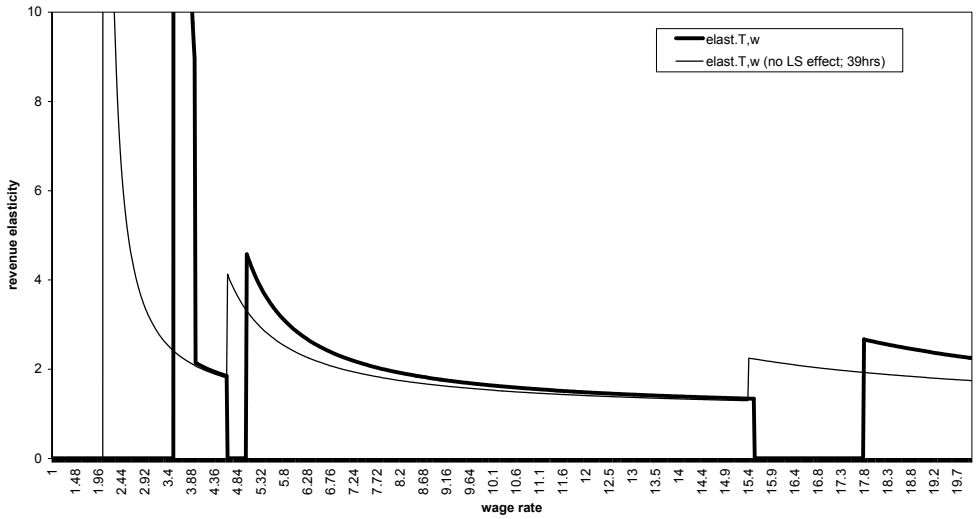

Figure 2: Tax-Wage Elasticities

distribution might be affected. Figure 3 shows that, given endogenous labour supply, $\eta_{T,y}$ can be expected to be less than $\eta_{T,w}$ at all wage rates except at corners where both $\eta_{T,w}$ and $\eta_{T,y}$ are zero. Both elasticities can be very large for individuals immediately above tax thresholds. However, it can be seen that positively valued elasticities are generally in the range of 1 to 3.

Figure 4 shows the ratio of the tax-wage and tax-income elasticities, providing an indication of the extent of underestimation at different wage levels if $\eta_{T,y}$ is used as a proxy for $\eta_{T,w}$. Apart from low wage levels, where $\eta_{T,w}$ can exceed $\eta_{T,y}$ by as much as 70-80 per cent, for most wage rates $\eta_{T,w}$ exceeds $\eta_{T,y}$ by less than 20 per cent, and more typically by less than 10 per cent.

4.4 Benefit Elasticities

Figure 5 provides information about the variation in the benefit elasticity, that is, the responsiveness of transfer expenditures to changes in wage rates. This is negative, reflecting the withdrawal of benefits as wages rates rise, and values obviously depend on the effective marginal rate applied (0.5 in this case). The elasticity changes from around -0.5, when the benefit taper begins to apply, to around -3.5 when income tax becomes payable. This temporarily increases the benefit elasticity which continues to fall thereafter until benefits are completely withdrawn (at the point where the individual jumps to the next segment of the

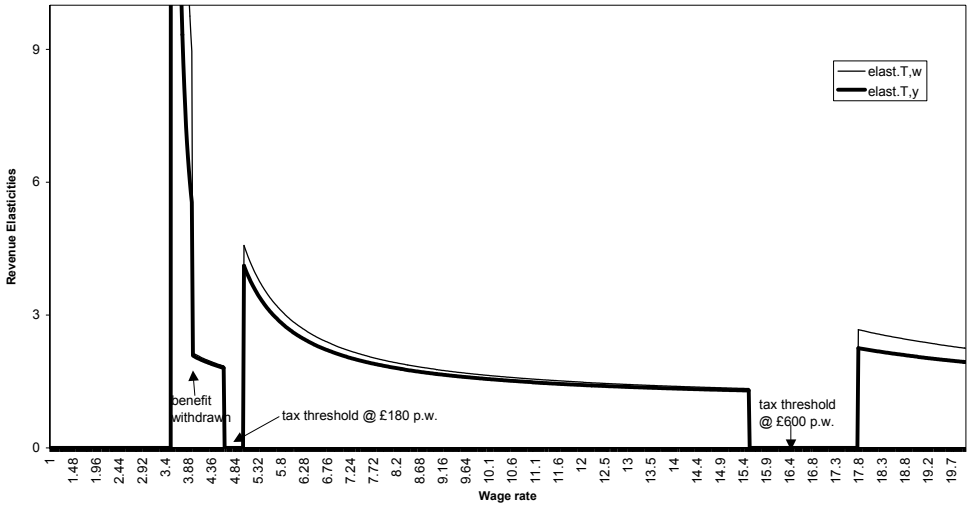

Figure 3: Tax-Income and Tax-Wage Elasticities

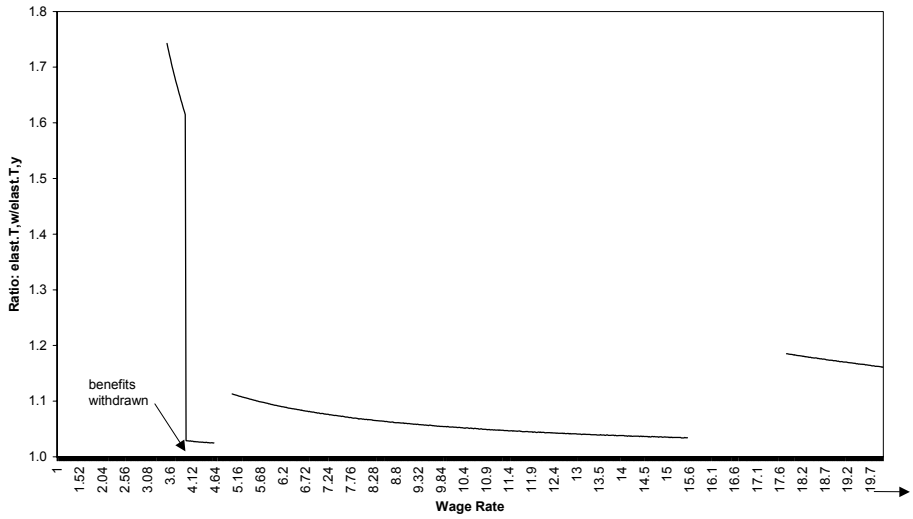

Figure 4: Elasticity Ratio

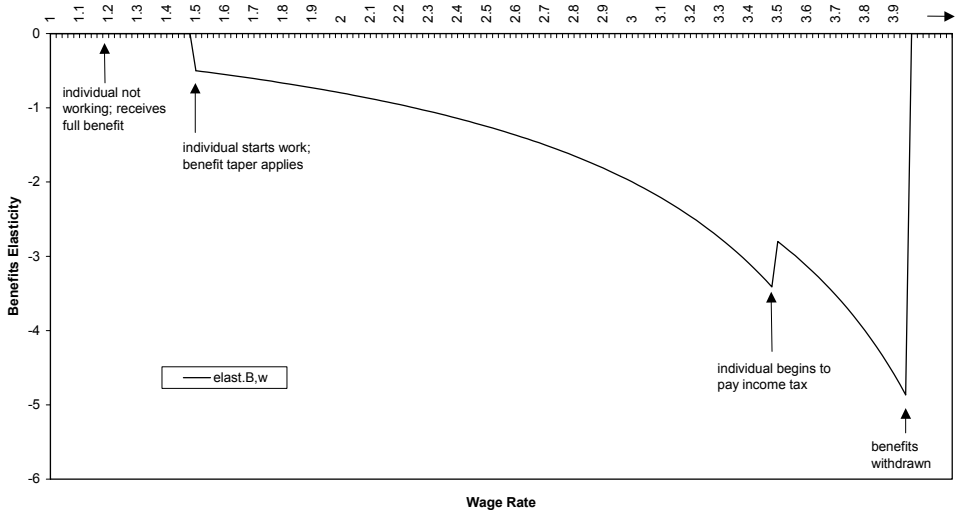

Figure 5: Individual Benefit Elasticity

budget constraint). These values suggest that, even with a fairly modest taper rate of 0.5, the benefit elasticity can be large in absolute value, causing transfer expenditures to decline relatively quickly as wage rates rise.

5 Aggregate Elasticities

This section examines aggregate revenue elasticities. The first subsection presents the basic concepts and the second subsection reports simulation results.

5.1 Basic Relationships

For a continuous wage rate distribution, where the distribution function is $F(w)$, total tax revenue, T_Y, can be expressed as:

$$T_Y = \int T(y(w))\, dF(w) \tag{28}$$

The elasticity of aggregate revenue, with respect to a change in the arithmetic mean wage, W, can be written as:

$$\eta_{T_Y,W} = \int \eta_{T,w}\eta_{w,W}\frac{T(y(w))}{T_Y}dF(w) \tag{29}$$

where $\eta_{w,W}$ is the elasticity of w with respect to the average wage. In the equiproportional case where all wage rates change in equal proportions, $\eta_{w,W} = 1$ and the aggregate elasticity is the tax-share weighted average of individual elasticities:

$$\eta_{T_Y,W} = \int \eta_{T,y}\eta_{y,w}\frac{T\left(y\left(w\right)\right)}{T_Y}dF\left(w\right) \tag{30}$$

The further analysis of equation (30) must allow for the number of different linear segments in the tax function, each having a different m_k and t_k, and the wage thresholds over which each combination is applicable. As mentioned above, for those who are in ranges of w that place them at corners, small changes in w have no effect on y, so $\eta_{y,w} = 0$ over that range. In general it would be extremely complex to derive the precise thresholds since, for example, the switching wage cannot be obtained explicitly except for very simple utility functions.[10] For this reason, simulation methods are used in this section where revenue elasticities are based on a random sample of individuals drawn from a specified wage rate distribution.

5.2 Simulation Results

Simulation of the aggregate tax revenue elasticity requires a form for the wage distribution. The following examples are based on the use of a lognormal distribution, $\Lambda(\mu, \sigma^2)$, where μ and σ^2 are respectively the mean and variance of the logarithms of hourly wage rates. Values of $\mu = 2.0$ and $\sigma^2 = 0.5$ were chosen, giving a mean hourly wage rate of $\exp(\mu + \frac{1}{2}\sigma^2) = 9.49$. Results are reported for a random sample of 5000 individuals drawn from this distribution.[11] For convenience, simulations assume a common value of the preference parameter, α, across all individuals, though in practice this may vary if, for example, high wages are associated with a lower leisure preference.[12]

Table 2 shows values of both $\eta_{T_Y,W}$ and $\eta_{T_Y,Y}$ for alternative values of α, together with the ratio, $\eta_{T_Y,W}/\eta_{T_Y,Y}$. Column 5 provides an alternative measure of the degree of underestimation arising from the use of $\eta_{T_Y,W}$ as a proxy for $\eta_{T_Y,Y}$. This measures the ratio of the proportionate change in tax revenues predicted by the two tax elasticities, and is given by $\left(\eta_{T,W} - 1\right) / \left(\eta_{T,Y} - 1\right)$. For example, with $\alpha = 0.5$, column 4 indicates that the aggregate tax-wage elasticity is approximately 9 per cent greater than the tax-income elasticity. However,

[10] For further analysis of switching wage rates, see Creedy (1996).

[11] Simulations excluded the small number of individuals for whom the small wage increase (of 0.2%) led to a large discrete jump in labour supply from the first to the second segment of the budget constraint, or a move from a corner to a tangency solutions. Elasticities for these individuals are disproportionately large.

[12] Creedy (2001) discusses the appropriate simulation method for the case where w and α are jointly lognormally distributed.

Table 3: Aggregate Elasticities

α	$\eta_{T_Y,W}$	$\eta_{T_Y,Y}$	$\frac{\eta_{T_Y,W}}{\eta_{T_Y,Y}}$	$\frac{\eta_{T_Y,W}-1}{\eta_{T_Y,Y}-1}$
0.1	2.095	1.844	1.14	1.31
0.3	1.776	1.597	1.11	1.30
0.5	1.636	1.502	1.09	1.27
0.7	1.544	1.463	1.06	1.18
0.9	1.468	1.442	1.02	1.06
1.0	1.426	1.426	1.00	1.00

column 5 shows that this implies that tax revenue increases using $\eta_{T_Y,W}$, for a given increase in wage rates, are around 27 per cent greater than would be obtained using $\eta_{T_Y,Y}$. For $\alpha = 1$, there are no labour supply effects and both elasticities are identical.

Two features stand out in Table 2. Firstly, allowing for endogenous labour supply has a noticeable effect on the magnitudes of both elasticity measures: $\eta_{T_Y,W}$ rises from 1.4 to 2.1 as α is reduced from 1.0 to 0.1, with a slightly smaller increase in $\eta_{T_Y,Y}$. Secondly, the extent to which $\eta_{T_Y,W}$ exceeds the more commonly calculated $\eta_{T_Y,Y}$ also depends on the nature of leisure preferences. With a strong preference for consumption (high α) the increase in tax revenues is only about 6 per cent higher using $\eta_{T_Y,W}$ compared with the use of $\eta_{T_Y,Y}$, whereas with a stronger leisure preference (low α), this figure is around 30 per cent. This suggests that the accurate measurement of $\eta_{T_Y,W}$ in practice is likely to require an allowance for population heterogeneity.

6 Conclusions

This paper has suggested that the usual practice of estimating the revenue responsiveness of income tax using the elasticity of tax revenue with respect to income (earnings) may be misleading as a guide to how tax revenue responds to changes in wage rates. Until now, analytical expressions for these elasticities have typically treated earnings as exogenous, so that they do not accommodate the endogenous response of labour supply to the income tax system. This paper has shown how revenue elasticity expression can be adapted to allow for endogenous labour supply. The primary objective was to identify how far, and in what circumstances, labour supply effects are quantitatively important for revenue responsiveness estimates, both for individual taxpayers and in aggregate.

Analytical elasticity expressions in section 3 showed the importance of the income tax structure (tax rates and thresholds), the elasticity of income tax allowances with respect to wages, $\eta_{a,w}$, and the labour supply elasticity, $\eta_{h,w}$. The

last of these is, in turn, a function of the tax structure and leisure preferences, in addition to the wage structure. In order to quantify these various determinants, a numerical exercise was used based on a stylised version of the UK income tax and transfer system. For individuals, this showed that even a relatively simple tax-benefit structure can produce labour supply responses which considerably alter tax revenue elasticity calculations. Especially for individuals on low wages where income taxes and transfers interact, and for those close to income tax thresholds, tax-income elasticities can severely misrepresent tax-wage responses. At most wage levels, however, and with modest leisure preferences ($\alpha = 0.5$), tax-wage elasticities were between 3 per cent and 17 per cent higher than tax-income elasticities, when allowing for endogenous labour supply.

Aggregate tax revenue elasticities were also computed for a simulated sample of individuals, using a lognormal distribution of wage rates. The results suggested that, in aggregate, tax-income elasticities may provide a reasonable approximation of tax-wage responses but only in the presence of a strong preference for consumption over leisure. With α around 0.5 or lower, aggregate income tax revenue growth associated with a given increase in wage rates can be as much as 30 per cent greater than the response of tax revenues to earned incomes. Recent estimates of the labour supply responses of men on average in the UK are generally small and positive, which might suggest that an exogeneity assumption is a reasonable approximation when estimating tax elasticities. However, it is also known that labour supply responses for particular groups of individuals (such as women, pension recipients, and low-wage men) can differ widely.[13] This suggests that empirical estimation of tax revenue elasticities may have to be aware of the differential impact of wage growth for different types of individual. Unlike the computation of standard income tax revenue elasticities, which requires only information about the tax structure and the income distribution, a full evaluation of the implications for revenue forecasting of endogenous labour supply responses to wage rate changes requires considerable information.

[13]See for example, Disney and Smith (2002) who find more substantial hours responses by pensionable men in the UK; and Bingley and Lanot (2002) who find a labour supply elasticity around 0.14 for a sample of Danish private sector workers.

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